CHAPTER 2

EXPERIMENTAL PARAMETERS AND THE RATE EQUATIONS
GOVERNING THE POPULATION OF THE LASER STATES:

2.1 EXPERIMENTAL PARAMETERS:

The output of the CVL is characterised by the pulse width, pulse shape, pulse height, number of pulses per second i.e. the pulse repetition rate, intensity distribution across the laser beam etc. For many applications the laser beam of high peak power having uniform intensity distribution across the laser beam is desired. Building of the high power lasers need the high volume of the laser cavity. The use of high volume leads to the annular shape of the output laser beam. Thus the use of large bore tube may not give uniform power across the laser beam, which is desired for several applications. The properties of the output of the laser beam are determined by the discharge parameters like 1) the discharge current pulse shape, 2) the temperature of the discharge electrons, 3) the pressure of the buffer gas, 4) the temperature and partial pressure of the copper in the tube, 5) the density of the metastable state, 6) diameter of the discharge tube, 7) the laser cavity.

2.1.1) CURRENT PULSE SHAPE:

The discharge current pulse shape plays an important role in the excitation of the laser states which in turns controls the power output of the laser. As the
life time of the upper laser state is about 40 nsec, the discharge pulse of the fast rise time is desired. The shape of the current pulse depends upon the charge Q=C/V stored by the capacitor, the impedance of the spark gap or thyratron and the variation of the resistivity of the discharge across the tube. The pulse rise time is solely controlled by the properties of the spark gap like the distance between the spark electrodes, shape of the electrodes, material of the electrodes, the gas between the electrodes etc. In case of the thyratron switches the rise time is determined by the characteristics of the thyratron. After firing the discharge pulse, the current rises to a peak value and then it decreases and becomes zero. The decay time of the current pulse depends upon the rate at which the electrons are cooled and disappear in recombination or near the cold walls. The discharge looses the energy through the spectral emission and to the walls through collisions.

2.1 2) THE ELECTRON TEMPERATURE:

The electron temperature play vital role in several processes like ionisation, recombination, excitation, deexcitation etc. The electron temperature also influences the gas temperature. The discharge current pulse width is of the order of 300 nsec (1-4). The current pulse width of the transversely excited laser is comparatively shorter ( may be order of 20 nsec ) (5-6).

The electron temperature in the discharge tube is
controlled by heating due to the electric field and losses by the spectral emission and losses to the walls. As the diameter and length of the discharge tube differs from system to system, the impedance in the circuit differs from system to system and the electron temperature also differ accordingly. The electron temperature is not uniform across the discharge tube since the walls act as the infinite sink of the energy. The plasma on the laser axis is away from the walls so it is least affected by the cooling due to the walls. The portion of the plasma at the axis of the discharge tube is hottest and the plasma electron temperature go on decreasing towards the walls. During the period of the discharge current pulse the discharge electrons get heated. After the current pulse is over the electrons are cooled down by the different processes. In the afterglow of the discharge the emission is observed for about 200 μsec (7-8). This may be because of the combined effect due to the electron impact excitation and the lifetime of the states. The lifetimes of the atomic states are not as long as 200 μsec. The characteristic decay time may be the time required for cooling down the electrons. The electron temperature in the discharge tube lies between 1 eV to 6 eV. Infact the electron temperature is not same for the discharge tube of different diameter.

2.1 3) THE GAS TEMPERATURE:

The gas temperature determines the Penning transfer
rate coefficient and charge transfer rate coefficient and de-excitation rate of the metastable states. The gas particles also act as a sink for the electron temperature. The average gas temperature is governed by the average discharge current, the material of the discharge tube, the enclosure of the discharge tube etc. The partial pressure of the copper in the discharge tube depends upon the gas temperature. The operating temperature of the CVL is high, since high temperature is required to obtain desired partial pressure. It may be maintained by using proper insulation and by passing high average discharge current.

2.1 4) RESISTIVITY OF THE PLASMA:

The resistivity of the plasma column is expressed by the equation (9)

\[ \eta = \frac{52 \times 10^{-3} \ln(\lambda)}{(T_e)}^{3/2} \quad \text{[Te is in eV]} \quad (2.1) \]

The logarithmic term \( \ln(\lambda) \) is a function of electron density and it is a very slowly varying function. For the conditions in gas discharge plasmas the value of \( \ln(\lambda) \) is about 10.

Using this value of the term the expression for the resistivity is written in the final form as

\[ \eta = \frac{52 \times 10^{-2}}{(T)^{3/2}} \quad \text{[ohm-cm]} \quad (2.2) \]
The plasma electrons are cooled by the collisions with the walls of the container and heated by the electric current. The plasma on the axis is far away from the walls and least affected by the cooling due to walls. The plasma near the walls is cold and it goes on increasing towards the tube axis. This gives rise to high current density on the axis and low current density near the walls. This effect further increases the non uniform distribution of the electron temperature across the discharge column. This leads to the non uniform distribution of 1) the electron density, 2) the ion density, 3) the ion temperature, 4) the density of the laser upper state, 5) the density of laser lower state.

2.2 DENSITIES:

The processes of excitation and deexcitation of the laser states and ionisation and recombination are controlled by the electron temperature, the electron density and the density of copper atoms. In absence of the current pulse density of electrons is low and it is of the order of $10^{13} \text{ cm}^{-3}$ (13). Since the plasma segments are neutral the ion density must be same as the electron density. The current pulse ionises the gas particles and increases the electron density as well as the ion density. The electron density has been obtained by Kushner (13) it is about $4 \times 10^{13} \text{ cm}^{-3}$.

The density of the lower laser state plays vital role in the determination of the pulse characteristics as it is metastable state having lifetime of 700 nsec. If
the atoms are accumulated in this state the laser output power is reduced to large extent. The distribution of the metastable state is not uniform across the discharge cavity (10-11). The radial distribution of the lower state is a function of time.

The properties of the laser output beam are determined by the factors like; 1) the distance between two electrodes, 2) the charging voltage, 3) the value of the capacitor which is discharged through the active medium, 4) the pulse repetition rate, 5) total pressure and partial pressures in the discharge tube, 6) the kind of gas used as the buffer gas, 7) the impedance of the thyatron etc. The effect of the parameters on the qualities of the output of the laser has been studied in details by Kushner and his group(12-14). They have solved the rate equations and obtained the solutions under various conditions. They concluded that theory developed by them may be employed to study several experimental results. However, in no case they refer the concepts of computation of the fractional abundance of the different species in the discharge. In fact solving the rate equations exactly is not possible but the equations may be solved by applying some approximations. In obtaining solutions in any way needs some knowledge of the fractional abundance.

In the present work we introduce the concept of fractional abundance for the first time for obtaining
the results in the CVL branch. The detailed rate equations have been obtained and regourous treatment of the equations is discussed.

Based on the quantities for which the rates are expressed the rate equations are divided into three categories. Each category contains several rate equations. Before considering the rate equations we shall list the reactions taking place in the discharge column.

2.3 THE POSSIBLE REACTIONS IN THE DISCHARGE:

The electrons and other particles in the discharge column colloid among themselves and among each other. In the process of collision many things happen. Particles may transfer kinetic or potential energy to another particle and transferred energy may get converted into kinetic energy or potential energy. Few reactions which take place in the discharge tube are listed below:

\[ e + M \longrightarrow M^+ + e + e - \Delta E \] Ionisation

\[ e + M \longrightarrow M^* + e - \Delta E \] Excitation

\[ e + M^* \longrightarrow M + e + \Delta E \] Superalastic collision

\[ e + M^+ \longrightarrow M^{**} \] Collisional recombination

\[ e + e + M^+ \longrightarrow M^{**} + e + \Delta E \] Radiative recombination

\[ He^+ + Cu^* \longrightarrow Cu^+ + He \] Charge exchange

\[ He^+ + Cu^* \longrightarrow Cu^+ + He + e + \Delta E \] Penning reaction
\[ \text{M}^* + \text{M} \rightarrow \text{M} + \text{M} \quad \text{Collisional deactivation} \]

\[ \text{M}^* + \text{wall} \rightarrow \text{M} \quad \text{Diffusion and deactivation} \]

The rates of the reaction depend upon the densities of the particles taking part in the reaction and the rate coefficient of the reaction.

2.4 RATE EQUATIONS:

2.4 1) RATE EQUATIONS GOVERNING THE GAS PRESSURE:

The gas temperature of the laser tube walls is constant and the pressure inside the tube is also constant. But when the discharge pulse is passed through the gas the temperature of the gas suddenly increases which in turn increases the local pressures of the gases. The rate of change of buffer gas pressure \( P_g \) and laser gas pressure \( P_c \) are expressed as

\[
\frac{dP_g}{dt} = \frac{dT_g}{dt} \frac{P_g}{T_g} \frac{(P_g - P_{\text{ex}})}{1} 2v_S
\]

\[
\frac{dP_c}{dt} = \frac{dT_c}{dt} \frac{P_c}{T_c} \frac{(P_c - P_{\text{vp}})}{R} v_d
\]

where \( v_S \) is the speed of the sound in the gas

\( v_d \) is the diffusion velocity of copper

\( P_{\text{ex}} \) is external gas pressure

\( P_{\text{vp}} \) is the vapour pressure of the copper based on the temperature of the wall

\( l \) is the length of the discharge

\( R \) is the radius of the tube
Parameters in one set depend upon the parameters in another set directly or indirectly. The density of the copper atoms relaxes to the cold walls and the buffer gas relaxes to the external pressure with the time rates \( v_d/R \) and \( 1/2v_B \) respectively. The rate equations for the corresponding densities may be written as

\[
\frac{\delta n_g}{\delta t} = - \frac{n_g}{T_g} \frac{dT_g}{dt} + \frac{n_g}{P_g} \frac{dP_g}{dt} \\
= -(P_g - P_{ex}) \frac{n_g^2v_B}{1P_g}
\]

\[
\frac{\delta n_c}{\delta t} = - \frac{n_c}{T_g} \frac{dT_g}{dt} + \frac{n_c}{P_c} \frac{dP_c}{dt} \\
= -(P_c - P_{vp}) \frac{n_c v_d}{^vP_c}
\]

The rate of change of the densities of the buffer gas and the copper may be studied during the laser discharge pulse.

2.4 2) RATE EQUATIONS GOVERNING THE CURRENTS AND VOLTAGES:

A) VOLTAGES AND CURRENTS:

The discharge circuits used for the excitation of the laser states are shown in the figure(2.1). One circuit is for the ideal laser and another circuit is for real laser. In this case the exciting field is axial.
Before firing the exciting pulse the power supply charges the capacitor of the circuit with certain rate and the charged capacitor gets discharged through the laser medium. Thus the time rate of change of voltage across the capacitor is written as

\[
\frac{dV}{dt} = f(T_s) V_0 - V \quad \frac{1}{R_d C}
\]

where \( f(T_s) \) is the voltage turn on function

\( V_0 \) is the maximum voltage on the capacitor

\( R_d \) is the instantaneous discharge impedance

\[
R_d = \frac{1}{l} \frac{m_e v}{n_e A e^2}
\]

where \( l \) is the distance between electrodes

\( v \) is the electron collision frequency

\( n_e \) is the density of the electrons

\( A \) is the effective cross sectional area of the discharge tube

The another set of rate equations for the circuit currents and voltages are written as

\[
\frac{dI_1}{dt} = \frac{(V_1 - V_2 + V_0 f(T_s))}{L_1}
\]

\[
\frac{dI_2}{dt} = \frac{(V_2 - V_4)}{L_2}
\]

\[
\frac{dV_1}{dt} = \frac{-I_1}{C_1}
\]
\[
\frac{dV_2}{dt} = \frac{I_1 - I_2}{C_2}
\]

where current \( I_1 \) flows through \( L_1 \) and \( V_i \) is the voltage across \( C_i \).

The voltage across the discharge tube is

\[ V_d = \frac{I_2 R_c R_d}{R_c + R_d} \]

while the current through the discharge is

\[ I_d = \frac{I_2 R_c}{R_c + R_d} \]

The voltages and currents through the different parts of the circuit may be studied with the help of these equations. The more important parameter in the equation is the impedance of the discharge tube and the current passing through the tube. The current passing through the discharge tube heats the discharge electrons which in turn excite the laser medium. The impedance of the gas between the electrodes is time dependent as well as the space dependent. When the gas is not in breakdown condition the impedance of the gas is almost infinity and no current passes through the gas. When the voltage across the gas is increased the gas between electrodes breaks down and the impedance decreases suddenly and the heavy current starts flowing through the discharge. The current density across the discharge tube is not uniform owing to the fact that the electron temperature is not uniform across the discharge tube. At this stage one can
talk about the resistivity of the plasma given by the equation (2.1).

Near the walls the electron temperature is almost zero and the resistivity at the wall is infinity. Moreover, the electron density is very near to zero at the discharge tube walls. The resistivity of the discharge tube decreases towards the axis of the tube. Thus more and more current would pass through the plasma segment on the axis and the current density goes on decreasing towards the walls. While talking about the resistivity of the discharge plasma it is to be divided into coaxial cylindrical shell of infinitesimal thickness. Since the parameters on which the resistivity depends are uniform for the plasma in the cylindrical shell the resistivity of the plasma may be treated as constant for the shell. The equations written for the gas density must be modified for the spatial variation of the current density. M J Kushner and B E Warner (14) studied the radial distribution of the electric field, electron temperature, the power deposited into the gas etc for small bore tubes and large bore tubes. In their work it has been shown that the electric field is more near the wall than that at the axis for the large bore tube. The energy deposited near the walls is more than that deposited on the axis. This is true for the large bore tubes and for the leading edge of the laser pulse. As the pulse advances the radial dependance of the electric
field disappears. However, in the theory of Kushner and Warner the radial dependence of the conductivity $\sigma$ is not taken into account. The conductivity $\sigma$ of the plasma varies from point to point in the discharge tube. While applying skin effects the radial dependence of $\sigma$ must be taken into account. After considering the radial dependence of $\sigma$ the electric field and the electron temperature should be calculated.

2.4 3) THE RATE EQUATIONS FOR THE DENSITIES OF THE LASER STATES:

The pertinent partial energy level diagram of the copper atoms is shown in the figure 2.2. The exciting and deexciting processes may be visualised and understood well with the help of the diagram. The current passing through the laser active medium heats the discharge electrons by collisions between electrons. The electrons also heat the gas by the process of collision. The heated electron colloid with copper atoms and atoms of the buffer gas and excite them into different states depending upon the corresponding cross section.

The time rate of heating of electron is

$$\frac{d (3/2 \text{kTe})}{dt} = \frac{e^2 \cdot 2}{\text{me} \cdot \text{v}} \sum \frac{\text{HemNijr}_{ij} \Delta \text{E}_{ij}^p}{n_e}$$

$$+ \sum_{\text{i}, \text{j}, \text{k}} \text{Nijr}_{ij} \Delta \text{E}_{ij}^S \sum_{\text{i}, \text{j}, \text{k}} \text{Nik}_{ijk} \Delta \text{E}_{ijk} - \sum_{\text{i}, \text{j}, \text{k}} \text{Nij}_{ij} \Delta \text{E}_{ij}^k$$

$$- 3/2 \text{Da kTe} \sqrt[4]{2n_e} - \sum_{\text{i}, \text{j}, \text{k}} \text{Nij}_{ij} \Delta \text{E}_{ij}^l \frac{3/2 \text{kTe}}{n_e}$$
\[- \frac{3}{2} k(T_e - T_g) \sum_{\substack{ij \in \mathcal{N} \setminus i \setminus j \setminus n_i}} N_{ij} n_e Z_{me} \]

The terms in the equation are defined in the Appendix A. The first three terms represent the heating of the plasma electrons by the electric field, the Penning ionisation of the species and the super elastic relaxation of the different states respectively. The discharge electrons loose their energies in the processes; 1) Excitation and ionisation, 2) Diffusion to the walls, 3) heating of the electron produced in the process of ionisation and 4) heating of the gas atoms which are at relatively at low temperature.

The discharge electrons derive their energy primarily from the electric field applied to the discharge tube and secondly from the super elastic collision. The discharge electrons excite the helium atoms to the state which in Penning collision transfer energy to the electrons. When the laser system is not put on the gas enclosed in the laser cavity contains no electrons. Although there are few electrons the electron density is very low. The firing of the the spark gap for the first time produces electrons by the acceleration of stray electrons and subsequent ionising collisions with the gas particles. The electrons grow in number by the process of avalanche. After sometime the charge on the capacitor gets exhausted consequently there is no further acceleration of electrons. The discharge
The energetic electrons when colloided with atoms either ionise them or excite the atoms to various levels depending upon the excitation cross section. The excited atom loose their energies by the several processes. The rate equations for the laser upper state and the lower state are written as

\[ \frac{d[Cu]}{dt} = n_e \left( r_{12}[Cu] + r_{32}^S[Cu^*] + r_{42}^S[Cu^{**}] - (r_{21}^S[Cu^*]) - \right) 
\]

\[ + r_{23}^I[Cum]) + Cu^* + Cu^{**} - (r_{21,Cu}[Cu]_{T32} + r_{42,Cu}[Cu]_{T42}) + r_{21,He}[He] - r_p[He]^P - r_{CE}[He^+] - D_{Cu}[Cum]_{\Lambda^2} 
\]

\[ + J_B([Cu^*] - g_3[Cum]) \]

\[ \frac{d([Cu^*])}{dt} = n_e \left( r_{13}[Cu] + r_{23}[Cum] + r_{43}^S[Cu^{**}] - \right) 
\]

\[ - (r_{21}^S + r_{32}^S + r_{34} + r_{31}^I)[Cu^*] + Cu^{**}/T_{43} (r_{32,Cu}[Cu] + r_{32,He}[He] - r_p[He]^P - r_{CE}[He^+] - D_{Cu}[Cum]_{\Lambda^2} \]
electrons start cooling down because of the processes mentioned above. The electron temperature as well as the electron density go on depleting depending upon the decay time. The laser medium may retain a certain number of electrons till the second pulse comes (13). The firing of the second pulse may experience more electron density than that experienced by the first pulse. The electron density present at the time of firing of the discharge go on increasing as the number of pulses are fired and the value of the electron density may reach a saturation level. Furthermore, the increase in the repetition rate increases the value of the electron density present at the time of the firing of the pulse. Running of the discharge for some time increases the overall temperature of the system. This increase in the temperature enhances the average value of the electrons present in the discharge tube.

When the discharge is fired the electrons are produced by the ionization of the atoms of the buffer gas and copper and they are destroyed by the recombinations. The process of Penning ionisation also produces the electrons. Few electrons are lost due to the diffusion process. The net rate of production of electrons is

\[
\frac{dn_e}{dt} = n_e \left( \sum_i r_{ij}^I N_{i,j} - \frac{D_a}{\lambda^2} \right) - \sum_i \frac{N_i^+ (r_{RR} + n_e r_{CRR})}{\lambda} + H_{em} \sum_j \gamma_{ij} N_{i,j}
\]
The electron impact excitation and deexcitation rates are denoted by $r$ with corresponding subscript. Superscript $S$, $C$ and $I$ are used for super elastic collision, collisional activation and ionisation respectively. The laser states are populated by the processes: 1) electron impact excitation from the ground state and low lying energy state, 2) super elastic collisions with high lying energy state, 3) radiative decay of high lying energy state, 4) recombination of the ions into the state. The atoms in the laser state are destroyed by the processes like: 1) radiative decay of the state, 2) super elastic collision of the atoms in the state and 3) ionisation of the atoms present in the state. Furthermore, the atoms in the upper state are destroyed and the atoms in the lower state are produced by the process of stimulated emission.

The time rate of change of the intensity of the laser beam is

\[
\frac{dI}{dt} = \frac{h \nu C}{3} \left( [\text{Cu}^+] - g_3 [\text{Cum}] \right) \frac{l_a}{l_c} g_2
\]

\[- CI \left( \frac{1 - R}{l_c} + r \right) + h \nu A \left[ \text{Cu}^+ \right] \alpha\]

The first term is the rate of increase in intensity by the process of stimulated emission and absorption.
The second term represents the loss at output coupling mirror. The third term is addition of the photons to the laser beam because of the spontaneous emission of the upper state.
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Figure 2.1 Equivalent circuit of the pulse shaping line.
Figure 2.2 Pertinent partial energy level diagram of copper atom.
Figure 2.3 Cross section of discharge tube.