Chapter 5

Fibonacci Cordial Labeling

5.1 Introduction

As cordial labeling is considered as weaker version of graceful labeling, it was a source of motivation for us whether we can consider the condition of cordial labeling by replacing graceful labeling using Fibonacci numbers. This idea lead us to define a new labeling called Fibonacci cordial labeling. Fibonacci cordial labeling can be considered as a frontier between graph theory and number theory.

Definition 5.1.1. The Fibonacci numbers can be defined by the linear recurrence relation

\[ F_n = F_{n-1} + F_{n-2}, \quad n \geq 3. \]

This generates the infinite sequence of integers beginning

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots

In this chapter we introduce Fibonacci cordial labeling and we discuss Fibonacci cordial labeling of different graphs.

Definition 5.1.2. Assume \( G \) to be a simple connected graph with \( n \) vertices. An injective function \( g : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\} \), where \( F_j \) is the \( j^{th} \) Fibonacci number (\( j = 0, 1, \ldots, n \)), \( F_0 = 0, F_1 = 1, F_2 = 2 \), is said to be Fibonacci cordial
labeling if the induced function $g^* : E(G) \rightarrow \{0,1\}$ defined by 
\[
g^*(xy) = (g(x) + g(y)) \pmod{2}
\]
satisfies the condition $|e_g(0) - e_g(1)| \leq 1$, where $e_g(0)$ is the number of edges with label $0$ and $e_g(1)$ is the number of edges with label $1$. A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph.

### 5.2 Fibonacci Cordial Labeling of Some Graphs

**Theorem 5.2.1.** Petersen graph is Fibonacci cordial.

**Proof.** Assume $x_1, x_2, x_3, x_4, x_5$ as the internal vertices and $x_6, x_7, x_8, x_9, x_{10}$ as the external vertices of Petersen graph such that each $x_i$ is adjacent to $x_{i+5}, 1 \leq i \leq 5$.

We define labeling function $g : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{10}\}$ as follows:
\[
g(x_1) = F_0, g(x_2) = F_1, g(x_i) = F_{i-1}, 3 \leq i \leq 10.
\]
Then we have $e_g(0) = 7$ and $e_g(1) = 8$. Therefore $|e_g(1) - e_g(0)| = 1$.

Hence Petersen graph is Fibonacci cordial.

**Example 5.2.1.** Fibonacci cordial labeling of Petersen graph is given in Figure 5.1.

![Figure 5.1: Fibonacci cordial labeling of Petersen graph.](image)

**Theorem 5.2.2.** Wheel $W_n$ is Fibonacci cordial for all $n \geq 3, n \in \mathbb{N}$.

**Proof.** Assume $x_1, x_2, \ldots, x_n$ as successive rim vertices and $x_0$ as the apex vertex of $W_n$. Here $|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$. 

---

AMIT H. ROKAD 91
To define labeling $g : V(W_n) \to \{F_0, F_1, F_2, \ldots, F_{n+1}\}$, we consider following two cases.

**Case 1:** $n \equiv 0 \pmod{3}$.

$g(x_0) = F_1$, $g(x_i) = F_{i+1}$, $1 \leq i \leq n$.

**Case 2:** $n \not\equiv 0 \pmod{3}$.

$g(x_0) = F_1$, $g(x_1) = F_0$, $g(x_i) = F_{i+1}$, $2 \leq i \leq n$.

Then in each case we have $e_g(1) = e_g(0) = n$.

Hence wheel $W_n$ is Fibonacci cordial for all $n \geq 3, n \in \mathbb{N}$.

**Example 5.2.2.** Fibonacci cordial labeling of $W_9$ is given in Figure 5.2.

![Figure 5.2: Fibonacci cordial labeling of $W_9$.](image)

**Theorem 5.2.3.** Shell $S_n$ is Fibonacci cordial for all $n \geq 3, n \in \mathbb{N}$.

**Proof.** Assume $x_1, x_2, \ldots, x_n$ as successive vertices of shell $S_n$, where $x_1$ is the apex vertex of shell $S_n$.

We define labeling $g : V(S_n) \to \{F_0, F_1, F_2, \ldots, F_n\}$ as

$g(x_1) = F_1$, $g(x_2) = F_0$, $g(x_i) = F_{i-1}$, $3 \leq i \leq n$.

With this labeling the edge labels produced will satisfy the condition as given in following table.

Assume $n = 3c + d$, $c, d \in \mathbb{N}$.
Table 5.1: Edge conditions for Fibonacci cordial labeling of $S_n$.

<table>
<thead>
<tr>
<th>b</th>
<th>Edge conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,1</td>
<td>$e_g(1) = e_g(0) + 1$</td>
</tr>
<tr>
<td>2</td>
<td>$e_g(1) + 1 = e_g(0)$</td>
</tr>
</tbody>
</table>

From the above table $|e_g(1) - e_g(0)| \leq 1$.

Hence shell $S_n$ is Fibonacci cordial for all $n \geq 3, n \in \mathbb{N}$.

Example 5.2.3. Fibonacci cordial labeling of $S_{11}$ is given in Figure 5.3.

![Figure 5.3: Fibonacci cordial labeling of $S_{11}$](image)

**Theorem 5.2.4.** Bistar $B_{n,n}$ is Fibonacci cordial, for all $n$.

**Proof.** Assume $x_0, y_0$ as the apex vertices of $B_{n,n}$. Assume $x_1, x_2, \ldots, x_n$ as the pendant vertices adjacent to the vertex $x_0$ and $y_1, y_2, \ldots, y_n$ as the pendant vertices adjacent to the vertex $y_0$.

We define labeling function $g : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows:

**Case I:** $n \equiv 2 \pmod{3}$

- $g(x_0) = F_1, g(x_1) = F_2, g(x_i) = F_{i+1}, 2 \leq i \leq n$.
- $g(y_0) = F_0, g(y_i) = F_{n+i+1}, 1 \leq i \leq n$.

Then we have $e_g(1) = n$ and $e_g(0) = n + 1$.

**Case II:** $n \not\equiv 2 \pmod{3}$

- $g(x_0) = F_1, g(x_i) = F_{i+2}, 1 \leq i \leq n$.
- $g(y_0) = F_0, g(y_1) = F_2, g(y_i) = F_{n+i+1}, 2 \leq i \leq n$.
Then we have $e_g(1) = n$ and $e_g(0) = n + 1$.

Hence $B_{n,n}$ is a Fibonacci cordial graph for all $n$.  

**Example 5.2.4.** Fibonacci cordial labeling of $B_{7,7}$ is given in Figure 5.4.

![Figure 5.4: Fibonacci cordial labeling of $B_{7,7}$](image)

**Theorem 5.2.5.** $G =< B_{n,n} : x >$ is Fibonacci cordial graph.

**Proof.** Assume $G =< B_{n,n} : x >$. Assume $u_0, v_0$ as the apex vertices of $B_{n,n}$ and $x$ as the vertex added as a result of subdivision of the edge joining $u_0$ and $v_0$. Assume $u_1, u_2, \ldots, u_n$ as the pendant vertices adjacent to the vertex $u_0$ and $v_1, v_2, \ldots, v_n$ as the pendant vertices adjacent to the vertex $v_0$.

We define vertex labeling $g : V(G) \to \{F_0, F_1, F_2, \ldots, F_{2n+3}\}$ as follows:

**Case I:** $n \equiv 0 \pmod{3}$

$g(x) = F_3, g(u_0) = F_1, g(u_1) = F_{2n+3}, g(u_i) = F_{i+2}, 2 \leq i \leq n.$

$g(v_0) = F_0, g(v_i) = F_{n+i+2}, 1 \leq i \leq n.$

Then we have $e_g(1) = e_g(0) = n + 1$.

**Case II:** $n \equiv 1 \pmod{3}$

$g(x) = F_3, g(u_0) = F_1, g(u_1) = F_2, g(u_i) = F_{i+2}, 2 \leq i \leq n.$

$g(v_0) = F_0, g(v_n) = F_{2n+3}, g(v_i) = F_{n+i+2}, 1 \leq i \leq n - 1.$

Then we have $e_g(1) = e_g(0) = n + 1$.

**Case III:** $n \equiv 2 \pmod{3}$

$g(x) = F_3, g(u_0) = F_1, g(u_1) = F_2, g(u_i) = F_{i+2}, 2 \leq i \leq n.$
Chapter 5. Fibonacci Cordial Labeling

\[ g(v_0) = F_0, \quad g(v_i) = F_{n+i+2}, \quad 1 \leq i \leq n. \]

Then we have \( e_g(1) = e_g(0) = n + 1. \)

Hence \( G = B_{n,n} : x > \) is Fibonacci cordial graph.

**Example 5.2.5.** Fibonacci cordial labeling of \( B_{7,7} : x > \) is given in Figure 5.5.

![Figure 5.5: Fibonacci cordial labeling of \( B_{7,7} : x > \).](image)

**Theorem 5.2.6.** The graph \( K_{2,n} \odot x_2(K_1) \) is Fibonacci cordial.

**Proof.** Assume \( G = K_{2,n} \odot x_2(K_1) \). Assume \( V = V_1 \cup V_2 \) as the bipartition of \( K_{2,n} \) such that \( V_1 = \{ x_1, x_2 \} \) and \( V_2 = \{ y_1, y_2, \ldots, y_n \} \). The pendant vertex \( w \) is adjacent to vertex \( x_2 \) in \( G \).

We define vertex labeling \( g : V(G) \rightarrow \{ F_0, F_1, F_2, \ldots, F_{n+3} \} \) as follows:

\[ g(w) = F_3, \quad g(x_1) = F_1, \quad g(x_2) = F_2, \quad g(y_i) = F_{i+3}, \quad 1 \leq i \leq n. \]

Then \( e_g(1) = n + 1 \) and \( e_g(0) = n. \)

Therefore, \( |e_g(1) - e_g(0)| = 1. \)

Hence, \( G = K_{2,n} \odot x_2(K_1) \) is a Fibonacci cordial graph.

**Example 5.2.6.** Fibonacci cordial labeling of \( K_{2,6} \odot x_2(K_1) \) is given in Figure 5.6.
Theorem 5.2.7. The graph \( C_n \oplus K_{1,n} \) is Fibonacci cordial.

Proof. Assume \( V(C_n \oplus K_{1,n}) = Y_1 \cup Y_2 \), where \( Y_1 = \{x_1, x_2, \ldots, x_n\} \) is the vertex set of \( C_n \), \( Y_2 = \{y, y_1, y_2, \ldots, y_n\} \) is the vertex set of \( K_{1,n} \), \( y_1, y_2, \ldots, y_n \) are pendant vertices and \( y = x_1 \). Note that \( |V(C_n \oplus K_{1,n})| = |E(C_n \oplus K_{1,n})| = 2n \).

We define \( g : V(C_n \oplus K_{1,n}) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n}\} \) as follows:

**Case I:** \( n \equiv 0, 1 \pmod{3} \).

\[
g(x_1) = F_1, \quad g(x_i) = F_{n+i}, \quad 2 \leq i \leq n.
\]

Then \( e_g(1) = e_g(0) = n \).

**Case II:** \( n \equiv 2 \pmod{3} \).

\[
g(x_1) = F_1, \quad g(x_i) = F_{n+i-1}, \quad 2 \leq i \leq n.
\]

Then \( e_g(1) = e_g(0) = n \).

Therefore \( e_g(1) = e_g(0) = n \) in each case.

Hence \( C_n \oplus K_{1,n} \) is Fibonacci cordial.

Example 5.2.7. Fibonacci cordial labeling of \( C_9 \oplus K_{1,9} \) is given in Figure 5.7.
Theorem 5.2.8. Square of $B_{n,n}$ is Fibonacci cordial.

Proof. Consider bistar $B_{n,n}$ with vertex set $\{x, y, x_i, y_i/1 \leq i \leq n\}$, where $x_i, y_i$ are pendant vertices and $x, y$ are apex vertices. Assume $G$ as the square of $B_{n,n}$. Here $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$.

We define vertex labeling $g : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows:

$g(x) = F_1$,
$g(x_i) = F_{i+1}$, $1 \leq i \leq n$.
$g(y) = F_0$,
$g(y_i) = F_{n+i+1}$, $1 \leq i \leq n$.

Then $e_g(1) = 2n + 1$ and $e_g(0) = 2n$.

Therefore, $|e_g(1) - e_g(0)| = 1$.

Hence square of $B_{n,n}$ is Fibonacci cordial. $\square$
Example 5.2.8. The Fibonacci cordial labeling of square of $B_{5,5}$ is given in Figure 5.8.

![Figure 5.8: Fibonacci cordial labeling of square of $B_{5,5}$.](image)

Theorem 5.2.9. Vertex switching of cycle $C_n$ is Fibonacci cordial.

Proof. Assume $x_1, x_2, \ldots, x_n$ as the successive vertices of cycle $C_n$. Assume $(C_n)_{x_1}$ denote the switching of vertex of $C_n$ with respect to any arbitrary vertex $x_1$ of $C_n$.

We define labeling $g : V((C_n)_{x_1}) \to \{F_0, F_1, F_2, \ldots, F_n\}$ as

$g(x_1) = F_1, g(x_2) = F_0, g(x_i) = F_{i-1}, 3 \leq i \leq n.$

With this labeling the edge labels produced will satisfy the condition as given in following table.

Assume $n = 3c + d$, $c, d \in \mathbb{N}$.

Table 5.2: Edge conditions for Fibonacci cordial labeling of vertex switching of cycle $C_n$

<table>
<thead>
<tr>
<th>b</th>
<th>Edge conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$e_g(0) + 1 = e_g(1)$</td>
</tr>
<tr>
<td>0, 2</td>
<td>$e_g(0) = e_g(1) + 1$</td>
</tr>
</tbody>
</table>

From the above table $|e_g(1) - e_g(0)| \leq 1$.

Hence the vertex switching of cycle $C_n$ is Fibonacci cordial. □
Example 5.2.9. Fibonacci cordial labeling for vertex switching of cycle $C_{10}$ is given in Figure 5.9.

Figure 5.9: Fibonacci cordial labeling for vertex switching of $C_{10}$.

Theorem 5.2.10. Vertex switching of any vertex of cycle $C_n$ having one chord is Fibonacci cordial, where chord forms a triangle with two edges of $C_n$.

Proof. Assume $G$ as the cycle having one chord. Assume $x_1, x_2, \ldots, x_n$ be consecutive vertices of cycle $C_n$ and $e = x_2x_n$ as the chord of cycle $C_n$. The vertices $x_1, x_2, x_n$ form a triangle with chord $e$. If $x_i$ and $x_j$ are of same degree then the graph obtained by switching of vertex $x_i$ and the graph obtained by switching of vertex $x_j$ are isomorphic to each other. Hence we require to discuss two cases: (i) vertex switching of a vertex of degree 2 and (ii) vertex switching of a vertex of degree 3. Assume $(G)_{x_1}$ denote the switching of a vertex of $G$ with respect to an arbitrary vertex $x_1$.

To define labeling function $g : V((G)_{x_1}) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\}$ we conceive the below cases.

Case 1: $\deg(x_1) = 2$

Subcase I: $n \equiv 0 \pmod{3}$

$g(x_1) = F_1, g(x_2) = F_n, g(x_3) = F_0, g(x_4) = F_2,$

$g(x_i) = F_{i-2}, 5 \leq i \leq n.$

Then $e_g(1) = e_g(0) = n - 2$.

Subcase II: $n \equiv 1 \pmod{3}$
Chapter 5. Fibonacci Cordial Labeling

\[ g(x_1) = F_1, \ g(x_n) = F_0, \ g(x_i) = F_i, \ 2 \leq i \leq n - 1. \]
Then \( e_g(1) = e_g(0) = n - 2. \)

**Subcase III:** \( n \equiv 2 \pmod{3} \)
\[ g(x_1) = F_1, \ g(x_2) = F_0, \ g(x_i) = F_{i-1}, \ 3 \leq i \leq n. \]
Then \( e_g(1) = e_g(0) = n - 2. \)

**Case 2:** \( \text{deg}(x_1) = 3 \)
\[ g(x_1) = F_1, \ g(x_2) = F_0, \ g(x_i) = F_{i-1}, \ 3 \leq i \leq n. \]
Then in each case \( e_g(1) = e_g(0) = n - 3. \)
Therefore \( |e_g(1) - e_g(0)| = 0 \) in each case.

Hence vertex switching of cycle \( C_n \) having one chord is Fibonacci cordial. \( \Box \)

**Example 5.2.10.**

(a) Fibonacci cordial labeling of switching of a vertex of degree 2 in cycle \( C_{10} \) with one chord is given in Figure 5.10(a).

(b) Fibonacci cordial labeling of switching of a vertex of degree 3 in cycle \( C_{10} \) with one chord is given in Figure 5.10(b).

![Figure 5.10](image-url)

Figure 5.10: Fibonacci cordial labeling of vertex switching of \( C_{10} \) with one chord with respect to vertex of degree 2 and degree 3.
Theorem 5.2.11. Vertex switching of cycle having twin chords $C_{n,3}$ is Fibonacci cordial.

Proof. Assume $G$ as the cycle having twin chords $C_{n,3}$. Assume $x_1, x_2, \ldots, x_n$ as the successive vertices of cycle $C_n$. Assume $e_1 = x_n x_2$ and $e_2 = x_n x_3$ as the chords of cycle $C_n$. If $x_i$ and $x_j$ are of same degree then the graph obtained by switching of vertex $x_i$ and the graph obtained by switching of vertex $x_j$ are isomorphic to each other. Hence we require to discuss three cases: (i) vertex switching of a vertex of degree 2, (ii) vertex switching of a vertex of degree 3 and (iii) vertex switching of a vertex of degree 4. Assume $(G)_{x_1}$ denote the vertex switching of $G$ of an arbitrary vertex $x_1$.

To define labeling $g : V((G)_{x_1}) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\}$ we conceive the below cases.

Case I: $\deg(x_1) = 2$

Subcase I: $n \equiv 0, 1 \ (mod \ 3)$

$g(x_1) = F_1, g(x_i) = F_i, 2 \leq i \leq n.$

Subcase II: $n \equiv 2 \ (mod \ 3)$

$g(x_1) = F_1, g(x_2) = F_0, g(x_i) = F_{i-1}, 3 \leq i \leq n.$

Case II: $\deg(x_1) = 3, \deg(x_1) = 4$

$g(x_1) = F_1, g(x_2) = F_0, g(x_i) = F_{i-1}, 3 \leq i \leq n.$

In each case $|e_g(1) - e_g(0)| \leq 1.$

Hence vertex switching of cycle having twin chords $C_{n,3}$ is a Fibonacci cordial graph.

Example 5.2.11.

(a) Fibonacci cordial labeling of switching of a vertex of degree 2 in cycle with twin chords $C_{13,3}$ is given in Figure 5.11(a).

(b) Fibonacci cordial labeling of switching of a vertex of degree 3 in cycle with twin chords $C_{13,3}$ is given in Figure 5.11(b).

(c) Fibonacci cordial labeling of switching of a vertex of degree 4 in cycle with twin chords $C_{13,3}$ is given in Figure 5.11(c).
Chapter 5. Fibonacci Cordial Labeling

Figure 5.11: Fibonacci cordial labeling of vertex switching of $C_{13}$ with twin chords with respect to vertex of degree 2, degree 3 and degree 4.

**Theorem 5.2.12.** The joint sum of two copies of fan $F_n$ (with respect to apex vertex) is Fibonacci cordial.

**Proof.** Assume $\{x_0, x_1, x_2, \ldots, x_n\}$ and $\{y_0, y_1, y_2, \ldots, y_n\}$ as the vertices of first and second copy of $F_n$ respectively, where $x_0$ is the apex vertex of first copy of $F_n$ and $y_0$ is the apex vertex of second copy of $F_n$. Assume $G$ as the joint sum of two copies of fan $F_n$.

We define $g : V(G) \to \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows:

**Case I:** $n \equiv 0, 1 \pmod{3}$.

$g(x_0) = F_1, g(x_1) = F_0, g(x_2) = F_2, g(x_i) = F_{i+1}, 3 \leq i \leq n.$

$g(y_0) = F_3, g(y_i) = F_{n+i+1}, 1 \leq i \leq n.$

**Case II:** $n \equiv 2 \pmod{3}$.

$g(x_0) = F_1, g(x_1) = F_0, g(x_2) = F_2, g(x_i) = F_{i+1}, 3 \leq i \leq n.$

$g(y_0) = F_3, g(y_n) = F_{2n+2}, g(y_i) = F_{n+i+1}, 1 \leq i \leq n - 1.$

In all cases, $|e_g(1) - e_g(0)| \leq 1$.

Hence the joint sum of two copies of fan $F_n$ (with respect to apex vertex) is Fibonacci cordial graph.

**Example 5.2.12.** The Fibonacci cordial labeling of joint sum of two copies of $F_5$ is given in Figure 5.12.
Theorem 5.2.13. The joint sum of two copies of wheel $W_n$ (with respect to apex vertex) is Fibonacci cordial.

Proof. Assume $\{x_0, x_1, x_2, \ldots, x_n\}$ and $\{y_0, y_1, y_2, \ldots, y_n\}$ as the vertices of first and second copy of $W_n$ respectively, where $x_0$ is the apex vertex of first copy of $W_n$ and $y_0$ is the apex vertex of second copy of $W_n$. Assume $G$ as the joint sum of two copies of $W_n$.

We define labeling function $g : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows:

**Case I:** $n \equiv 0 \pmod{3}$

$g(x_0) = F_1, \ g(x_1) = F_0, \ g(x_2) = F_2, \ g(x_i) = F_{i+1}, \ 3 \leq i \leq n.$
$g(y_0) = F_3, \ g(y_i) = F_{n+i+1}, \ 1 \leq i \leq n.$
Then $e_g(1) = 2n + 1$ and $e_g(0) = 2n$.

**Case II:** $n \equiv 1 \pmod{3}$

The labeling pattern is same as Case-I.
Then $e_g(1) = 2n$ and $e_g(0) = 2n + 1$.

**Case III:** $n \equiv 2 \pmod{3}$

$g(x_0) = F_1, \ g(x_1) = F_2, \ g(x_i) = F_{i+2}, \ 2 \leq i \leq n.$
$g(y_0) = F_3, \ g(y_n) = F_{2n+1}, \ g(y_{n-1}) = F_{2n+2}, \ g(y_i) = F_{n+i+2}, \ 1 \leq i \leq n - 2.$
Then $e_g(0) = 2n + 1$ and $e_g(1) = 2n$.

Therefore $|e_g(1) - e_g(0)| = 1$ in each case.

Hence joint sum of two copies of $W_n$ (with respect to apex vertex) is Fibonacci cordial. □
Example 5.2.13. The Fibonacci cordial labeling of joint sum of two copies of \( W_9 \) is given in Figure 5.13.

![Figure 5.13: Fibonacci cordial labeling of joint sum of two copies of \( W_9 \).](image)

Theorem 5.2.14. The joint sum of two copies of Petersen graph is Fibonacci cordial.

Proof. Assume \( G \) as the joint sum of two copies of Petersen graph. Assume \( \{x_1, x_2, \ldots, x_{10}\} \) is the vertex set of first copy of Petersen graph and \( \{y_1, y_2, \ldots, y_{10}\} \) is the vertex set of second copy of Petersen graph.

We define labeling function \( g : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{20}\} \) as follows:

\[
g(x_1) = F_0, \quad g(x_2) = F_1, \quad g(x_i) = F_{i-1}, \quad 3 \leq i \leq 10.
\]

\[
g(y_i) = F_{9+i}, \quad 1 \leq i \leq 10.
\]

Then \( e_g(1) = 16 \) and \( e_g(0) = 15 \).

Therefore, \( |e_g(1) - e_g(0)| = 1 \).

Hence the joint sum of two copies of Petersen graph is Fibonacci cordial.

Remark: Here joint sum is taken with reference to external vertices. One can consider joint sum between any two arbitrary internal vertices and by different combinations of labels, one can prove the result.

Example 5.2.14. Fibonacci cordial labeling of joint sum of two copies of Petersen graph is as given in Figure 5.14.
5.3 Conclusion

We introduce here Fibonacci cordial labeling which connects graph labeling with number theory. It is the work carried out by the motivation from existing labeling namely Fibonacci graceful labeling. Here we have investigated fourteen graph families satisfying the condition of Fibonacci cordial labeling. As it is a novel concept in the field of graph labeling there is a vast scope of research for the researchers in this field. It is important to see that whether all Fibonacci graceful graph are Fibonacci cordial or not.

5.4 Open Problems

- To investigate necessary and sufficient conditions for a graph to admit a Fibonacci cordial labeling.

- To investigate some new graph or graph families which admit Fibonacci cordial labeling.

- To characterize the graph families admitting Fibonacci cordial labeling.