Chapter - 6
INTRODUCTION:

Queueing theory has been recognized as one of the most powerful mathematical tools for making quantitative analysis of communication network. In the early 1960s it was realized that queueing theory would prove to be an effective tool for studying the throughput, response time, and other measures of performance of the network. Since that time, the literature of queueing theory and network applications have virtually exploded with analytical models for different networks. Such as for the telephone system the data transmission rates were not sufficient to meet the demand, consequently, planning for special data network began earlier in Europe then it did in United States. Since there was not yet a great demand for data services, planning was directed towards to meet the future requirement. The planners of some European nation were in many ways more ambitious than those announced by AT & T in United States, whose digital network were expected to be un-switched. The stop and wait, and continuous error detection and re-transmission schemes are widely used in such network. The researchers primary focus was the development of an appropriate model for analyzing the delays encountered in establishing a virtual circuit through a switch based LAN.

The method developed were applied to many related problems i.e. multiple link, telephone calls, receiver transmitter interaction between
switches and shared peripherals in networks. One of such approaches developed by Wilhelm, Neil C. [145] to analyze a disk subsystem in which all of the disks had the same load. Whitt, W. studied a similar method applied to markovian queueing networks as a way of extending the approximations developed by Whitt, W. Kelly, F.P. considered an approximation technique for a general class of circuit switched tele traffic models.

In 1986, Fredericks, A.A. [60] developed an approximation methods for analyzing the performance of a virtual circuit switch based LAN. The basic system consists of N input and N output parts, each with it band width divided into L equal segments for the virtual circuit from a given input to output parts, it is necessary to obtain a time, slot in each part. The author suggested a simple analytical approximation methods for the delay encountered in setting up such a circuit. Kumar, A. Singh, M.P. and Kumar V. [100] also suggested a methods to the M/M/1 bulk service queue model for the analysis of mean response time of the communication network. Graph theory plays an important role in modeling and analyzing several problem concerns with various aspects of networking field. Also graph theoretic modeling find extensive applications in queueing theory for the networking problems. In this chapter we consider a bridge network, consists of four systems namely S₁, S₂, S₃ and S₄, which are inter connected by communication links. In this network we shall have to decided the path sets while a path in a multi-graph G consists of an alternating sequence of vertices and edges of the form v₀, e₁, v₁, e₂, v₂, ...........eₙ₋₁, vₙ₋₁, eₙ, vₙ where each
Edge e:\ corresponds to the vertices v_{e1} and v_{e2} (which appear on the sides of e\ in the sequence). The number of edges (n) is called length of the path. When there is no ambiguity. We denote a path by its sequence of vertices (v_0, v_1, ..., v_n). The path is said to be closed if v_0 = v_n otherwise we say that the path is from v_0 to v_n or between v_0 and v_n or connects v_0 to v_n.

A simple path is a path in which all vertices are distinct (A path in which all edges are distinct will be called a trail). A cycle is a closed path in which all vertices are distinct except v_0 = v_n. A cycle of length k is called a k-cycle. Here we assume that a network having “n” heterogeneous system, N = \{S_1, S_2, ..., S_n\}, which are interconnected by communication links. One of the “n” system is called a source system and one of the remaining “n-1” system is labeled as terminal system. At the source system, the jobs are arriving in the form of data packets and follows the poison process. The queueing model used here in the bulk arrivals (M^X/M/1) type. If the source system is busy, then these jobs are store in buffer and processed latter. The job propagates over the link from source towards terminals system till they reach to terminal system. Once a job reaches to terminals system, the terminals system sends an acknowledgment of correct processing of the job to the source system the aspects of the model is the service mechanism. Each arriving data packet of jobs has variable number of modules, which arrive in bulk and is then served by the system of the network. In this model, we have obtained the mean response time. For mean response time analysis we have listed the path sets of the
networks. The tasks follow any one path from processing for source to terminals system depending upon the allocation of path by the source system and it is assumed that arrivals of tasks occurs to as an ordinary poison process with mean \( \lambda \), and then they are served on the basis of first come first serve discipline. Each job of data packets has variable number of modules and their arrivals \( X \) at a time except when less then \( X \) is in the data packet. The amount of time the required for the service of any job is an exponentially distributed random variable.

In this chapter, a model for mean response time is analyzed, which is based on graph and queueing theories for making quantitative analysis in networking. Then the all the path sets of network have been listed and the \( M^\infty/M/1 \) bulk arrival queueing model is then applied for determining the mean response time of each path set.

6.1 THE PROPOSED METHODS:

Let us consider a bridge, network, consist of four system namely, \( S_1, S_2, S_3 \) and \( S_4 \), which are inter connected by communication.
6.3 MEAN WAITING TIME :

The Kolmogrov equation are given below:

\[ P^1_n(t) = -(\lambda + \mu) P_n(t) + \mu P_{n-1}(t) + \lambda \sum_{k=1}^{n} P_{n-k}(t) C_k \quad (n \geq 1) \]

(6.3.1)

\[ P^1_n(t) = -\lambda P_0(t) + \mu P_1(t) \]

(6.3.2)

As \( t \to \infty \) then the steady state differential difference equations are:

\[ 0 = -(\lambda + \mu) P_n + \mu P_{n-1} + \lambda \sum_{k=1}^{n} P_{n-k} C_k \quad (n \geq 1) \]

(6.3.3)

\[ 0 = -\lambda P_0 + \mu P_1 \]

(6.3.4)

Solving the equation with the help of generating function

\[ P(z) = \sum_{n=0}^{\infty} P_n z^n \quad (|z| \leq 1) \]

(6.3.5)
\[ C(z) = \sum_{n=0}^{\infty} C_n z^n = \sum_{n=1}^{\infty} C_n z^n (I_z I \leq 1) \quad (6.3.6) \]

Then using from equation (6.3.3)

\[ 0 = -\lambda \sum_{n=0}^{\infty} P_n z^n - \mu \sum_{n=1}^{\infty} P_n z^n + \frac{\mu}{z} \sum_{n=0}^{\infty} P_{n-1} z^{n-1} + \lambda \sum_{n=1}^{\infty} \sum_{k=1}^{n} P_{n-k} C_k z^n \quad (6.3.7) \]

Here we see that \( \sum_{k=1}^{n} P_{n-k} C_k \) is the probability function for the sum of

the steady state system size and batch size \( \sum_{n=1}^{\infty} \sum_{k=1}^{n} P_{n-k} C_k z^n = \sum_{k=1}^{n} C_k \)

\[ z^k \sum_{n=1}^{\infty} P_{n-k} z^{n-k} = C(z) P(z) \]

Then equation (6.3.7) we get

\[ 0 = -\lambda P(z) - \mu [P(z) - P_0] + \frac{\mu}{z} [P(z) - P_0] + \lambda C(z) P(z) \]

\[ P(z) [\lambda + \mu - \lambda C(z)] = \mu P_0 - \frac{\mu}{z} P_0 \]

\[ P(z) \left[ \frac{(\lambda + \mu)z - \mu - \lambda_z C(z)}{z} \right] = \mu P_0 \left( 1 - \frac{1}{z} \right) \]

\[ P(z) = \left[ \frac{\mu P_0 (z-1)}{(\lambda + \mu)z - \mu - \lambda_z C(z)} \right] \]

\[ P(z) = \left[ \frac{\mu P_0 (z-1)}{\mu (z-1) - \lambda_z (C(z)-1)} \right] \]

\[ P(z) = \left[ \frac{-\mu P_0 (z-1)}{\mu (1-z) - \lambda_z (1-C(z))} \right] \quad (|z| \geq 1) \quad (6.3.8) \]

and using the condition \( P(1) = 1 \) we get
Since numerator and denominator are both zero. So we use L’ Hospital rule

\[ X = \lim_{z \to 1} P(z) = \lim_{z \to 1} \left\{ \frac{-\mu P_0}{-\mu + \lambda E(X = \text{batch size})} \right\} \]

\[ = \frac{-\mu P_0}{-\mu + \lambda E(X)} \]

\[ 1 = \frac{-\mu P_0}{-\mu + \lambda E(X)} \]

\[ -\mu P_0 = -\mu + \lambda E(X) \]

\[ P_0 = \frac{-\mu + \lambda E(X)}{-\mu} \]

\[ P_0 = 1 - \frac{\lambda E(X)}{\mu} \]

\[ P_0 = 1 - \rho \]

Where \( \rho = \frac{\lambda E(X)}{\mu} \) (\( \rho < 1 \)) \hspace{1cm} (6.3.9)

Now we consider the batch size \( X \) is geometrically distributed i.e.

\[ C_x = (1-\alpha) \alpha^{x-1} \quad (0 \leq \alpha < 1) \hspace{1cm} (6.3.10) \]

Then using (6.3.4) we have

\[ C(z) = (1-\alpha) \sum_{n=1}^{\infty} \alpha^{n-1} z^n, \quad (|z| \leq 1) \hspace{1cm} (6.3.11) \]

\[ C(z) = (1-\alpha) \frac{z}{1-\alpha z} \]
From equation (6.3.8)

\[ P(z) = \frac{\mu P_0 (1-z)}{\mu (1-z) - \lambda z (1-C'(z))} \]

\[ P(z) = \frac{\mu P_0 (1-z)}{\mu (1-z) - \lambda z \left[ 1 - \frac{z(1-\alpha)}{1-\alpha z} \right]} = \frac{\mu (1-\rho)(1-z)}{\mu (1-z) - \lambda z \left[ 1 - \frac{z(1-\alpha)}{1-\alpha z} \right]} \]

\[ \frac{\mu (1-\rho)(1-\alpha z)}{\mu (1-\alpha z) \lambda z} = \frac{(1-\rho)(1-\alpha z)}{1-\alpha + (1-\alpha)\rho} \]

\[ P(z) = (1-\rho) \left\{ \frac{1}{1-\alpha + (1-\alpha)\rho} - \frac{\alpha z}{1-\alpha + (1-\alpha)\rho} \right\} \]

\[ P(z) = (1-\rho) \left\{ \sum_{n=0}^{\infty} [(\alpha + (1-\alpha)\rho)z]^{n} - \sum_{n=0}^{\infty} [\alpha(\alpha + (1-\alpha)\rho)]^{n-1} z^{n+1} \right\} \]

So that,

\[ P_n = (1-\rho) \left\{ \alpha + (1-\alpha)\rho \right\}^{n} - \alpha \left\{ \alpha + (1-\alpha)\rho \right\}^{n-1} \]

\[ P_n = (1-\rho)[\alpha + (1-\alpha)\rho]^{n-1}(1-\alpha)\rho \quad (n \geq 0) \quad (6.3.13) \]

**6.4 LENGTH QUEUE:**

The line length of queue (L_s) of the system is given by

\[ L_s = \sum_{n=1}^{\infty} n P_n \quad (6.4.1) \]

\[ = \sum_{n=1}^{\infty} n(1-\rho)[\alpha + (1-\alpha)\rho]^{n-1}(1-\alpha)\rho \]

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the waiting time of data packet of jobs in system for each path is as

\[ W_s(i, j) = \frac{1}{\lambda} \sum_{j=1}^{N-1} L_s(i, j) \quad i = 1, 2, \ldots \text{ PS} \] (6.4.3)

Therefore Mean waiting time of data packet of the job, is defined as

\[ T_w(i) = \sum_{j=1}^{N-1} W_s(i, j), \quad i = 1, 2 \ldots \text{ PS} \] (6.4.4)

6.5 MEAN SERVICE TIME:

Service of the system, which bulkily transmits the job from one system to another, depends upon a batch of size \( K \), which is exponentially distributed with the mean \( \mu_s(i, j) \). Therefore the mean service time for each path, out going from the transmitting systems given as:

\[ T_s(i) = \sum_{j=1}^{N-1} \left( \frac{1}{\mu_s(i, j)} \right), \quad i = 1, \ldots \text{ PS} \] (6.5.1)

6.6 MEAN RECEIVING TIME:

Similarly, for the service of the system, whose receives the job, it is assumed that the receiving time of this system with the mean, \( \mu_R(i, j) \) obeys exponential law and is as,

\[ T_r(i) = \sum_{j=1}^{N-1} \left( \frac{1}{\mu_R(i, j)} \right), \quad i = 1, \ldots \text{ PS} \] (6.6.1)
6.7 MEAN RESPONSE TIME:

Mean Response Time ($T_{MRT}$) is defined as the sum of the mean Waiting Time and Mean Service Time of processors, which transmit, plus the Mean Receiving Time of the processor, which receives, which receives the jobs i.e.

$$T_{MRT}(i) = [T_W(i) + T_S(i) + T_R(i)]$$

(6.7.1)

Where $T_W(i)$, $T_S(i)$ and $T_R(i)$ are given in the equation (6.4.4.), (6.4.1), (6.5.1), respectively

6.8 Computation of Numerical Results:

For the computation of numerical results, Mean Response Time has been obtained for different values of $x$ i.e. different data packet size, which are shown through table 1 to 4. Let us assume the service rate for $S_1$, $S_2$, $S_3$, $S_4$ are 1.5, 2.0, 2.5 and 2.5 respectively and receiving rate for $S_1$, $S_2$, $S_3$, $S_4$ are 0.5, 1.0, 1.5 and 2.0 respectively. For the various path sets of the network the different values of batch size $x$ and $\lambda$ are 1, 2, 3 E $(x)$ are 0.5, 0.7, 0.8, and 0.9, 0.5, 0.7 for $\alpha$. The Mean Waiting Time for each has been obtained. It is given in the following table-1.

<table>
<thead>
<tr>
<th>$T_W(i)$</th>
<th>Mean Waiting Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X = 1$</td>
</tr>
<tr>
<td>$T_W(1)$</td>
<td>0.0234</td>
</tr>
<tr>
<td>$T_W(2)$</td>
<td>0.0182</td>
</tr>
<tr>
<td>$T_W(3)$</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

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given in the following table – 2:

<table>
<thead>
<tr>
<th>$T_S$ (i)</th>
<th>Mean Service Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_S$ (1)</td>
<td>1.0666</td>
</tr>
<tr>
<td>$T_S$ (2)</td>
<td>1.5666</td>
</tr>
<tr>
<td>$T_S$ (3)</td>
<td>1.4666</td>
</tr>
</tbody>
</table>

The Mean Receiving Time for each have been calculated. It is given in the following table – 3:

<table>
<thead>
<tr>
<th>$T_R$ (i)</th>
<th>Mean Receiving Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_R$ (1)</td>
<td>2.6666</td>
</tr>
<tr>
<td>$T_R$ (2)</td>
<td>3.6666</td>
</tr>
<tr>
<td>$T_R$ (3)</td>
<td>3.1666</td>
</tr>
</tbody>
</table>

Finally the Mean Response Time, for the different values of batch size $x$ and $\lambda$ i.e., 1,2,3 for each path have been obtained. It is shown in the following table-4

<table>
<thead>
<tr>
<th>$T_R$ (i)</th>
<th>Mean Receiving Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_R$ (1)</td>
<td>2.6666</td>
</tr>
<tr>
<td>$T_R$ (2)</td>
<td>3.6666</td>
</tr>
<tr>
<td>$T_R$ (3)</td>
<td>3.1666</td>
</tr>
<tr>
<td>$T_{MRT}(i)$</td>
<td>Mean Response Time</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>$T_{MRT}(1)$</td>
<td>3.7566 3.8103 3.7563</td>
</tr>
<tr>
<td>$T_{MRT}(2)$</td>
<td>5.2514 5.2690 5.2418</td>
</tr>
<tr>
<td>$T_{MRT}(3)$</td>
<td>4.6397 4.6393 4.6382</td>
</tr>
</tbody>
</table>

6.9 DISCUSSION:

The mean response time $M^N/M/1$ bulk arrival queue model to the various path sets for the bridge network as shown in figure-1, has been obtained for the different value of batch size and arrival rate i.e. 1, 2, 3, respectively are shown in Table-4. Graphical representation of the batch size/arrivals rate vs mean response time has been shown in graph-1. It can be stated, that the mean response time is lesser for the path set-1. Further for the path set-2, the mean response time is higher as the service rate is poor. In the case the path set-3, it is between the path set-1 and path set-2, shows that the mean response time decrease with the increase service rate. The model discuss in this chapter, would be useful to the network system designer working in the field of networking. The decision of routing strategy in the design of network may also be facilitated by the present.