PREFACE

Solid mechanics deals with the mechanical behaviour of the deformed bodies (bars, plate, shafts, cylinder, discs, sphere, shell etc.) subjected to various types of external forces and goes by a variety of names, such as strength of material, mechanics of deformable solids etc. The research in solid mechanics is essentially not only for basic understanding of mechanical phenomenon but also to advance engineering methodology in most of areas throughout mechanical and structural technology. Advances in the subject are central to assure safely, reliability and economy in the design of device, structure and complete systems, and hence to the continued development of power generation technology such as fusion, nuclear and gas turbine power, aerospace and surface transportation vehicles, earthquake resistant design, offshore structures, orthopaedic devices, material processing and manufacturing technologies.

Recent development in the field of solid mechanics, which has space and nuclear engineering as it's leading parts, has required not only the exploitation of super high temperature alloys such as gas turbines, hypersonic planes, jet engines, missiles, nuclear reactor etc. which endure to severe conditions of high temperature and high stress but has also required the research of analytical methods of stress and strain in
the various equipment or structure employed in such condition. In these applications the influence of high temperature on the material properties is an important design consideration. Considerations of the plastic state of matter are today of interest to many branches of sciences and engineering. The material engineer is interested in plastic flow as it relates forming processes, such as rolling, forging, bending, stretching and deep drawing. The design engineer is interested in deformation behaviors in connection with his responsibility for avoiding excessive deflection or distortion in machine parts and engineering structures. The steal and metal workers desires to control more accurately the mechanical process of forming metals at forging temperature. Because of the large quantity of energy, at the present time, consumed in steal mills during the process of rolling, a more economical use of energy is needed.

Many industrial and military developments including the introduction of new material and the need for better design from the standpoint of both safety and economy have made it necessary to consider more thoroughly the properties of the materials. A designer's efforts will concentrate on compromising between what is structurally and materially desirable and what is technologically feasible or economically possible. He is not only interested in more exact knowledge of the limiting conditions of stress at which, in his parts, permanent deformation begins to
develop and danger of yielding or fracture is to be expected or
fatigue cracks begin to form but also in several cases he will have
to consider the possible deformation in machine parts exposed to
long duration of stressing.

Since 1950 the theory of elasticity for
anisotropic bodies has been continually developed and enriched
with new investigations of both serious problems of a general
nature and individual aspects of these problems. Thus, the general
theory has been placed on a rigorous scientific basis and a
number of laws have been established, with the result that this
theory, first worked out by B. de Saint-venant and P.V. Bekhterev,
has been revived. Of great importance is the development and
construction of many entirely new anisotropic materials, which
possess a number of advantages over those previously known (e.g.
glass-fiber reinforced plastics). Thus, over a quarter of a century
this branch of science has made great progress, both in a
theoretical and a purely practical way, i.e. in constructing new
anisotropic materials.

For the last several decades, anisotropic
materials have been widely used in many areas because of their
excellent static and dynamic behaviour and low strength-to-weight
ratio. The study of anisotropic elasticity has attracted attention of
scientists because of its increasing application to engineering
problems. The latest engineering structures and machineries are
often made of the composite materials, whose elastic behaviour may be analyzed macroscopically by the theory of anisotropic elasticity. Unidirectional laminated material belongs to the group of transversely isotropic materials. For transversely isotropic materials (materials which possesses the axis of symmetry in the sense that all the rays of light at right angle to this axis are equivalent), there are only five independent elastic constants. Finally, there are only two independent elastic constants known as Lame's parameters for isotropic elastic materials (materials in which elastic properties are independent of orientation of axes).

The demand of high-speed technology in transportation, communication and energy conversion have forced us to take serious notice of non-linearity. But some of us still find it difficult to get rid of the habit of a century old habit of analytical and experimental research in continuum mechanics, which leisurely leaned very heavily on linearization. If a medium ‘A’ changes into ‘B’ through a transition state T, ‘A’ and ‘B’ may be almost linear but T is non-linear. Since this non-linearity is difficult to investigate, workers have taken to the artifice of replacing it by singular, non-differentiable or discontinuous surfaces. This piece wise treatment necessitates the use of ad-hoc and semi-empirical law, which may or may not exist.

In classical treatment different constitutive equations are used for each state, which are based on some
hypothesis that simplifies the problem to much extent. First, the deformations are assumed to be small to make infinitesimal strain theory applicable. Secondly, the constitutive equations of the material are simplified by assuming the material incompressible, and in some cases without this assumption, it is not even possible to find the solution of the problem in the closed form. Seth has defined the generalized strain measure which when combined with the transition point analysis of the governing differential equation of the medium, not only eliminates ad-hoc assumptions like incompressibility, creep strain law and yield condition but also employs the same constitutive equations to give elastic-plastic and creep results through some transition functions.

In this thesis an attempt has been made to study elastic-plastic and creep problems in transversely isotropic materials by using Seth’s transition theory. It has been shown that the asymptotic solution through the principal stress leads from elastic to plastic state and through the principal stress difference from elastic to creep state. Results obtained have been discussed numerically and depicted graphically. The thesis is divided into seven chapters:

CHAPTER I

The first chapter is introductory in which a brief history of the theory of elasticity, plasticity and creep is given. Elastic-plastic and creep phenomenon has been described
briefly. The characteristics of strain rate in four stages of creep-elastic, transient, stationary and rupture has been discussed. A brief review of Seth's generalized strain measure has been presented. The fundamental constitutive equations have been given.

**CHAPTER II**

The second chapter deals with the problem of “Elastic-plastic Transition in a Transversely Isotropic Disc Having Variable Thickness under Internal Pressure”. The thickness of disc is assumed to vary radially i.e. \( h = h_o \left( \frac{r}{b} \right)^{-K} \) where \( h_o \) is the thickness at the external surface & \( K \) is the thickness parameter. Results obtained have been discussed numerically and depicted graphically. It has been observed that for a transversely isotropic disc with variable thickness, under internal pressure, the yielding occurs at the internal or external surface of the disc depending upon material and thickness ratio. The disc having variable thickness and made of transversely isotropic material yields at a less pressure as compared to disc made of isotropic material and transversely isotropic disc requires high percentage increase in pressure to become fully plastic from initial yielding as compared to isotropic material.
CHAPTER III

The third chapter deals with the problem of “Elastic-plastic Transition for Transversely Isotropic Rotating Disc of Variable Density with Edge Loading”. The effects of angular speed have been discussed for initial yielding and fully plastic state. A thin rotating disc made of isotropic material (Brass) whose density increases radially requires higher percentage increase in angular speed to become fully-plastic as compare to rotating disc having constant density or whose density decreases radially and made of transversely isotropic material. Rotating disc having variable density and made of isotropic material have a tendency to fracture at the bore i.e., it is where the largest tensile stress occurs as compare to rotating disc made of transversely isotropic material. The tendency of fracture at the bore increases with the increase in edge load.

CHAPTER IV

The fourth chapter deals with the problem of “Thermo Elastic-plastic Transition in a Transversely Isotropic Thick-walled Cylinder under Internal Pressure”. Elastic-plastic transitional stresses have been derived, by using Seth’s transition theory. The combined effects of pressure and temperature on the cylinder have been discussed numerically and depicted graphically. It has been observed that at room temperature, thick-
walled cylinder made of isotropic material yields at a high pressure at the internal surface as compare to cylinder made of transversely isotropic material. With the introduction of thermal effects isotropic/transversely isotropic cylinder yields at a lower pressure whereas cylinder made of isotropic material requires less percentage increase in pressure to become fully-plastic from its initial yielding as compare to cylinder made of transversely isotropic material.

CHAPTER V

The fifth chapter contains the problem of “Creep Transition in a Transversely Isotropic Disc Having Variable Thickness under Internal Pressure”. The thickness of disc is assumed to vary radially i.e. \( h = h_o \left[ \frac{r}{b} \right]^{-K} \) where \( h_o \) is the thickness at the external surface & K is the thickness parameter. Creep stresses and strain rates have been obtained for transversely isotropic disc having variable thickness subjected to internal pressure by using Seth’s transition theory. Results obtained have been discussed numerically and depicted graphically. It has been observed that the disc made of variable thickness for transversely isotropic material has maximum circumferential stress at the outer surface in comparison to disc having constant thickness and this value further increases with the increase in measure N and K.
Strain rates are maximum at the internal surface for flat disc made of isotropic material for measure $N=1$ and pressure $P_i = 0.1$ but strain rates decrease at the internal surface for measure $N > 1$. The strain rate for flat disc further increases with the increase in pressure at the internal surface. The disc having variable thickness ($K = 1.5$) made of isotropic material have strain rates maximum at the external surface for $P_i = 0.1$ and measure $N=1$. These values of strain rates further increase at the external surface with the increase in pressure and variable thickness ratio but decrease with the increase in measure $N$.

CHAPTER VI

The sixth chapter deals with the problem of “Creep Transition for Transversely Isotropic Rotating Disc of Variable Density with Edge Loading”. Creep stresses and strain rates have been obtained for a transversely isotropic thin rotating disc having variable density $\rho = \rho_o (r/b)^m$ where $\rho_o$ is the density at $r = b$ and $m$ is the density parameter by using Seth’s transition theory. Results obtained have been discussed numerically and depicted graphically. It has been observed that a disc made of isotropic material with edge load, rotating with higher angular speed and whose density decreases radially ($m=1$) increases the possibility of fracture at the bore as compare to a disc made of transversely isotropic material whereas rotating disc whose density
increases radially \((m=-1)\) recedes the possibility of fracture at the bore and the possibility of fracture further decreases with the increase in measure \(N\). Therefore, it can be concluded that a rotating disc made of transversely isotropic material with edge load, and whose density decreases radially, is on the safer side of the design in comparison to an isotropic rotating disc. It has been observed from strain rate curves that an isotropic rotating disc with edge load, whose density decreases radially experiences a significant deformation at higher angular speed as compare to a rotating disc made of transversely isotropic material.

**CHAPTER VII**

The seventh chapter has been devoted to the problem of “Thermo Creep Transition in a Transversely Isotropic Thick-walled Cylinder under Internal Pressure”. Creep stresses and strain rates have been obtained for transversely isotropic thick-walled cylinder under internal pressure at steady state temperature by using Seth’s transition theory. Results obtained have been discussed numerically and depicted graphically. It has been observed that circumferential stress is maximum at the external surface for thick-walled cylinder made of isotropic material under internal pressure at room temperature for measure \(N=7\) (i.e. \(n=1/7\)), whereas at high temperature though the circumferential stress has lower value yet it is maximum for thick-
walled cylinder made of transversely isotropic material. Introduction of thermal effects reduces the stresses at the external surface of thick-walled cylinder made of transversely isotropic/isotropic material under internal pressure.

It has been observed from strain rate curves that with the introduction of thermal effects, the creep rates have larger value at the internal surface of the thick-walled cylinder made of transversely isotropic material and the values of creep rates further increases with the increase in pressure for measure N= 3 (i.e. n = 1/3) as compare to measure N= 7 (i.e. n = 1/7) and cylinder made of isotropic material.