CHAPTER 9

ANGLE OF DIVERGENCE OF THE COPPER VAPOUR LASER BEAM

The angle of divergence plays central role where the high radiation density is required in the laser applications. The field of the investigation of the laser induced plasma requires high energy density radiations. The laser diagnosis and treatment in the medical science also needs the intense radiation with a good control over the intensity of the laser beam. The high power lasers having low angle of divergence give the required energy density after focusing by the good quality lenses. Thus it is essential to investigate the behaviour of the angle of divergence of the laser beam at different values of the laser parameters.

The stimulating action in the laser medium starts with a signal produced by spontaneous emission and subsequent amplification of the radiation by the laser medium. If the gain of the amplifying medium is high the amplification rate is high and the intensity builds up immediately after the production of spontaneous emission. As in the cases of copper vapour laser(1,2), nitrogen laser(3,4), lead vapour laser(5) the gain of the medium is high and the production of the population inversion is transient phenomenon the single pass of the radiation is sufficient for the building up of the laser intensity. On the other hand in the lasers like He-Se$^+$ (6), He-Cd$^+$ (7) and He-Zn$^+$ (8) the optical gain of the

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medium is small and multiple pass of the radiation is essential for building up of the laser intensity. In case of transient laser the spontaneous radiation gets amplified and comes out of the cavity in a very short interval of time consequently the radiation having large angle of divergence gets amplified. However in case of multiple pass laser the radiation having less angle of divergence gets amplified. So the pulsed lasers having narrow pulse width have large angle of divergence and the CW lasers have low angle of divergence (9).

So far there is not much work on the theory part as well as experimental part of the investigation of the angle of divergence. Pawar et al (10) carried out few computations in the branch of angle of divergence of the pulsed lasers.

In the present work the analytical expressions are obtained for the peak power of the output of the copper vapour laser without mirror, the intensity of the laser radiation across the laser beam, half peak power angle of divergence of the copper vapour laser beam. The angle of divergence is obtained by graphical method. The peak power of the laser beam at various spatial points across the laser beam is studied.

The copper vapour laser medium may be pumped by an AE (axial electric) field(11,12) and TE (transverse electric) field(1,2). The copper vapour laser pumped by transverse electric field is similar to the cyclic lasers like 3371 A nitrogen laser (3,4), 5401A neon
laser(13), magnesium laser(14), lead vapour laser(15), cadmium ion laser(16) etc. The angle of divergence of the TE field pumped laser output beam is rather larger than the angle of divergence of the AE field pumped output laser beam. In the case of TE pumped laser the pumping pulse is very narrow. During the period of pumping pulse the laser radiation hardly makes few round trips in the medium. Moreover, many times the lifetime of the population inversion is less than the pumping pulse width. Various laser discharges may be studied under the categories, which are described below.

9.1 SHORT PUMPING PULSE APPROACH:

The laser media pumped by the transverse electric field have pumping pulse of very narrow time duration. In general a Blumlein line has been employed for the purpose of the excitation of the laser medium. In Blumlein line two parallel plate condensers are connected in parallel and one of the capacitor is discharged through the spark gap. The other capacitor holds the high voltage. The ends of the capacitors are closed inside a vacuum chamber, which act as the electrodes igniting the discharge in the laser medium. The pumping pulse width may be neglected in comparison with the laser pulse width(17).

9.2 LONGER PUMPING PULSE APPROACH:

When the copper vapour laser is pumped by the axial electric field, the discharge operates at about 10 torr
pressure. The discharge pulse width is of the order of 1 usec. It may be considered that there is a continued pumping and during the continued pumping laser pulse is formed. Wagner and Lengyl (18) studied this type of the pulsed laser and derived theoretical expressions for different laser parameters. It seems that theory developed by them may be employed to the CVL pumped by axial electric field because of the following reasons.

The pumping pulse is of the order of microsecond and the laser pulse is formed in the initial part of the pumping pulse. The inversion density produced by the electron impact excitation is almost constant over a wide range of the electron temperature i.e. from 2 eV to 10 eV (fig. 5.1). During the discharge pulse although the electron temperature varies the inversion density produced by the electron impact excitation is constant.

9.3 FORMATION OF LASER PULSE:

When the discharge pulse is passed through the CVL active medium it produces population inversion. The upper laser state decays spontaneously and the radiation at laser wavelength is produced. The radiation may travel in all directions making an angle 4π radians. But the fraction of radiation travelling in the direction parallel to the laser axis undergoes amplification by the stimulated emission. The angle of divergence of the stimulated emission is restricted by the geometry of the laser active medium and the laser cavity. When the radiation comes out of the cavity for
the first time the angle of divergence is maximum. The mirror put at the end of the cavity reflects the radiation and now the beam having less angle of divergence would be amplified. As the radiation makes more and more round trips in the medium the angle of divergence go on decreasing. In case of the CVL the leading part of the laser beam has large angle of divergence. And the beam coming after sometime has low angle of divergence. For obtaining the angle of divergence of the leading part of the beam the pulse laser output without mirror has to be obtained.

9.4 COPPER VAPOUR LASER WITHOUT MIRROR:

The development of the following theory we assumes that bore discharge tube is excited by the current pulse and no mirror is placed at the either ends of the laser cavity to feed the photon flux. Moreover, it is also assumed that the pulse width of the discharge current is shorter as compared to the laser pulse width and the loss of inversion due to spontaneous emission is negligible compared to the loss of inversion due to the stimulated emission. In other words we may say that the rate of excitation of the laser states is negligible while the process of the formation of the laser pulse.

With these assumptions the rate equations governing the population densities of the laser states and the photon flux are written as
\[
\frac{d\theta_+ (z,t)}{dt} = C \frac{d\theta_+ (z,t)}{dz} + \theta_+ (z,t) \omega_0 C_n (z,t) \tag{9.1}
\]

\[
\frac{d\theta_- (z,t)}{dt} = C \frac{d\theta_- (z,t)}{dz} + \theta_- (z,t) \omega_0 C_n (z,t) \tag{9.2}
\]

And for the inversion density neglecting the spontaneous emission

\[
\frac{dn(z,t)}{dt} = -2 \omega_0 C_n(z,t) [\theta_+(z,t) + \theta_-(z,t)] \tag{9.3}
\]

where,

\[
n = \frac{N_2 - N_1}{N_2 + N_1} = \frac{N_2 - N_1}{N_2 + N_1} \tag{9.4}
\]

is the normalised inversion density.

\(N_1\) is the population density of the lower laser level,

\(N_2\) is the population density of the upper laser level, and

\(N_0 = N_1 + N_2\)

\[
\theta_0 = \frac{\theta}{N_0} = \frac{\theta_+ + \theta_-}{N_0} \tag{9.5}
\]

is the normalised photon flux.

\(\theta_0\) is the photon density.

\(\theta_+\) is the photon density travelling in positive direction.

\(\theta_-\) is the photon density travelling in negative direction.
\kappa = \kappa_0 n

(9.6)

\kappa_0 \text{ is absorption coefficient of laser medium at laser wavelength.}

The rate equations are same as the rate equations of W G Wanger and B A Lengyl (18) and P Richter et al (19) but the boundary conditions are different. Because there is no mirror at any end of the discharge tube, photon fluxes entering from the two ends of the discharge cavity are zero.

i.e \= \varnothing_+(0, t) = \varnothing_-(1, t) = 0

where 1 is the length of the laser medium.

If the laser plasma tube is uniformly wide, initial inversion along the laser tube may be assumed to be uniform across and along the discharge tube. The laser pulse emerging from the two ends have identical photon fluxes.

i.e

\\varnothing_+(1, t) = \varnothing_-(0, t) = \varnothing(t) = \varnothing_+(z, t) + \varnothing_-(z, t)

Here, we assume that photon density increases linearly with laser length, using these boundary conditions, if equation 9.1 and 9.2 are added and the resulting equation is divided by equation 9.3, we get

\frac{d\varnothing}{dn} = \frac{m_T}{2n} \frac{1}{2}

where
\[ n_T = \frac{2}{\alpha_{01}} \]

is threshold inversion density and

\( n(t_i) \) is initial inversion density.

The integration of the equation leads to the result

\[ \Phi(t) - \Phi_i = \frac{1}{2} \frac{\{n_T \log[n(t)] - [n(t) - n(t_i)]\}}{n(t_i)} \quad (9.7) \]

\( \Phi_i \) is the initial photon flux is assumed to be negligible.

It is to be noted that the resulting equation for the photon flux is similar to the corresponding equations obtained by W G Wanger and B A Lengyl (18) and P Richter et al (19) except the threshold inversion density \( n_T \).

The threshold inversion is equal to the loss per pass divided by \( \alpha_{01} \). It is implied that in the present work loss per pass is 2. It is obviously understood that because there is no mirror at either end of the laser tube the photon fluxes are completely lost from both the ends.

From equation (9.7) the expression for the peak power of the laser pulse may be calculated by putting

\[ n(t) = n_T = \frac{2}{\alpha_{01}} \]

The equation for the peak power of the laser output beam may be written as,
\[ \Phi_p = \frac{1}{2} \left\{ n(ti) - \frac{2}{x_01} [1 + \log \frac{n(t)}{x_01}] \right\} \quad (9.8) \]

The peak power of the laser output is determined by the factors like the initial inversion density, the length of the passage of the radiation through the laser medium, the gain of the medium. If one measures the above mentioned parameters, the peak power of the laser output pulse may be obtained. Unless there is influence of the travelling wave excitation (20) on the excitation of the laser medium the two pulses generated by the discharge tube have identical properties.

When the laser pulse leaves the medium, the inversion density left behind is practically very much less than the initial inversion density. The total energy stored by the laser medium is equally shared by two simultaneously generated laser pulses travelling in opposite directions. Hence the energy of the laser pulse may be expressed by the equation.

\[ E = \frac{n(ti)NhvW}{4} \quad (9.9) \]

where,

\( N \) is density of laser levels,
\( V \) is the volume of the excited laser medium,
\( h \) is energy of a photon.

9.5 VARIATION OF PEAK POWER ACROSS LASER BEAM:

To study the variation of peak power of laser pulse
across laser beam, first we have to recall the mechanism of pulse formation. After pump pulse operates, noise is generated due to spontaneous emission of radiation and that gets amplified, if there is population inversion present in the medium. In single pass laser the amplified noise which is not obstructed by walls of laser medium and the boundaries of the electrodes in the discharge tube emerges as laser beam. In the following, we shall study the spatial variation of laser peak power across the laser beam.

The geometry of the laser system and the paths of the radiations required for the present computations are shown in the figure 9.1. ABCD is the section of the laser plasma passing through the laser axis and the diameter. We wish to calculate the peak power of the laser output pulse at different points along the line XX' in plane perpendicular to the laser axis. The points between C' and B' will receive laser power emitted by the laser medium having same lasing length, the length of the laser plasma. For the point of observation beyond C' and B' on XX' line, the lasing length changes from point to point. At first it increases up to D' and A' as the point of observation shifts from B' to D' and from C' to A'. The points beyond D' receive the laser power emitted by laser medium of decreasing length. Now the problem is to calculate the length of the passage of the laser radiation through laser medium, which contributes
to power received by point \( p \) at a distance \( X \) from \( B \) on \( XX' \) line. We assume that the medium along the line joining the point of observation \( 'P' \) and the nearest edge of the laser tube contribute in delivering the laser power received by the point.

**If the point \( 'P' \) is between \( B \) and \( D' \) (I-Case)**

i.e. \[ 0 < X < \frac{Ld}{L} \]

The lasing length may be given by

\[ l = \left[ \frac{X^2d^2}{L^2} + \frac{L^2}{d^2} \right]^{1/2} \quad \text{(9.10)} \]

and

**If the point \( 'P' \) is beyond \( D' \) (II-Case)**

i.e. \[ X > \frac{Ld}{L} \]

the lasing length may be given by

\[ l = \left[ \frac{X^2d^2}{L^2} + d^2 \right]^{1/2} \quad \text{(9.11)} \]

Where \( L \) is the distance between exit end of laser and observation plane.

If the laser length \( (L) \) is large and the width \( (d) \) of the laser beam is small, the equations 9.4 and 9.5 are respectively reduced to,

**For** \[ 0 < X < \frac{Ld}{L} \]

\[ l = L \quad \text{(9.12)} \]
and

\[
\frac{Ld}{X} = \frac{Ld}{X} = (9.13)
\]

Using these values of the lasing length, peak power received by a point \( P \) at distance \( X \) from laser edge or the peak power of laser ray making an angle \( \theta \), with laser axis may be calculated using relations.

For \( X > \frac{Ld}{L} \)

\[
\theta < \frac{Ld}{L} \quad \text{where} \quad \theta = \frac{X}{L}
\]

\[
\phi_p = \frac{1}{2} \left\{ n(t) - \frac{2}{\kappa_0 \left( \frac{X^2}{L^2} + L^2 \right)^{1/2}} \right. \\
+ \frac{n(t)}{\kappa_0 \left( \frac{X^2}{L^2} + L^2 \right)^{1/2}} \\
\left[ 1 + \log \frac{2}{2} \right] \quad (9.14)
\]

\[
\phi_p = \frac{1}{2} \left\{ n(t) - \frac{2}{\kappa_0 \left( \frac{L^2d^2}{X^2} + d^2 \right)^{1/2}} \right. \\
+ \frac{n(t)}{\kappa_0 \left( \frac{L^2d^2}{X^2} + d^2 \right)^{1/2}} \\
\left[ 1 + \log \frac{2}{2} \right] \quad (9.15)
\]
It has to be mentioned here that contribution to the pulse peak power due to oblique radiation is neglected. It is clear from the equation 9.15 that the peak power decreases as \( X \) increases. The laser power becomes zero when lasing length decreases to threshold length.

\[
\frac{2}{\kappa_{on}(ti)}
\]

i.e for \( X = X_0 \), \( \theta_p = 0 \)

\[
\frac{2}{\kappa_{on}(ti)} = \frac{L \theta}{X_0} = \frac{d}{\theta_0}
\]

or

\[
\theta_0 = \frac{X_0}{L} = \frac{d \kappa_{on}(ti)}{2}
\]

(9.16)

If we measure \( \theta_0 \) or \( X_0 \) for which the laser power becomes zero, it might help in deducing initial inversion density using equation,

\[
\frac{X_0}{L} = \frac{2}{\kappa_{on}d} = \frac{\theta_0}{\kappa_{on}d}
\]

(9.17)

9.6 ANGLE OF DIVERGENCE OF THE LASER BEAM:

We define half peak power angle of divergence as the angle between the edge of the cavity and the direction of the laser beam, where the peak power is half the peak power of laser beam emerging parallel to the axis of the
laser plasma tube.

From the figure 9.2 and 9.3 desired angle of divergence may easily be obtained. Half peakpower or zero peak power angle of divergence may directly be read from the figures.

Half peak power angle of divergence $\theta_{1/2}$ may be calculated from equation (9.8) by

$$1 = \frac{Ld}{X_{1/2}} = \frac{d}{\theta_{1/2}}$$

and equating the ratio of the two peak powers to 2.

The equation for the half peak power angle of divergence looks like

$$2 = \frac{n(ti) \frac{\gamma_0 L}{\kappa_0 L}}{2 \left[ 1 + \log \frac{n(ti) \gamma_0 L}{\kappa_0 L} \right]}$$

or

$$\frac{4\theta_{1/2}}{\kappa_0 d} = \frac{n(ti) \gamma_0 d}{2 \theta_{1/2}}$$

$$\frac{n(ti) \gamma_0 L}{\kappa_0 L} = n(ti) + \frac{\gamma_0 d}{2 \theta_{1/2}}$$

The equation 9.8 may be solved if,

$$\theta < \frac{n(ti) \gamma_0 d}{2 \theta_{1/2}} < 2$$

to obtain
$$\theta_{1/2} = \frac{\kappa_{0d}}{4} \left\{ 4n(ti) - k \pm [k^2 - 12n^2(ti) - 8kn(ti)]^{1/2} \right\}$$

where

$$K = \frac{4\theta_{1/2}}{\kappa_{0d}} \left\{ 1 + \log \frac{n(ti)\kappa_{0d}}{2\theta_{1/2}} \right\}$$

There are two solutions of the equation 9.18 shown below

$$\theta_1 = \frac{\kappa_{0d}}{4} \left\{ 4n(ti) - k + [k^2 - 12n^2(ti) - 8kn(ti)]^{1/2} \right\}$$

$$\theta_2 = \frac{\kappa_{0d}}{4} \left\{ 4n(ti) - k - [k^2 - 12n^2(ti) - 8kn(ti)]^{1/2} \right\}$$

Exact value of $\theta_{1/2}$ may be obtained by plotting right hand side of equation (9.19) against $\theta$ and getting value of $\theta$ from intercept of $K$ with the plotted curve.

9.7 RESULTS AND DISCUSSIONS:

We calculate spatial variation of peak power across the laser beam of 5106 A copper vapour laser assuming that the initial inversion density is constant throughout the laser medium. The length of the laser is taken to be about 100 cm and the diameter is taken to be 1 cm (we are taking the above mentioned values because our laser has these parameters). The plane of observation is assumed to be located at a distance of 100 cms from the exit end of the laser. The absorption cross section $\kappa_0$ for the copper atoms at wavelength
5106 Å is about 0.393 cm\(^{-1}\). If the laser discharge conditions are optimised, the initial inversion density of about 0.25 can be obtained. Using equation (9.15), we have calculated peak power for different X along the diameter of the discharge tube and the results are plotted in the figures 9.2 and 9.3 for the initial inversion densities of 0.2 and 0.4 respectively. The figures show that if the value of X is increased the peak power increases very slightly till the point comes to a position for which \(X = Ld / L\). For the values of X more than \(Ld / L\) the peak power of the laser radiation go on decreasing as the value of X is increased. Ultimately the peak power goes to zero for a certain value of X.

Laser power across laser beam is nearly constant between points B and C in figures 9.2 and 9.3, if the inversion density is uniform across the laser medium. As the observation point P shifts from B to D, (i.e., from \(X=0\) to \(X=Ld / L\)) the beam intensity increases very slowly. Beyond D the beam power goes on decreasing as X increases and it becomes zero for \(X = X_0 = Ld \alpha(0n(t)/2\) or for a beam travelling in the direction making an angle \(\Theta_0 = d \alpha(0n(t)/2\) with the direction of the nearest edge of the laser tube. It is obvious that if \(\Theta_0\), \(d\) and \(\alpha(0\) are known, \(n(t)\) may be deduced using above relation. Hence the value of initial inversion density may be obtained by measuring angle \(\Theta_0\).
inversion experienced by photon flux travelling parallel to laser axis and the photon flux travelling in oblique direction is the same. The energy stored in terms of inversion density is shared by two fluxes. Hence the inversion density obtained by this method may be slightly lower than the actual value of the inversion density.

From the calculations of peak power across the laser beam desired angle of divergence may be obtained. The half peak power angles of divergence for n(ti) = 0.2 and 0.4 are respectively 20 mrad and 30 mrad. The TE field pumped CVL may have this angle of divergence. In this case also the leading part of the output laser beam has large angle of divergence and the lagging part of the output laser beam has less angle of divergence. In case of the AR field pumped CVL the angle of divergence would be much less than the above mentioned figures. This is because the laser beam makes many passes before coming out of the laser cavity.

We also calculate the half intensity angle of divergence using equation 9.19 and display the results in the figure 9.4.

The results of the experimental measurements (21) of the angle of divergence may be compared with the computations carried out in the present work. The comparison shows good agreement between the theoretical calculations and the experimental results.

In the results presented in this chapter it is
assumed that the inversion density in the laser medium is constant across the discharge tube. However, in last chapter it has been shown that the electron temperature is not constant across the discharge tube and the population density of the upper and lower laser states are different. This leads to the variation in the inversion density across the laser tube. The study of angle of divergence now becomes crucial. As long as the beam is not annular the calculation of the angle of divergence is easy. But as soon as the beam becomes annular the calculation of angle becomes complicated. As it has been mentioned in other chapters the laser discharge consists of thin hollow cylindrical shells of constant inversion densities figure 7.1. This indicates that a part of the cylindrical shell parallel to the laser axis treated as segment of discharge and the angle of divergence may be calculated. The divergence of the laser beam emitted by this segment would have less angle of divergence because the plasma has less thickness.
REFERENCES:

1) S Gabay, I Smilansky and Z Karny,
2) J J Kim and K Im,
3) W A Fitzsimmons, L W Anderson, C E Reidhauser,
   and Jan M Vertilek,
4) T Kasuya, and D R Lide,
5) G R Fowels and A W T Silfvast,
6) K G Hernqvist, and D C Pultorak,
7) M Mori, M Murayama, T Goto, and S Hattori,
8) P Gill, and C E Webb,
9) S N Thakur,
10) B H Pawar, S P Bhandari, and L V Thakre,
11) D B McDonald, and C D Jonah,
12) T W Alger, W L Bennett,
13) D A Leonard, R A Neal, and E T Gerry,

14) W W Smith, and A Gallagher,

15) G R Fowles, and W T Silfvast,

16) W T Silfvast,

17) A Kh Manatsakanyan, G V Naidis, and N P Shternov,

18) W G Wanger, and B A Lengyl,

19) P Richter, J D Kimal, and G C Molton,

20) J D Shipman,
Figure 9.1 Cross section of laser plasma and beam passing through laser axis.
Figure 9.2 Peak power $P$ across the laser beam for $n(t_1) = 0.2$. 
Figure 9.3 Peak power $P$ across the laser beam for $n(t_1) = 0.4$. 
Figure 9.4 Half peak power angle of divergence $\theta_{1/2}$ as a function of $n(t_1)$. 