CHAPTER 6

PROPOSAL OF SCATTERING MATRIX FOR TIR SWITCH
AND SWITCH FABRIC

6.1 INTRODUCTION

In the previous chapters, to study the capabilities of the QW/QD based TIR electro-optic switch and Spanke switch fabric, a study of QW structures, synthesis of QD, circuit model analysis and performance analysis were carried out. The results of these investigations reveal remarkable and useful information about the parameters of the switch and switch fabric, namely, number of switch elements required, number of crossovers, maximum loss and minimum loss, bandwidth and switching time. However, these investigations do not give details about the behavior of the switch and switch fabric, with variations in the incident power, impedance of the ports, applied potential to the electrodes, the incident angle etc. To explore the consequence of these effects, the device under study should be formulated using a method, which can reveal all these behaviors. The scattering matrix method is used efficiently to explore such sort of behavioral studies in the devices operating at
microwave frequency range (Pozar 1998, and Overfelt and White 1989). Due to the non-availability of such analysis for optical devices, an attempt is made to extend the theory of scattering matrix to characterize the TIR electro-optic switch (Hunsperger 1991) and the Spanke switch fabric (Ramasamy and Sivarajan 2000). The mathematical relationships between the reflected power and the incident power using the scattering matrix is also necessary for a complete understanding of the behavior of the switch and switch fabric.

Presented here is the scattering matrix derivation for both TIR electro-optic switch and Spanke switch fabric, which is followed by establishment of their mathematical relationships. Before concluding this, the numerical simulation results are discussed.

6.2 S-MATRIX FORMULATION FOR TIR SWITCH

The TIR electro-optic switch shown in Figure 6.1 is considered for the S-matrix formulation. It is a four-port device. The incident field and reflected field associated with each port ‘i’ are named as a_i and b_i, respectively. Based on these scattering filed, the scattering matrix is formed. The various steps involved in this formulation are presented as a flow diagram (Figure 6.2).
Figure 6.1 Schematic diagram of the TIR electro-optic switch

Figure 6.2 Flow diagram for TIR switch analysis
The scattering matrix of any four-port network is given by the equation (Pozar 1998),

\[
(S) = \begin{pmatrix}
S_{11} & S_{12} & S_{13} & S_{14} \\
S_{21} & S_{22} & S_{23} & S_{24} \\
S_{31} & S_{32} & S_{33} & S_{34} \\
S_{41} & S_{42} & S_{43} & S_{44}
\end{pmatrix}
\]  

(6.1)

In the matrix element \( S_{ij} \), ‘i’ refers the reflected port (output) number and ‘j’ refers the incident (input) port number.

The equation (6.1) is reduced to equation (6.2) by applying the switch principle discussed in the Sections 1.4 and 4.5.

\[
(S) = \begin{pmatrix}
0 & 0 & S_{11} & S_{14} \\
0 & 0 & S_{23} & S_{24} \\
S_{31} & S_{32} & 0 & 0 \\
S_{41} & S_{42} & 0 & 0
\end{pmatrix}
\]  

(6.2)

Thus from this equation, it is inferred that the light which is incident at port 1, is reflected to ports 3 and 4 and not to ports 1 and 2 (i.e. to the ports placed opposite to the incident port). The same kind of incidence and reflection are observed at all other ports. The zero and unitary properties of the matrix are applied to equation (6.2) that becomes

\[
S_{13} \times S_{13}^* = S_{24} \times S_{24}^*
\]  

(6.3)
which indicates that the magnitude and phase of the signal reflected to port 3 due to the light incidence at port 1, is the same as the magnitude and phase of the signal reflected to port 4 due to the light incidence at port 2.

**Signal Flow Graph of TIR switch**

This is a pictorial representation of equation (6.2). To compute the Transfer Function of a port of TIR switch, the Signal Flow Graph (SFG) is derived from its S matrix. It is solved using Mason’s gain formula (Nagrath and Gopal 1992) considering one port at a time. The scatterings to all other ports are also taken into account.

\[
T = \frac{1}{\Delta} \sum_{i=1}^{N} P_i \Delta_i
\]  \hspace{1cm} (6.4)

where \( P_k \) is the forward path gain of the \( k^{th} \) forward path.

\[
\Delta = 1 - \text{(sum of individual loop gain)} + \text{(sum of gain products of all possible combinations of 2 non-touching loops)} - \text{(sum of gain products of all possible combinations of 3 non-touching loops)} + \ldots
\]

\[
\Delta_k = \text{the } \Delta \text{ for the part of graph which is non touching with the } k^{th}
\]

forward path
Thus, the reduction of the signal flow graph for the switch results in four transfer functions. The TFs of the electro-optic TIR switch, obtained by the signal flow graph (Figure 6.3) reduction is given as

\[
\begin{align*}
T_1 &= S_{31} + S_{41} \\
T_2 &= S_{32} + S_{42} \\
T_3 &= S_{13} + S_{23} \\
T_4 &= S_{14} + S_{24} 
\end{align*}
\]

(6.5)
The above transfer functions correspond to the light incidence in the four different ports of the switch. These functions help in determining the characteristics of the switch. The transfer function for port 1 contains the scattering components of the signals from port 3 and port 4. Similarly, the TFs for other ports have the scattered components as given in equation (6.5).

6.3 S-MATRIX FORMULATION FOR SWITCH FABRIC

This section deals with the S-matrix formulation for the Spanke switch fabric. The two main aspects to be considered for the S-matrix formulation of the switch fabric are the type of architecture in which the switch is placed and the type of cascading process used in the switch elements.

![Figure 6.4 Schematic diagram of switch fabric](image)

Here, the Spanke switch fabric (4x4) discussed in Chapter 4 is considered. In this architecture (Figure 4.20 of Chapter 4), the signal at input port has to travel through one switch element at each stage (resulting in a maximum of 4 switch elements traversal) to reach any output port. In order to
simplify the cascading process of the Spanke switch fabric, it is assumed that at any moment only one input port is active. Thus, the resultant switch fabric used in this study consists of only four cascaded switches as shown in Figure 6.4.

The procedure involved in the analysis of the switch fabric is presented in the form of a flow diagram, which is shown in Figure 6.5. The analysis process of the switch fabric and the switch (Figure 6.2), differ only at the initial stages. The switch fabric is studied for its operation. The S-matrix of each individual block is formed. Then, the analysis is continued using either of the following two methods: star product or signal flow graph. The error mentioned in the flow diagram is checked manually with the expertise of the designer.

In the star product method, the cascading of individual switches is performed, to formulate the S-matrix of the entire fabric. Whereas in the signal flow graph method, the SFG of individual switches is formed, and they are port coupled to get the SFG of the entire fabric (Pozar 1998). Otherwise, the SFG of the switch matrix may be formed directly. This SFG is solved using Mason’s gain formula from which the TF of the fabric is obtained. In the present work, the star product method (Bandler and Seviora 1970, and Chu and Itoh 1986) is implemented. From the fabric shown in Figure 6.4, two switches are taken at the first stage. The resultant switch fabric is shown in Figure 6.6 for the initial consideration.
Derive the scattering matrix

No

Error?

SGF 1?

Yes

Formulate individual SFG

Cascading of network

Obtain transfer function

No

Cascading of network

Formulate Signal flow graph

Obtain transfer function

SFG 2?

No

Yes

Formulate Signal flow graph of Fabric

Obtain transfer function

Error?

Plot the response

Figure 6.5 Flow diagram of switch fabric analysis
Figure 6.6 Schematic structure of cascading two 4-port switches (First stage)

The S-matrix of the above switches and waveguides are formulated as

\[
\begin{pmatrix}
\mathbf{b}_1 \\
\mathbf{b}_2 \\
\mathbf{b}_3 \\
\mathbf{b}_4
\end{pmatrix} =
\begin{pmatrix}
S_{11} & S_{12} & S_{13} & S_{14} \\
S_{21} & S_{22} & S_{23} & S_{24} \\
S_{31} & S_{32} & S_{33} & S_{34} \\
S_{41} & S_{42} & S_{43} & S_{44}
\end{pmatrix}
\begin{pmatrix}
\mathbf{a}_1 \\
\mathbf{a}_2 \\
\mathbf{a}_3 \\
\mathbf{a}_4
\end{pmatrix}
\quad \text{(for } S_1) \quad (6.6)
\]

\[
\begin{pmatrix}
\mathbf{b}_5 \\
\mathbf{b}_6 \\
\mathbf{b}_7 \\
\mathbf{b}_8
\end{pmatrix} =
\begin{pmatrix}
S_{55} & S_{56} & S_{57} & S_{58} \\
S_{65} & S_{66} & S_{67} & S_{68} \\
S_{75} & S_{76} & S_{77} & S_{78} \\
S_{85} & S_{86} & S_{87} & S_{88}
\end{pmatrix}
\begin{pmatrix}
\mathbf{a}_5 \\
\mathbf{a}_6 \\
\mathbf{a}_7 \\
\mathbf{a}_8
\end{pmatrix}
\quad \text{(for } S_2) \quad \text{and} \quad (6.7)
\]

\[
\begin{pmatrix}
\mathbf{a}_9 \\
\mathbf{a}_a \\
\mathbf{a}_b \\
\mathbf{a}_c
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 & S(l) \\
0 & 0 & S(l) & 0 \\
0 & S(l) & 0 & 0 \\
S(l) & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\mathbf{b}_1 \\
\mathbf{b}_2 \\
\mathbf{b}_3 \\
\mathbf{b}_4
\end{pmatrix}
\quad \text{(for waveguide)} \quad (6.8)
\]

These matrices are represented in the form of equations. The waveguide parameters are substituted in the equations (6.6) and (6.7), and the resultant
The S-matrix of the switch fabric at the first stage is obtained as follows.

\[
\begin{pmatrix}
  b_1 & b_1 & b_1 & b_1 \\
  a_1 & a_2 & a_3 & a_4 \\
  b_2 & b_2 & b_2 & b_2 \\
  a_1 & a_2 & a_3 & a_4 \\
  b_3 & b_3 & b_3 & b_3 \\
  a_1 & a_2 & a_3 & a_4 \\
  b_4 & b_4 & b_4 & b_4 \\
  a_1 & a_2 & a_3 & a_4
\end{pmatrix}
\]

The individual elements of the matrix are defined in the following equations.

\[
\begin{align*}
  b_{1i} &= S_{i1} + S_{i1}S(l)U_{i}S_{a}S(l)S_{a1} + S_{i1}S(l)U_{i}S_{a}S(l)S_{a}S_{a1} + S_{i1}S(l)U_{i}S_{a}S(l)S_{a}S_{a}S_{a1} + S_{i1}S(l)U_{i}S_{a}S(l)S_{a}S_{a}S_{a}S_{a1} \\ 
  a_{1} & \\
  b_{1i} &= S_{i2} + S_{i2}S(l)U_{i}S_{a}S(l)S_{a2} + S_{i2}S(l)U_{i}S_{a}S(l)S_{a}S_{a2} + S_{i2}S(l)U_{i}S_{a}S(l)S_{a}S_{a}S_{a2} + S_{i2}S(l)U_{i}S_{a}S(l)S_{a}S_{a}S_{a}S_{a2} \\ 
  a_{2} & \\
  b_{1i} &= S(l)S_{i1} + S(l)S_{a1}, \\ 
  a_{3} & \\
  b_{1i} &= S_{i4}S(l)U_{i}S_{a}S(l) + S_{i4}S(l)U_{i}, \\ 
  a_{4} &
\end{align*}
\]

Similarly, the rest of the matrix elements are also obtained. The resultant network for the second stage of the switch fabric after cascading the third switch to the first stage network is shown in Figure 6.7.
Now, the S-matrix of the switch element \((S_3)\) to be cascaded with the first stage is given by

\[
\begin{pmatrix}
    b_s \\ b_b \\ b_c \\ b_d \\
\end{pmatrix} = \begin{pmatrix}
    S_a & S_{ab} & S_{ac} & S_{ad} \\
    S_{ba} & S_{bb} & S_{bc} & S_{bd} \\
    S_{ca} & S_{cb} & S_{cc} & S_{cd} \\
    S_{da} & S_{db} & S_{dc} & S_{dd} \\
\end{pmatrix} \begin{pmatrix}
    a_s \\ a_b \\ a_c \\ a_d \\
\end{pmatrix}
\]

(6.11)

By substituting the equation for waveguide (6.8) in switch element equations (6.9) and (6.11), and simplifying them, the S-matrix of the second stage network is obtained which can be given as

\[
\begin{pmatrix}
    b_s \\ b_b \\ b_c \\ b_d \\
\end{pmatrix} = \begin{pmatrix}
    b_s & b_b & b_c & b_d \\
    a_s & a_b & a_c & a_d \\
    b_s & b_b & b_c & b_d \\
    a_s & a_b & a_c & a_d \\
\end{pmatrix} \begin{pmatrix}
    a_s \\ a_b \\ a_c \\ a_d \\
\end{pmatrix}
\]

(6.12)
where the various elements of the matrix are

\[
\begin{align*}
\frac{b_2}{a_1} &= ([7] + S(0)[9] U_r \{S_\omega S(0)[19] + S_\omega S(0)[13]\} + S(0)[10] U_r \{S_\omega S(0)[19] + S_\omega S(0)[13]\}) \\
\frac{b_2}{a_2} &= ([8] + S(0) U_r [9] \{S_\omega S(0)[20] + S_\omega S(0)[14]\} + S(0) U_r [10] \{S_\omega S(0)[20] + S_\omega S(0)[14]\}) \\
\frac{b_2}{a_3} &= (S(0)[9] U_r S_\omega + S(0) U_r [10] S_\omega) \\
\frac{b_2}{a_4} &= (S(0) U_r [9] S_\omega + S(0) U_r [10] S_\omega)
\end{align*}
\]  

(6.13)

The numerals used in the equations (given in square brackets) are for the simplification of the mathematical analysis. Their values are defined in the equation (6.14). These are the parameters in the first stage analysis and become a part of the second stage equations.

\[
\begin{align*}
[7] &= \frac{b_2}{a_1} & [13] &= \frac{b_1}{a_1} \\
[8] &= \frac{b_2}{a_2} & [14] &= \frac{b_2}{a_2} \\
[9] &= \frac{b_2}{a_3} & [19] &= \frac{b_3}{a_3} \\
[10] &= \frac{b_2}{a_4} & [20] &= \frac{b_4}{a_4}
\end{align*}
\]  

(6.14)

In the final stage, the analysis of the switch fabric is done by adding the fourth switch element \((S_\omega)\) to the second stage network that is shown in Figure 6.8 to find the scattering matrix.
Figure 6.8 Cascading of fourth switch of switch fabric to the second stage network (Third stage)

The S-matrix of the fourth switch element ($S_4$) found by the switch operation is given as

$$
\begin{pmatrix}
    b_s \\
    b_i \\
    b_s \\
    b_h
\end{pmatrix} =
\begin{pmatrix}
    S_{sr} & S_{ri} & S_{sr} & S_{rh} \\
    S_{is} & S_{ii} & S_{is} & S_{ih} \\
    S_{ns} & S_{ni} & S_{ns} & S_{nh} \\
    S_{ns} & S_{ni} & S_{ns} & S_{nh}
\end{pmatrix}
\begin{pmatrix}
    a_i \\
    a_i \\
    a_i \\
    a_h
\end{pmatrix}
$$

(6.15)

Similar to the previous cascading process, the scattering matrix computation for the Spanke switch fabric is carried out and the result is shown in equation (6.16).

$$
\begin{pmatrix}
    b_i \\
    b_s \\
    b_s \\
    b_h
\end{pmatrix} =
\begin{pmatrix}
    b_i & b_i & b_i & b_h \\
    a_i & a_i & a_i & a_h \\
    b_i & b_i & b_i & b_i \\
    a_i & a_i & a_i & a_h
\end{pmatrix}
\begin{pmatrix}
    a_i \\
    a_i \\
    a_i \\
    a_h
\end{pmatrix}
$$

(6.16)
If the switch element of the Spanke switch fabric is TIR switch then the equation (6.16) is reduced to equation (6.17). This equation is represented in the signal flow graph, which is similar to the SFG of the single switch element, except the change in the magnitude of the S-parameter value.

\[
\begin{pmatrix}
  b_1 \\
  b_2 \\
  b_g \\
  b_h \\
\end{pmatrix} =
\begin{pmatrix}
  0 & 0 & \frac{b_1}{a_s} & \frac{b_1}{a_h} \\
  0 & 0 & \frac{b_2}{a_s} & \frac{b_2}{a_h} \\
  \frac{b_g}{a_s} & \frac{b_g}{a_h} & 0 & 0 \\
  \frac{b_h}{a_s} & \frac{b_h}{a_h} & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2 \\
  a_g \\
  a_h \\
\end{pmatrix}
\] (6.17)

The various elements of the matrix are

\[
\frac{b_1}{a_s} = S(l)[30] U_{12} s_{f_g} + S(l)[31] U_{11} s_{e_g}
\] (6.18)

\[
\frac{b_1}{a_h} = S(l)[30] U_{12} s_{f_h} + S(l)[31] U_{11} s_{e_h}
\]

\[
\frac{b_2}{a_s} = S(l)[38] U_{12} s_{f_g} + S(l)[39] U_{11} s_{e_g}
\] (6.19)

\[
\frac{b_2}{a_h} = S(l)[38] U_{12} s_{f_h} + S(l)[39] U_{11} s_{e_h}
\]

\[
\frac{b_g}{a_s} = S(l)[49] U_{10} s_{f_g} + S(l)[41] U_{i4} s_{g_f}
\]

\[
\frac{b_g}{a_h} = S(l)[50] U_{10} s_{f_h} + S(l)[42] U_{i4} s_{g_l}
\]

\[
\frac{b_h}{a_s} = S(l)[49] U_{10} s_{f_g} + S(l)[41] U_{i4} s_{g_f}
\]

\[
\frac{b_h}{a_h} = S(l)[50] U_{10} s_{f_h} + S(l)[42] U_{i4} s_{g_l}
\]

and

\[
\frac{b_h}{a_s} = S(l)[49] U_{10} s_{f_g} + S(l)[41] U_{i4} s_{g_f}
\]

\[
\frac{b_h}{a_h} = S(l)[50] U_{10} s_{f_h} + S(l)[42] U_{i4} s_{g_l}
\]
The numerals used in the equations are for mathematical simplification and their values are given in terms of scattering parameters in the following equations.

\[
\begin{align*}
\frac{b_h}{a_h} &= S(l)[49]U_{10}S_{he} + S(l)[41]U_{ys}S_{hl} \\
\frac{b_h}{a_l} &= S(l)[50]U_{10}S_{he} + S(l)[42]U_{ys}S_{hl}
\end{align*}
\]

There are some additional numerals used in this equation, which are illustrated as follows:

\[
\begin{align*}
[30] &= S(l)[3]U_sS_{sc} + S(l)[4]U_sS_{sc} \\
[31] &= S(l)[3]U_sS_{sd} + S(l)[4]U_sS_{sd} \\
[38] &= S(l)[9]U_sS_{sc} + S(l)[10]U_sS_{sc} \\
[39] &= S(l)[9]U_sS_{sd} + S(l)[10]U_sS_{sd} \\
[41] &= S(l)[19]U_sS_{sc} + S(l)[13]U_sS_{sc} \\
[42] &= S(l)[20]U_sS_{sc} + S(l)[14]U_sS_{sc} \\
[49] &= S(l)[19]U_sS_{sd} + S(l)[13]U_sS_{sd} \\
[50] &= S(l)[20]U_sS_{sd} + S(l)[14]U_sS_{sd}
\end{align*}
\]

There are some additional numerals used in this equation, which are illustrated as follows:
\[ [13] = S_{25}S(l)U_2S_{41} + S_{76}S(l)U_1S_{31} \]
\[ [14] = S_{75}S(l)U_2S_{42} + S_{76}S(l)U_1S_{32} \]
\[ [19] = S_{85}S(l)U_2S_{41} + S_{86}S(l)U_1S_{31} \]
\[ [20] = S_{85}S(l)U_2S_{42} + S_{86}S(l)U_1S_{32} \] (6.22f)

\[ U \] represents the inverse sub-matrices present in the scattering matrix and their values in terms of the S-parameter are given in the following equation.

\[
\begin{align*}
U_1 &= \{I - S_{15}S(l)S_{65}S(l) - S_{16}S(l)S_{66}S(l)\}\^{-1} \\
U_2 &= \{I - S_{25}S(l)S_{65}S(l) - S_{26}S(l)S_{66}S(l)\}\^{-1} \\
U_3 &= \{I - S_{35}S(l)S_{45}S(l) - S_{36}S(l)S_{46}S(l)\}\^{-1} \\
U_4 &= \{I - S_{45}S(l)S_{65}S(l) - S_{46}S(l)S_{66}S(l)\}\^{-1} \\
U_5 &= \{I - S_{55}S(l)[15]S(l) - S_{56}S(l)[16]S(l)\}\^{-1} \quad (6.23a) \\
U_6 &= \{I - S_{65}S(l)[21]S(l) - S_{66}S(l)[22]S(l)\}\^{-1} \\
U_7 &= \{I - S_{75}S(l)[22]S(l) - S_{76}S(l)[16]S(l)\}\^{-1} \\
U_8 &= \{I - S_{85}S(l)[21]S(l) - S_{86}S(l)[15]S(l)\}\^{-1} \\
U_9 &= \{I - S_{95}S(l)[44]S(l) - S(l)[45]S(l)S_{96}S(l)\}\^{-1} \\
U_{10} &= \{I - S(l)[52]S(l)S_{fe} - S(l)[53]S(l)S_{ef}\}\^{-1} \\
U_{11} &= \{I - S(l)[53]S(l)S_{fe} - S(l)[45]S(l)S_{ef}\}\^{-1} \quad (6.23b) \\
U_{12} &= \{I - S(l)[52]S(l)S_{fe} - S(l)[44]S(l)S_{ef}\}\^{-1}
\end{align*}
\]

Using Mason's gain formula, the transfer functions of the switch fabric are obtained. The resultant transfer functions (Nakkeeran 2003c) are shown in the following equation.
where $T_1$, $T_2$, $T_3$, and $T_4$ are the transfer functions corresponding to the light incidence in ports 1, 2, 3 and 4 respectively. These transfer functions are used to estimate the response of the switch fabric. The right hand side of this equation is already defined in equations (6.18) to (6.21).

6.4 MATHEMATICAL RELATIONSHIPS OF SWITCH AND SWITCH FABRIC

This section deals with the mathematical relationship between the incident power and the reflected power of the TIR switch, which is necessary for a complete understanding of the switch and the switch fabric. Though many mathematical relationships (Hunsperger 1991) about the parameters of TIR electro-optic switch and methods to study its performance are available, the operation principle of this switch can also be analyzed in terms of scattered parameters and power relations (Sisodia 1987). Then, with the help of relationship (Kurokawa 1965) between the scattered fields ($a_i$ & $b_i$) and the scattered parameters ($S_{ij}$), it is possible to bring out explicit relations for
reflected power in terms of incident power, incident angle, applied potential to the electrodes, port impedance, port voltage, port current and operating wavelength (Nakkeeran 2004). With these newly arrived relations, the switch can be easily analyzed in different perspective. In addition, this model can be used for the calculation of switching time.

6.4.1 Relationship between incident power and reflected power

The analysis in Sections 6.2 and 6.3 result in the formulation of the transfer function for the behavior of the switch and the switch fabric. As the transfer function of the TIR switch is defined by set of equations for four ports, the transfer function of port 1 of the TIR switch is considered in this portion of work.

\[ T_1 = S_{31} + S_{41} \]  \hspace{1cm} (6.25)

Where,

\[ S_{31} = \sqrt{\frac{P_3}{P_1}} \] \hspace{1cm} and \hspace{1cm} \[ S_{41} = \sqrt{\frac{P_4}{P_1}} \]  \hspace{1cm} (Sisodia 1987) \hspace{1cm} (6.26)

\( P_1 \) is incident power at port 1.

\( P_3 \) and \( P_4 \) are reflected power to port 3 and port 4 respectively.

The scattering parameter \((S_{ij})\) can be related (Kurokawa 1965 and Pozar 1998) to the scattering fields \((b_i \) and \(a_j)\) as follows

\[ S_{ij} = \frac{b_i}{a_j} \]
Hence $S_{31} = \frac{b_3}{a_1}$ and $S_{41} = \frac{b_4}{a_1}$

(6.27)

Where,

$$b_4 = \frac{V_4 - I_4 Z_4}{2 \sqrt{\text{Re}(Z_4)}} e^{i \theta_4}$$

and

$$a_1 = \frac{V_1 + I_1 Z_1}{2 \sqrt{\text{Re}(Z_1)}} e^{i \theta_1}$$

$V_1$, $I_1$, $Z_1$ and $\theta_1$ are the incident voltage, incident current, port impedance and incident angle at port 1, $V_3$, $I_3$, $Z_3$ and $\theta_3$ are the reflected voltage, reflected current, port impedance and reflected angle at port 3 and $V_4$, $I_4$, $Z_4$ and $\theta_4$ are the reflected voltage, reflected current, port impedance and reflected angle at port 4.

Consider the following assumptions of the port parameter value.

i. The reflected voltage, $V_r = V_4 = \frac{V_1}{2}$

ii. The reflected current, $I_r = I_4 = \frac{I_1}{2}$ and

iii. The port impedance, $Z_1 = Z_3 = Z_4$

The formulation of this relationship is attempted in order to get relative information of the reflected signal with respect to the variation in
incident signal. Indeed, it tries to explore the overall envelope shape of the variation but not an absolute value of this parameter. When \( V_3 = V_4 = V_1/2 \), this assumption only affects the magnitude (very small amount) of the response not the ultimate shape. Hence, this assumption is taken such that the derivation of this equation is made simple. This justification is also applicable for the parameters of the switch like port current and impedance. A linear increase in the reflected power is observed when it is plotted against the ratio of port currents (Figure 6.19). On the other hand, a linear decrease in the reflected power is observed when it is plotted against the ratio of port voltages (Figure 6.19). This also confirms the assumption. In Figure 6.20, the amplitude response of TIR switch (for \( V_3 = V_1/2 \), \( V_3 = V_1/4 \) and \( V_3 = V_1 \)) is plotted, which shows a slight change only in the peak value of the response but not in the shape.

Before proceeding to the relationship between the incident power (\( P_1 \)) and reflected power (\( P_3 \) or \( P_4 \)) of the TIR optical switch, let \( V_{\text{TIR}} \) be defined as the required voltage to get power along the x-axis (Figure 6.9) for the given angle of incidence. For this applied voltage to the electrode, the light signal will undergo neither reflection nor refraction. By increasing the applied voltage beyond this \( V_{\text{TIR}} \) leads to total internal reflection. Usually, the critical angle (incident angle) and the applied voltage are inversely proportional.

\[
V_{\text{wR}} = \frac{2d(1-\text{Sin}\theta)}{n_1^2\text{r}_1} \quad (\text{Tsai 1978}) \tag{6.29}
\]
where,

\begin{align*}
  d & \quad \text{distance between the electrodes} \\
  \theta_1 & \quad \text{incident angle} \\
  n_1 & \quad \text{reflective index of the medium} \\
  r_{33} & \quad \text{electro-optic coefficient of the medium}
\end{align*}

Figure 6.9 Schematic representation of incident and reflected light beams

Using the above said assumptions for the following two switch conditions (‘ON’ and ‘OFF’) and substituting equation (6.28) in equation (6.25) through equations (6.26) and (6.27), the incident (power is incident at port 1) and reflected power relationship is derived.
i. 'ON' state (output power at port 3)

\[ V > V_{\text{TIR}}; S_{41} = 0; P_4 = 0; \theta_4 = 0; \]

From Snell’s law and equation (6.29)

\[ \theta_i = \frac{\pi}{4} + \sin^{-1}\left[1 - \left(\frac{n_i^2 \times r_{ij} \times V}{2d}\right)^2\right] \]  \hspace{1cm} (6.30a)

where,

\[ \frac{\pi}{4} \] is the initial angle of reflection when the power is through port 3.

\[ P_3 = P_1 \left[\left(\frac{V_i - I_iZ_i}{V_i + I_iZ_i}\right)e^{j(\theta_1 - \theta_i)}\right]^2 \]  \hspace{1cm} (6.30b)

Thus, the reflected power \( (P_3) \) is made available at port 3 from port 4 by applying voltage \( (V > V_{\text{TIR}}) \) to the electrodes. It corresponds to ‘ON’ states of port 3.
ii. ‘OFF’ state (output power at port 4)

From Snell’s law and equation (6.29)

\[ \theta_4 = \sin^{-1} \left[ \sin \theta_i + \left( \frac{n_i^2 \times r_{ii} \times V}{2d} \right) \right] \]  

(6.31a)

\[ P_4 = P_1 \left[ \left( \frac{V_i - I_1 Z_1}{V_i + I_1 Z_1} \right) e^{j(\theta_i - \theta_4)} \right]^2 \]  

(6.31b)

Thus, the reflected power \((P_3)\) is made zero at port 3 by applying voltage \((V < V_{TIR})\) to the electrodes, which corresponds to ‘OFF’ state of port 3.

Figure 6.11 Schematic representation of incident and reflected light beams when \(V <= V_{TIR}\)
6.4.2 Frequency response of the TIR switch

In order to obtain the frequency response of the TIR switch, the following equations given by Hunsperger (1991) are considered.

\[ \Delta n = n_1 - n_2 \geq \frac{(2m + 1)^2 \lambda^2}{32n_1 t^2} \]  \hspace{1cm} (6.32a)

\[ \Delta n = n_1 r n_1 \left( \frac{V}{2d} \right) \]  \hspace{1cm} (6.32b)

where,

\[ n \] small change in refractive index of the medium

\[ n_1 \] refractive index of the medium

\[ n_2 \] reduced refractive index of the medium because of applied voltage to the electrode.

\[ m \] mode value in integer \((>0)\)

\[ \lambda_0 \] operating wavelength signal

\[ t \] thickness of the medium

By rearranging the equations (6.32a) and (6.32b), the applied voltage to the electrodes is defined in terms of operating wavelength.

\[ V = \frac{(2m + 1) \lambda_0 \sqrt{\Delta n}}{2n_1 r n_1 \sqrt{2n_1}} \]  \hspace{1cm} (6.33)
By substituting the equation (6.33) into (6.30a) and (6.30b), the reflected power to port 3 in terms of switch parameters is deduced as follows.

$$ P_3 = P_1 \left\{ \left( \frac{V_1 - I_1 Z_1}{V_1 + I_1 Z_1} \right) e^{j \left( \frac{\pi}{4} + \sin^{-1}\left[ 1 - n_1^2 \times r_{33} \times \left( \frac{(2m+1) \lambda_0 \sqrt{\lambda n}}{4dn_1^3 r_{33} \sqrt{2n_1}} \right) - \theta_1 \right] \right)^2} \right\} $$

(6.34)

For various values of operating wavelength (400nm to 900nm), the frequency response (amplitude and phase responses) of the TIR electro optic switch is obtained as shown in Figures 6.20 and 6.21.

### 6.4.3 Impact of waveguide parameters

To determine the response of the TIR switch for various values of waveguide parameters, the transfer function of the switch when light incidence at port 1 (equation 6.25) is considered. It becomes equation (6.35), when the reference plane is shifted in its position.

$$ T_1' = S_{\prime,1} + S_{\prime,2} $$

(6.35)

After incorporating the waveguide parameters ($\beta, l$) in equation (6.35) through equation (6.27) and by simplification it becomes,

$$ T_1' = \frac{b_3 e^{-2i\beta_1 l_1} + b_4 e^{-2i\beta_2 l_2}}{a_1 e^{2i\beta_1 l_1}} $$

(6.36)
if $\beta_s = \beta_s = \beta_s = \beta_s$ and $l_s = l_s = l_s = l_s$, then

$$T_1' = \frac{b_3 + b_4}{a_1} \left( e^{-4 j \beta_s l_s} \right)$$

$$T_1' = P_1 \left[ \frac{V_1 - I_1 Z_1}{V_1 + I_1 Z_1} \right] \left( e^{j(\theta_i - \theta_i)} \right) \times \left( e^{-4 j \beta_s l_s} \right)$$

Where,

$T_1'$ is the resulting transfer function due to change in waveguide parameters

$S_{31}'$ and $S_{41}'$ are the resulting scattering parameters due to change in waveguide parameters

$a_1$ is the incident field from the port 1,

$b_3$ and $b_4$ are the scattered fields from the port 3 and 4 respectively,

$l_1, l_3$ and $l_4$ are the length of the ports 1, 3 and 4 respectively and

$\beta_i, \beta_s$ and $\beta_s$ are the propagation constant values at the ports 1, 3 and 4.

This is solved to get $|T_1'|$ and $|T_1'|$ of the TIR switch for variations in the waveguide parameters.

6.4.4 Magnitude and phase of Spanke switch fabric

The magnitude and phase response of the Spanke switch fabric is derived from its TF (equation 6.24). It is assumed that all the intermediate
switch ports of the cascaded switch fabric are identical except the input and output switch ports, so that the incident and scattered beam are identical in all these ports.

\[
|T_i| = \tan^{-1}\left( \frac{\sin(6\theta_i)}{-\cos(6\theta_i)} \right) \quad (6.39a)
\]

\[
|T_i| = 4S(l)^5 U_{in} (U_i + U_o) \left( U_i + U_o \right) \left( \frac{(V/2 - 1/2 \times Z)^3}{(V/2 + 1/2 \times Z)^2 (V + 1 \times Z)} \right) \times \left( \frac{V/2 - 1/2 \times Z}{V/2 - 1/2 \times Z} \right) \times \left( \frac{\Re(Z)^3}{\Re(Z_i) \times \Re(Z_o)} \right) \times \sqrt{2\cos^2 6\theta_i} - 1 \quad (6.39b)
\]

All the variables of this equation are already defined in equation (6.23) and in the previous sections of this chapter. From these equations, the response of the switch fabric is obtained.

6.5 RESULTS AND DISCUSSION

**Inference from the S-matrix**

From the S-matrix of the four port network (equation 6.1), the S-matrix of the TIR switch (equation 6.2) is derived using its properties and the scattering matrix. This matrix has only eight non-zero elements. Similarly, the S-matrix of the Spanke switch fabric (equation 6.17) is derived. This also has only eight non-zero elements and they account for all the scattering between the input and output ports of the switch fabric.
Inference from mathematical relationship

The mathematical relationships exist between

(i) the scattered powers \((P_i, \text{ and } P_r)\) and the scattering parameters \((S_u)\),
(equation 6.26) and

(ii) the scattering parameters \((S_u)\) and the scattering fields \((a_i \text{ and } b_i)\)
(equation 6.27).

Using them, a relationship between the powers and fields is derived
(equation 6.30b and 6.31b). This relationship is a function of port current, voltage, impedance and incident angle. Further extending this, the equation (6.34) is established, which is also a function of operating wavelength, thereby the frequency response of the switch can be studied.

Similarly, relationships are also brought out to account waveguide parameter variations (equation 6.38) and to study the transfer function response of switch fabric (equation 6.39a and 6.39b). It is inferred that the waveguide parameters used in cascading the switches affect the transfer function of the Spanke switch fabric.
Inference from the simulation

All the mathematical formulations developed in the previous subsections are simulated using the Matlab 6.1 software package (APPENDIX IV). Their responses are shown in Figures 6.12 to 6.20.

- As expected, the reflected power is directly proportional (linear) to the incident power of the TIR optical switch (Figure 6.12). It also satisfies the assumed ideal insertion loss 0dB.

- The reflected power increases steadily and reaches maximum (approximately equal to incident power) at reflected angle $\theta_3 = 45^\circ$ (which is assumed in order to couple maximum power to the port 3), then decreases while increasing the reflected angle (Figure 6.13), which is also shown in Figure 6.14 for higher values of incident angle. The envelope follows the distribution of Gaussian

- From the principle of Pockel’s effect, it is understood that when the applied voltage to the electrode is less than $V_{TIR}$, the light beam simply passes (switch ‘OFF’) through port 4 (assuming that incident at port 1). On the other hand, if it is more than $V_{TIR}$, the light beam is reflected into the port 3 (switch ‘ON’). When the applied voltage is equal to $V_{TIR}$, the light beam is directed to neither port 3 nor
port 4 (no output). These situations are shown in Figure 6.15. It is observed that in this case $V_{\text{TIR}} = 157V/\mu m$ (approximately) and port 3 attains maximum output when applied voltage, $V = 538V/\mu m$ (approximately). These values are coinciding with Tsai (1978) work (here $\theta_c = 45^\circ$, but Tsai designed the TIR switch with $\theta_c = 87.8^\circ$). However, it is advisable to reduce the applied voltage to the electrodes by increasing the incident angle of light beam (these two are inversely proportional).

- The variation of reflected power to the port against the electrode voltage for various values of incident angles is illustrated in Figure 6.16. As according to previous argument, this response clearly reveals that the applied voltage to electrode increases linearly when the incident angle of the port decreases. It can be seen that for the incident angle $\theta_1 = 45^\circ$, the required applied voltage to the electrode is, $V_{\text{TIR}}=157V/\mu m$.

- The reflected power to the output port against the ratio of port impedance (port 3/port 4) is shown in Figure 6.17. This shows a steady response without any variation. It is quiet obvious in this case because in Figure 6.18, the reflected power is increasing while varying the port current and decreasing while varying the port voltage. Hence, the reflected power for the variation of impedance
ratio is to be a steady response without any variation (provided if it is kept in a proper reference plane). Figure 6.19 illustrates that how the response of transfer function of TIR switch varies for different (-π to π) βl values, where βl is product of propagation constant with length of waveguide.

- The frequency response of TIR switch is shown in Figures 6.20 and 6.21. From the amplitude response, it is observed that the 3dB bandwidth (Δf) of the switch is around 0.3120×10¹⁴Hz. This response is taken for various port voltages (i.e., \( V_3 = V_1/2 \), \( V_3 = V_1/4 \) and \( V_3 = V_1 \)). For all these voltages, the response is the same, but with a small change in the peak amplitude.

- The responses of the Spanke switch fabric to the various incident angles and intermediate port impedance values (Figures 6.22 and 6.23) reveal that the received light magnitude decreases with an increase in the port impedance and some steep reduction in magnitude is also observed. This is due to the mismatch in impedance between the waveguide and the switches. However, for increasing incident angle the phase response undergoes small change, which is shown for one full rotation.
The value of the parameters used in the simulation is presented in Table 6.1. The compound considered in this study is LiNbO$_3$. Though the results are given based on this compound, other compounds like GaAs etc are also attempted in this study. It is found that there is a small change in the amplitude of their response. This difference is because of their associated electro-optic coefficient value. For example, in the case of LiNbO$_3$ ($r_{33}=30.8\times10^{-10}\text{cm/V}$) the required voltage to achieve TIR condition is approximately 157V/μm, whereas for GaAs ($r_{41}=1.2\times10^{-12}\text{cm/V}$) it is approximately 100kV/μm (for the given other parameter values, as shown in Table 6.1). Therefore, it reveals, that the material property (electro-optic coefficient) is inversely proportional to the applied voltage required to achieve TIR condition.
Table 6.1 Parameter values used in the simulation

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Applied electric field to electrodes (Tsai 1978)</td>
<td>10V/μm for $\theta_{c} = 87.8^\circ$ (LiNbO₃)</td>
</tr>
<tr>
<td></td>
<td>In present work (the applied voltage increases as the angle is decreased)</td>
<td>157V/μm for $\theta_{c} = 45^\circ$ (LiNbO₃)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100V/μm for $\theta_{c} = 45^\circ$ (GaAs)</td>
</tr>
<tr>
<td>2.</td>
<td>Incident Power ($P_1$)</td>
<td>4mW</td>
</tr>
<tr>
<td>3.</td>
<td>Input port voltage ($V_1$)</td>
<td>0.1V</td>
</tr>
<tr>
<td>4.</td>
<td>Other port voltage ($V_3$ and $V_4$)</td>
<td>$V_3 = V_4 = V_1/2$</td>
</tr>
<tr>
<td>5.</td>
<td>Input port current ($I_1$)</td>
<td>20mA</td>
</tr>
<tr>
<td>6.</td>
<td>Other port currents ($I_3$ and $I_4$)</td>
<td>$I_3 = I_4 = I_1/2$</td>
</tr>
<tr>
<td>7.</td>
<td>Input port impedance ($Z_1$)</td>
<td>500Ω</td>
</tr>
<tr>
<td>8.</td>
<td>Other port impedance ($Z_3$ and $Z_4$)</td>
<td>$Z_3 = Z_4 = Z_1$</td>
</tr>
<tr>
<td>9.</td>
<td>Waveguide parameter ($\beta L$)</td>
<td>$-\pi$ to $+\pi$</td>
</tr>
<tr>
<td>10.</td>
<td>Thickness of the waveguide ($t$)</td>
<td>5μm</td>
</tr>
<tr>
<td>11.</td>
<td>Electro optic coefficient ($r_{33}$) for LiNbO₃</td>
<td>$30.8 \times 10^{-16}$cm/V</td>
</tr>
<tr>
<td></td>
<td>Electro optic coefficient ($r_{31}$) for GaAs</td>
<td>$1.2 \times 10^{-12}$cm/V</td>
</tr>
<tr>
<td>12.</td>
<td>Refractive index ($n_1$)</td>
<td>2.2</td>
</tr>
<tr>
<td>13.</td>
<td>Distance between electrodes ($d$)</td>
<td>4μm</td>
</tr>
<tr>
<td>14.</td>
<td>Length of the electrode pair ($l$)</td>
<td>3.4mm</td>
</tr>
<tr>
<td>15.</td>
<td>Wavelength of operation ($\lambda$)</td>
<td>633nm</td>
</tr>
<tr>
<td>16.</td>
<td>Angle of Incidence ($\theta_1$)</td>
<td>$45^\circ$</td>
</tr>
<tr>
<td>17.</td>
<td>Angle of reflection ($\theta_3$ and $\theta_4$)</td>
<td>$\theta_3 = 90^\circ - \theta_1$, $\theta_4 = 0$, when $V \neq 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_3 = 0$, $\theta_4 = 45^\circ$, when $V = 0$</td>
</tr>
</tbody>
</table>
Figure 6.12 Variation of reflected power Vs incident power

Figure 6.13 Variation of reflected power Vs reflected angle (1)
Figure 6.14 Variation of reflected power Vs reflected angle (11)

Figure 6.15 Variation of power for varying applied voltage
Figure 6.16 Variation of power Vs electrode voltage for various values of incident angles

Figure 6.17 Reflected power Vs ratio of port impedance
Figure 6.18 Variation of reflected power Vs port current and voltage

Figure 6.19 Variation of magnitude and angle of transfer function (TF) of TIR switch for various beta*L values
Figure 6.20 Amplitude response of TIR switch

Figure 6.21 Phase response of TIR switch
Figure 6.22 Magnitude variation with impedance of switch fabric

Figure 6.23 Phase variation with incident angle of switch fabric
Calculation of switching time

The important fundamental parameter of the TIR switch, the switching time, is calculated from the frequency response (Figure 6.20). By taking 3dB response, it is inferred that the bandwidth of the switch is approximately \(0.312 \times 10^{14}\) Hz. The switching time of the TIR switch is determined through the following equation.

\[
\text{Switching time} = \frac{2\pi}{\Delta f}
\]

(6.40)

where \(\Delta f\) is the bandwidth of the TIR switch (Hunsperger 1991). It is calculated approximately as 0.201 ps.

Table 6.2 Comparison of switching time

<table>
<thead>
<tr>
<th>Bulk Electro-optic</th>
<th>TIR switch</th>
<th>Quantum Well Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practical (only)</td>
<td>Circuit Model</td>
<td>S-matrix</td>
</tr>
<tr>
<td>5ns (Papadimitriou 2003)</td>
<td>1.93ns (present work)</td>
<td>0.201ps (present work)</td>
</tr>
</tbody>
</table>

The practical switching time of QW based TIR electro-optic switch is not reported. However, the QW electro-optic structure and bulk electro-optic switching times are reported. The calculated value through S-matrix analysis is
in reasonable agreement with these values as shown in Table 6.2. Whereas, the value evaluated using circuit model is not comparable with that of S-matrix. The circuit model analysis may not be that much suitable for the devices operating at this frequency.

6.6 CONCLUSION

In this chapter the derivation of the scattering matrix for both TIR electro-optic switch and Spanke switch fabric are presented. Using Mason’s gain formula, their transfer functions are formulated. From the transfer function, the frequency response of the switch is evaluated. The switching time is also calculated. In addition, the following mathematical relations are derived:

i. The relationship between the incident and the reflected power in terms of switch parameter

ii. The magnitude and the phase of the TIR switch TF with respect to incident angle, impedance and waveguide parameters

iii. The Frequency response of the TIR switch

iv. The magnitude and the phase of the Spanke switch fabric for a complete understanding of the switch and switch fabric

Thus, a new mathematical approach to analyze the TIR electro-optic switch and Spanke switch fabric by employing the scattering matrix method is evolved.