CHAPTER  -  5

Supergravity induced $E_8$ gauge hierarchies.
5.1. Introduction:

Inspired by the superstring phenomenology, here we study the hierarchical symmetry breaking of $N = 1$ local supersymmetry$^1$ coupled to $E_8$ gauge symmetry. Since local supersymmetry automatically includes gravity, these type of theories can be regarded as a possible candidate of unifying gravity with strong and electroweak interactions. In this model, we know that there are several intermediate symmetry breaking scales and the initial stage of breaking occurs at a scale of $10^{18-19}$ GeV. The model is constructed to lead to a breaking pattern of $E_8 \times \text{SUSY} \rightarrow E_6 \times \text{SUSY} \rightarrow SO(10) \times \text{SUSY} \rightarrow SU(5) \times \text{SUSY} \rightarrow G_{\text{std}} \times \text{SUSY} \rightarrow G_{\text{LE}}$. Here $G_{\text{std}} = SU(3)_C \otimes U(1)_Y \otimes SU(2)_L$ and $G_{\text{LE}} = SU(3)_C \otimes U(1)_Q$. Here $E_8$ breaks to $E_6$ gauge group in the zeroth order of supergravity at a scale of $10^{18}$ GeV and subsequent symmetry breakings are generated as the next four higher orders of supergravity corrections.

5.2. $E_8$ invariant superpotential, representations and symmetry breaking patterns:

The possibility of $E_8$ as a grand unified gauge group has been discussed before.$^2$ Here we want to study the local SUSY grand unification based on the largest exceptional group $E_8$. The fundamental fermions belong to the adjoint representation $248$ of $E_8$. The $E_8$ invariant superpotential can be
written as,

$$g = \lambda \cdot 248 \otimes 248 \otimes 248 + M248 \otimes 248$$  \hspace{1cm} (5.1)$$

While breaking up into other gauge groups constituting the symmetry breaking patterns, this gives rise to a superpotential,

$$g_4 = \lambda_4 \left( \frac{1}{2} M_{\text{Planck}}^2 - \frac{1}{3} P^3 \right) + \chi \left( \lambda_2 k^2 m^2 p^2 + \lambda_3 d^2 \right)$$

$$+ M k^6 m^6 q^2 + \lambda_4 Y k^4 m^4 R \overline{R} + \lambda_5 k^2 m^2 \nu R \overline{R}$$

$$+ M k^4 m^4 s^2 + \lambda_6 \overline{U} H H' + \lambda_7 k^2 m^4 z + M k^4 m^4 R \overline{R}$$

$$+ M k^4 m^4 H H' + \lambda_8 k^5 m^5 R \overline{R} S + \lambda_9 k^2 m^2 \nu S^2$$

$$+ \lambda_{10} k^3 m^3 H H' S + \lambda_{11} k^4 m^4 R^2 H + \lambda_{12} k^4 m^4 R^2 H' + B$$

\hspace{1cm} (5.2)$$

Here $\kappa = (8 \pi G)^{1/2} = 4.1 \times 10^{-19} \text{ GeV}^{-1}$ is the gravitational coupling constant. Under SU(3) $\otimes$ E\(_6\) decomposition, the 248 adjoint representation of E\(_6\) breaks as,

$$248 = (8, 1) + (1, 78) + (3, 27) + (\overline{3}, \overline{27})$$  \hspace{1cm} (5.2')$$

E\(_6\) has fundamental representation 27 and the adjoint repre-
sentation is 78. Under an SO(10) x U(1) decomposition, these representations break as,

\[ 27 = (1,4) + (10,-2) + (16,1) \]  

\[ 78 = (1,0) + (45,0) + 16,-3) + 16,3) \]

In equation (5.2), \( P \) has quantum numbers (8,1) under SU(3) x E_6 and it belongs to 248 representation of E_8. The nonzero vacuum expectation values of \( P \) breaks E_8 to E_6. \( Q \) has quantum numbers (1,4) under SO(10) x U(1), whose nonzero VEV breaks E_6 to SO(10). \( R \) belongs to the 16 representation of SO(10) and the \( \nu^L \) component having nonzero VEV breaks SO(10) to SU(5). In order to make the D-term vanish, we also take the 16 representation (\( \bar{R} \)), for which the component \( \nu^L \) gets the same VEV as \( \nu^L \). Here \((R \bar{R})_{i \cdots k} = R^T \gamma_i \cdots \gamma_k \bar{R}, i, j = 1, \ldots, 10\) are SO(10) tensor indices, \( \gamma^\alpha \) are the generalized Dirac matrices and \( C \) is the generalized charge conjugation matrix.

S, the 45 representation of SO(10) is used to break SU(5) to \( G_{\text{std}} \). The component (1,1,0) under SU(2)_L x SU(3)_C x U(1)_Y having nonzero VEV is responsible for this symmetry breaking. Finally \( H, H' \) belonging to the 10 representation of SO(10), along with a singlet super field \( U \) break \( G_{\text{std}} \) to \( G_{\text{LE}} \). The (2,1,1) component of 10 gets nonzero VEV. X and Y are two singlet superfields having zero VEV. \( V \) is also a singlet superfield to which we give a nonzero VEV. \( Z \) is a Polonyi
A singlet which belongs to the hidden sector and it is responsible for the spontaneous breakdown of supergravity. Quarks, leptons and Higgs superfields belong to the observable sector. Since these two sectors only communicate through the gravitational interactions, the fields in the hidden sector are singlets with respect to all gauge interactions in the observable sector. B is an additive constant, which can be finetuned to make the cosmological constant vanish.

The scalar potential is given by,

$$V = \frac{1}{2} E \left( |g, \phi|^2 + \frac{1}{2} k^2 \phi' \phi^\dagger \right) - \frac{3}{2} k^2 |g|^2 + \frac{1}{2} |D\phi|^2, \quad (5.3)$$

where $E = \exp \left[ \frac{k^2}{2} |\phi'|^2 \right]$ and summation over all the fields $\phi'$ is understood. We shall take in the present framework that the D term vanishes, so that Fayet-Illiopolos mechanism of symmetry breaking cannot be considered. Since we want to express the superpotential and scalar potential in terms of dimensionless quantities, we introduce a small hierarchy parameter $\mathcal{B} = k m$, which is of the order of $10^{-4}$. Here $k^{-1} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}$ GeV, which is the gravitational coupling constant and $m$ is such that $k^3 m^4 \approx 300$ GeV. $\mathcal{B} = k m$ gives mass scale, at which $E_8$ breaks to $E_6$. We shall expand the VEVs of various fields in terms of the small hierarchy parameter $\mathcal{B}$ and shall try to obtain the VEVs through an extremization procedure of the scalar potential. The various
Symmetries which break at different scales can be given by the chain,

\[ E_8 \times \text{SUSY} \xrightarrow{M} E_6 \times \text{SUSY} \xrightarrow{3^2 M} \text{SO}(10) \times \text{SUSY} \xrightarrow{3^3 M} \text{SU}(5) \times \text{SUSY} \xrightarrow{3^4 M} \]

\[ G_{\text{Std.}} \times \text{SUSY} \xrightarrow{3^4 M} G_{\text{LE}}. \]

(5.4)

Here supersymmetry and weak symmetry break at the scale \( 3^4 M \), which is of the order of \( 10^2 \) GeV.

After introducing the hierarchy parameter, the dimensionless superpotential can be written as,

\[
\tilde{W} = k^3 g = \lambda_1 \left( \frac{1}{2} \chi_0 \tilde{P}^2 - \frac{4}{3} \tilde{P} \tilde{P}^2 \right) + \tilde{X} \left( \lambda_2 \tilde{\phi}^2 \right) + \lambda_3 \tilde{A}^2 + \chi_0 \tilde{A}^2 + \lambda_4 \tilde{Y} \tilde{R} \tilde{R} + \\
\lambda_5 \tilde{\chi} \tilde{\chi} \tilde{R} \tilde{R} + \chi_0 \tilde{\chi} \tilde{\chi} \tilde{S}^2 + \lambda_6 \tilde{H} \tilde{H} + \lambda_{7} \tilde{\tilde{H}} \tilde{\tilde{H}} + \lambda_{8} \tilde{\tilde{Y}} \tilde{\tilde{S}}^2 + \lambda_{9} \tilde{Y} \tilde{S}^2 + \lambda_{10} \tilde{H} \tilde{H} \tilde{S} + \lambda_{11} \tilde{R} \tilde{R} \tilde{S} + \lambda_{12} \tilde{Y} \tilde{S} \tilde{S} \tilde{S} + B, \]

(5.5)
We also have $\tilde{\nu} = k^4 \nu$ and $\tilde{\phi} = k \phi$.

5.3. **Minimization of scalar potential:**

The minimization equation for $\tilde{\nu}$ in an arbitrary channel is given by,

$$\tilde{\nu}, \tilde{\phi} = \{ \tilde{g}, \tilde{\phi}^\dagger, \tilde{g}, \tilde{\phi}, \tilde{\phi}^\dagger + \frac{1}{2} [ (\tilde{g}^\dagger (\tilde{\phi}', \tilde{g}, \tilde{\phi}', \tilde{\phi}^\dagger - 2 \tilde{g}, \tilde{\phi}^\dagger ) + (\tilde{g}, \tilde{\phi}', \tilde{\phi}^\dagger ) \tilde{g}, \tilde{\phi}^\dagger ] + \frac{1}{q} [ (\tilde{\phi}', \tilde{\phi}^\dagger )^\dagger \tilde{g}^\dagger \tilde{g}, \tilde{\phi}^\dagger ] \} \tilde{\nu} + \frac{1}{2} \tilde{\nu} \tilde{\phi}^\dagger = 0. \tag{5.6}$$

In equation (5.6), there is a summation over dummy field variables $\tilde{\phi}'$ and the comma denotes partial differentiation.

Considering minimization equation in this fashion makes the algebra a lot simpler. Equation (5.6) is the basic working formula which will be solved in different channels in different orders of $q$. We also take the scalar potential $\tilde{\nu}$ to be equal to zero, so that the cosmological constant vanishes.

In fact we have the equation,

$$\tilde{\nu} = |\tilde{g}, \tilde{\phi}'|^2 + \frac{1}{2} [ (\tilde{g}, \tilde{\phi}', \tilde{\phi}' ) \tilde{g}^\dagger + h.c.]$$

$$+ \frac{1}{4} |\tilde{g}|^2 |\tilde{\phi}'|^2 - \frac{3}{2} |\tilde{g}|^2 = 0. \tag{5.7}$$
We now consider equation (5.6) in \( \tilde{p} \) channel. In the leading order (i.e., order \( \tilde{\alpha}^0 \)) this gives,

\[
x_0 \tilde{p} - \tilde{p}^2 = 0,
\]

which has a straightforward solution,

\[
\langle \tilde{p} \rangle = x_0,
\]

which breaks \( E_8 \) to \( E_6 \).

5.4. **Hierarchy structure:**

We consider the supergravity corrections of various fields in terms of the expansion parameter \( \tilde{\alpha} \). We take the expansion such that

\[
\langle \tilde{p} \rangle = x_0 + x_4 \tilde{\alpha}^4 + x_8 \tilde{\alpha}^8 + \cdots
\]

\[
\langle \tilde{a} \rangle = \tilde{\alpha} y_1 + \tilde{\alpha}^5 y_5 + \cdots
\]

\[
\langle \tilde{r} \rangle = \langle \tilde{\kappa} \rangle = \tilde{\alpha}^2 \gamma_2 + \tilde{\alpha}^6 \gamma_6 + \cdots
\]
Here we have expanded in increasing powers of $\beta^4$. The constant term $B$ and the superpotential $\tilde{g}$ can also be expanded as,

$$B = B_0 + B_4 \beta^4 + \ldots$$

(5.20)

$$\tilde{g} = \tilde{g}_4 + \tilde{g}_8 + \ldots$$

(5.21)

so that gravitino has a mass of the order of $\beta^4 M \approx 10^2\text{GeV}$.
The zeroth order expansion $B_0$ is obtained by substituting the value of $\langle \tilde{F} \rangle = \chi_0$. In fact the cosmological constant vanishes in the leading order, if we choose,

$$B_0 = -\frac{1}{6} \lambda_1 \chi_0^3 .$$

Then comparing terms of the order of $\tilde{F}^4$ and $\tilde{F}^8$ from the superpotential, we get,

$$g_{\tilde{y}} = \lambda_7 z_0 + B_4 ,$$

(5.23)

$$g_{\tilde{g}} = \lambda_1 \left( -\frac{1}{2} \chi_0 \chi_4^2 \right) + \chi_0^2 + \lambda_5 z_2^2 + \lambda_7 z_4^2 + \chi_0 \chi_4^2 .$$

(5.24)

The minimization equation in order $\tilde{F}^4$, in $\tilde{F}$ channel gives,

$$\chi_4 = g_{\tilde{y}} / 2 \lambda_1 .$$

(5.25)

Then the minimization equation in $\tilde{Z}$ channel and $\tilde{V} = 0$ condition in lowest order $\tilde{F}^8$ give two simultaneous equations in $g_{\tilde{y}}$,

$$- g_{\tilde{y}} + \frac{1}{2} \lambda_7 z_0 + \frac{g_{\tilde{y}}}{4} \left( \chi_0^2 + z_0^2 \right) + \frac{1}{4} \frac{z_0 g_{\tilde{y}}}{\lambda_7}$$

$$+ \frac{\lambda_1}{2} \left( -\chi_0 \chi_4 \right) = 0 ,$$

(5.26)
\[ \lambda_1^2 x_0 x_4^2 - \lambda_1 x_0^2 g_q x_4 + \frac{4}{4} g_q^2 (x_0^2 + z_0^2) + \lambda_7 (\lambda_7 + g_q z_0) - \frac{3}{4} g_q^2 = 0. \]  

(5.27)

Substituting the value of \( x_4 = g_q / 2 \lambda_1 \) in equations (5.26) and (5.27), we get,

\[ -g_q + \frac{1}{4} g_q^2 \left( \frac{z_0}{\lambda_7} \right) + \frac{\lambda_7}{2} z_0 + \frac{1}{4} g_q z_0^2 = 0, \]  

(5.28)

\[ \lambda_7^2 + \lambda_7 g_q z_0 + \frac{1}{4} g_q^2 z_0 - \frac{3}{2} g_q^2 = 0. \]  

(5.29)

Solving these two equations, we get\(^9\),

\[ z_0^2 = \left[ 8 \pm 4\sqrt{3} \right], \]  

(5.30)

\[ g_q / \lambda_7 = z_0 / (1 - \frac{1}{4} z_0^2). \]  

(5.31)

which implies that \( \lambda_7 \) is not an independent parameter.

Then the equation \( \tilde{\nu} = 0 \) in next higher order of \( \frac{1}{3} \) gives,
\[ 2\lambda_1 \chi_0 \chi^3 + (-2\lambda_1 \chi_0 \theta_1) \chi^2 + \left[ -\lambda_1 \chi_0 \theta_8 + \frac{1}{2} \chi_0 \theta_1 \right] \chi + \left[ \lambda_1 \chi_1 \theta_4 \theta_8 + \chi_0 \theta_1 \theta_4 \theta_8 + \frac{1}{4} \theta_4 \theta_8 \left( \chi_0^2 + \chi_0^2 \right) + \frac{1}{2} \theta_4 \left( \chi_0^2 + \frac{1}{2} \chi_0^2 + \frac{1}{2} \chi_0 \theta_2 \right) \right] = 0. \] 

We now substituting the value of \( g_4 \) and \( g_8 \) from equations (5.23) and (5.24) in equation (5.32) and get a cubic equation in \( x_4 \),

\[ C_1 \chi^3 + C_2 \chi^2 + C_3 \chi + C_4 = 0, \] 

where,

\[ C_1 = \left[ 3 \lambda_1 \chi_0 + \frac{4}{3} \lambda_1 \chi_0 - \frac{1}{3} \lambda_1 \chi_0 \chi_0 \chi_0 \right], \]

\[ C_2 = \left[ -\frac{1}{2} \lambda_1 \chi_0 \chi_0 + \lambda_1 \chi_0 \chi_0 + \lambda_1 \chi_0 \chi_0 \right]. \]
\[ c_3 = \left[ \lambda_1 y_4^2 \left\{ -\frac{1}{2} x_0^3 + \frac{4}{2} x_0 z_0^2 - 4x_0 \right\} + \lambda_1 \lambda_7 \left\{ -\frac{1}{2} x_0^2 z_y - 4z_y + \frac{4}{2} z_0 z_y \right\} + \lambda_1 \lambda_5 \right] \]

\[ \left\{ -\frac{1}{2} x_0^2 \varphi_2 \gamma_2^2 + \frac{4}{2} z_0^2 \varphi_2 \gamma_2^2 \right\} + \lambda_4 \left\{ -\frac{1}{2} x_0^3 \gamma_2^2 \right\} + \frac{4}{2} z_0^2 x_0 \gamma_2^2 - 2x_0 \gamma_2^2 \right\} \]

\[ c_4 = \left[ y_4^2 z_0 x_0 + \lambda_7 z_0 z_y + \lambda_5 \varphi_2 \gamma_2 z_0 + x_0 \varphi_2 \gamma_2 \right]. \quad (5.34) \]

Equation (5.33) can be solved to get the value of \( x_4 \).

Now equation (5.34) contains four unknown namely, \( y_4, z_4, \varphi_2 \) and \( \gamma_2 \). But the value of \( y_4 \) can be determined from lowest order \( x \)-channel equation, which is given by,

\[ y_4^2 = -\frac{\lambda_2}{\lambda_3} x_0^2. \quad (5.35) \]

Still there are three unknown parameters \( z_4, \varphi_2 \) and \( \gamma_2 \).

Now considering the minimization equation in \( \gamma \)-channel in next higher order i.e. \( O \left( \frac{\mathcal{G}}{12} \right) \), we get
\[- \lambda_7 \gamma_8 + \frac{1}{2} \lambda_7 \left[ -2 \gamma_0 \gamma_3^2 - \gamma_0^2 \gamma_8 + \gamma_1^2 + \lambda_7 \gamma_4 \right. \\
+ 3 \lambda_5 \gamma_5 \gamma_2 + 2 \gamma_0 \gamma_5 \gamma_2^2 \left. \right] + \frac{1}{4} \left( \gamma_0 \gamma_3 + \gamma_0 \gamma_2 \right) \gamma_4 \\gamma_8 + \left( \gamma_2^2 + \gamma_3^2 \right) \gamma_4 \gamma_8 \right] + \\
2 \gamma_0 \gamma_4 \gamma_8 + \gamma_4 \gamma_8^2 = 0 \quad (5.36)\]

Equation (5.36) contains four unknowns namely, \( \gamma_2, \gamma_2', \gamma_4 \) and \( \gamma_8 \). Now consider \( \gamma_3 = 0 \) in the lowest order i.e. \( O(\delta_{12}) \). This gives,

\[
4 \lambda \gamma_0 \beta_3^2 + \lambda_3 \beta_3 \left[ - \frac{1}{2} \lambda_1 \gamma_0^2 \gamma_4 + \frac{1}{2} \lambda_7 \gamma_0 \gamma_2 \right. \\
\left. + \frac{1}{4} \left( \gamma_0^2 + \gamma_5^2 \right) \gamma_4 + \lambda_4 \gamma_2^2 \left[ - \frac{1}{2} \lambda_1 \gamma_0^2 \gamma_4 + \frac{1}{2} \lambda_7 \gamma_0 \gamma_2 \right. \\
+ \frac{1}{4} \left( \gamma_0^2 + \gamma_5^2 \right) \gamma_4 \right] = 0 \quad (5.37)\]

Equation (5.37) has four unknowns \( \beta_3, \gamma_2, \gamma_2', \gamma_4 \). Then the minimization equations in \( R \) and \( V \) channel in lowest order \( O(\delta_{10}) \) are respectively given by,
\[ \lambda_5^2 \left( \gamma_2^2 + \psi_2^2 \right) - \lambda_1 \lambda_5 \chi_0^2 \chi_y \psi_2 + \lambda_5 \lambda_7 \chi_0 \psi_2 \]
\[ + \frac{1}{2} \lambda_5 \psi_2 \left( \chi_0^2 + \chi_0^2 \right) + 2 \lambda_5 \chi_0 \psi_2 + \chi_0^2 \gamma_1 \chi_0 \chi_y \]
\[ + \lambda_7 \chi_0 \chi_0 + \frac{1}{4} \psi_y \left( \chi_0^2 + \chi_0^2 \right) \left( \lambda_5 \psi_2 + \chi_0 \right) + \frac{4}{9} \psi_y^2 = 0 , \]
(5.38)
\[ 2 \lambda_5^2 \gamma_2 \psi_2 + 2 \lambda_5 \chi_0 \gamma_2 + \frac{\lambda_5}{2} \left( - \lambda_1 \chi_0 \chi_y \gamma_2 \right) \]
\[ + \frac{1}{2} \lambda_7 \chi_0 \gamma_2 \right) + \frac{\lambda_5}{4} \left( \chi_0^2 + \chi_0^2 \right) \gamma_y \gamma_2 + \]
\[ \frac{1}{4} \psi_2 \gamma_y^2 = 0 . \]
(5.39)

Now the minimization equation in \( \sim \)–channel in lowest order, i.e. in \( O(\gamma_1) \) gives,
\[ 4 \chi_0^2 - \psi_y \chi_0 - \lambda_1 \chi_0^3 \psi_y \chi_y + \lambda_7 \chi_0 \psi_0 \psi_y \]
\[ + \frac{1}{2} \left( \chi_0^2 + \psi_0^2 \right) \chi_0 \psi_y + \frac{4}{9} \psi_y^2 + 2 \lambda_7^2 \psi_3^2 \]
\[ + 2 \lambda_7 \lambda_7 \gamma_2^2 = 0 . \]
(5.40)
Equations (5.37), (5.38), (5.39) and (5.40) contain four unknown parameters \( \gamma_2, \vartheta_2, Z_4 \) and \( \beta_3 \). Hence these four parameters can be calculated by solving these four equations. Substituting these values in (5.36), we get the value for \( \chi_8 \). Once we know the values of \( \gamma_2, \vartheta_2, Z_4 \), we can calculate \( x_4 \). So \( x_4 \) and \( x_8 \) has been determined in this manner.

The lowest order analysis (i.e. in \( O(\lambda^3) \)) in \( \overline{Q} \)-channel was just a consistency check equation for the value of \( y_1 \). So in the next higher order correction in \( \overline{Q} \)-channel (i.e. in \( O(\lambda^7) \)), we get,

\[
y_5 = \left( \frac{2 \lambda_2}{\lambda_3} \right) \frac{x_4 x_0}{y_1} \tag{5.41}
\]

Substituting the values of \( x_4 \) and \( y_1 \), \( y_5 \) can be calculated. Then we consider the minimization equations in \( \overline{U} \)-channel and \( \overline{H} \)-channel in the leading order in \( \lambda^{12} \). The equations are given by

\[
2 \lambda_6 h_4^2 \chi_4 + \frac{1}{2} \left[ -\lambda_1 \chi_0^2 \chi_4 + \lambda_7 z_0 \right] \lambda_6 h_4^2 \\
+ \frac{1}{y} \lambda_6 g_y h_4 \left[ \chi_0^2 + z_0^2 \right] + \lambda_6 h_4 \left( 2 \chi_0 h_4 \right) \\
+ \lambda_{11} \chi_2^2 + \lambda_{12} \chi_2^2 \right) + \frac{1}{y} g_4 \frac{2}{\lambda} u \chi_4 = 0 , \tag{5.42}
\]
and

\[ \lambda_6^2 \rho_4^3 + \rho_4 \left[ (\lambda_6 u_4 + x_0)(\lambda_6 u_4 + x_0 - \frac{1}{2} \lambda_1 x_0^2 x_4 + \frac{1}{2} \lambda_7 z_0 + \frac{1}{4} (x_0^2 + z_0^2) g_4) + \lambda_4^2 \gamma_2^2 - \frac{1}{2} g_4 x_0 + \frac{1}{4} g_4^2 \right] + \gamma_2^2 \left[ \lambda_6 \lambda_1^2 u_4 - \lambda_1^2 x_0 - \frac{1}{2} \lambda_1 \lambda_1^2 x_0^2 x_4 + \frac{1}{2} \lambda_7 \lambda_1^2 z_0 + \frac{1}{4} \lambda_1^2 (x_0^2 + z_0^2) g_4 - \frac{1}{2} \lambda_1 \lambda_1^2 g_4 \right] = 0 . \] (5.43)

Equations (5.42) and (5.43) can be solved to obtain the values of \( u_4 \) and \( h_4 \).

Then we consider the next higher order correction in \( R \) and \( V \)-channel. So the minimization equations \( \widetilde{V}_R = 0 \) and \( \widetilde{V}_V = 0 \) in \( O(\partial^{14}) \) are respectively given by,

\[ \gamma_6 \left[ 3 \lambda_5^2 \gamma_2^2 - \lambda_5^2 \rho_2^2 + 2 \lambda_5 x_0 \rho_2 + x_0^2 + \frac{1}{2} \left[ -x_0 g_4 - \lambda_1 \lambda_5 x_0^2 x_4 \rho_2 + \lambda_5 \lambda_7 z_0 \rho_2 - \lambda_1 x_0^3 x_4 + \lambda_7 x_0 z_0 \right] + \frac{1}{4} \left[ (\lambda_5 \rho_2 g_4 + \right. \]
\begin{align*}
&x_0 y_4 \left( x_0^2 + z_0^2 \right) + g_4^2 \right\} + v_6 \left\{ 2 \lambda_5^2 \gamma_2 v_2 + 2 \lambda_5 x_0 \gamma_2 + \frac{1}{2} \left[ - \lambda_1 \lambda_5 x_0^2 y_4 \gamma_2 + \lambda_5 \gamma z_0 \gamma_2 \right] \\
&+ \frac{1}{4} \lambda_5 \gamma y \gamma_2 \left( x_0^2 + z_0^2 \right) \right\} + \frac{1}{2} \gamma_2^3 \left[ \lambda_5 x_0 v_2 \\
&+ 2 x_0^2 + 2 \lambda_5^2 v_2^2 + 2 \lambda_5 \gamma y v_2 + 2 x_0 g_4 + \\
&\lambda_4^2 \right\} + \gamma_2 \left\{ 2 \lambda_5^2 \gamma \beta_3 v_2 + \gamma \lambda_5 \gamma_1 \gamma y v_2 + \\
&2 \lambda_8 x_0 \beta_3 + 2 \lambda_1 \gamma \gamma_2 \gamma y + 2 \lambda_1 \gamma x_0 \gamma y + \lambda_8 x_0 \beta_3 \\
&+ \lambda_6 \lambda_1 \gamma \gamma_2 \gamma y + \lambda_1 x_0 \gamma y + \lambda_1 \lambda_1 \gamma y_2 + 2 \lambda_1 \gamma x_0 \gamma y \\
&+ \gamma \gamma \gamma \gamma_3^2 + \frac{1}{2} \left[ 2 \lambda_5 \gamma \gamma \gamma_3 v_2 + 2 \lambda_8 \gamma \gamma \gamma_3^2 + \\
&8 \lambda_1 \gamma \gamma y \gamma y - 2 \lambda_5 \gamma \gamma \gamma_3 v_2 - 2 x_0 g_8 - 2 \lambda_8 \gamma \gamma \gamma_3^2 - \\
&2 \lambda_1 \lambda_1 \gamma \gamma x_0 \gamma \gamma v_2 - 2 \lambda_1 x_0^2 \gamma y - \lambda_4 \lambda_5 x_0^2 \gamma y v_2 - \\
&\lambda_1 x_0^2 \gamma y + \lambda_5 \gamma \gamma z \gamma \gamma \gamma_2 + \gamma \gamma \gamma z_0 \gamma y + \lambda_5 \gamma \gamma \gamma_4^2 \gamma v_2 + \gamma \gamma x_0 \gamma \gamma \gamma_4 \\
&- \lambda_1 \gamma \gamma x_0 \gamma \gamma \beta_3 - 2 \lambda_1 \lambda_1 \gamma \gamma \gamma z \gamma \gamma \gamma y \gamma y + \gamma \gamma \gamma z_0 \gamma \beta_3 + \\
&2 \lambda_1 \lambda_1 \gamma \gamma z \gamma \gamma \gamma y \right\} + \frac{1}{4} \left[ \gamma \gamma \gamma \gamma \gamma_3 + 2 \lambda_1 \gamma \gamma \gamma \gamma \gamma y + \\
&\lambda_5 \gamma \gamma \gamma_3 v_2 + \gamma \gamma \gamma_3 x_0 \right] \left( x_0^2 + z_0^2 \right) + 2 \lambda_5 \gamma \gamma \gamma_2 \gamma x_0 \gamma y + \\
&2 \gamma \gamma v_2 \gamma z_0 \gamma y + \lambda_5 \gamma \gamma \gamma_2 \gamma v_2 + 2 x_0 \gamma \gamma y \gamma y + 2 \gamma \gamma \gamma z_0 \gamma \gamma \gamma y + \\
&4 x_0 \gamma \gamma \gamma z_0 \gamma \gamma \gamma y + 2 \lambda_5 \gamma \gamma \gamma z_0 \gamma \gamma \gamma y + \\
&2 \gamma \gamma \gamma \gamma \gamma y z_0 \gamma \gamma \gamma y + \lambda_5 \gamma \gamma \gamma \gamma \gamma y + 2 \gamma \gamma \gamma \gamma \gamma z_0 \gamma \gamma \gamma y + \\
&2 \lambda_8 \gamma \gamma \gamma \gamma y \gamma y + \lambda_8 \gamma \gamma \gamma \gamma z_0 \gamma \gamma \gamma y + \
\end{align*}
Now equations (5.44) and (5.45) contain two unknown parameters $\gamma_6$ and $\psi_6$, which can be determined for a range of values of $\lambda_1$, $\lambda_5$, $\lambda_6$, $\lambda_7$, $\lambda_8$ and $\lambda_{11}$.
Then we consider the minimization in $\tilde{S}$-channel in $O(\frac{\alpha_s}{3})$ to get a solution for $\beta_7$. The equation is given by,

$$\beta_7^2 (-\kappa_0 g_4) + \beta_7 \left[ 4\lambda x_0^2 - \lambda_1 x_0 x_4 + 2 \lambda_7 x_0 z_6 + \frac{1}{2} \kappa_0 g_4 (x_0^2 + z_6^2) + \frac{1}{4} g_4^2 \right]$$

$$+ \beta_3 \left[ 2\lambda_6 \gamma_0^2 + 2 \lambda_1 \gamma_7 \gamma_2^2 + 2 g_4 g_8 \right] + \left[ \lambda_{10} h_y \left( 2 \lambda_6 \kappa_0 \kappa_4 h_x + 2 \kappa_0 \kappa_4 h_y + \lambda_{11} \gamma_2^2 + \lambda_{12} \gamma_2^2 + 2 \kappa_0 \kappa_4 h_y + \frac{1}{2} \lambda \kappa_0 \kappa_4 h_y - \frac{1}{2} \lambda_2^2 \kappa_0 \kappa_4 h_y + \frac{1}{2} \left( x_0^2 + z_6^2 \right) \kappa_4 ^2 h_y \right) \right] = 0 \ . \quad (5.46)$$

Here all the quantities except $\beta_7$ have been previously determined. Hence $\beta_7$ can be calculated.

5.5. Conclusions:

We observe that by considering the minimization equations in successive orders of the expansion parameter $\frac{\alpha_s}{3}$ in different channels, we are successful in determining the VEVs of the fields which are responsible for the symmetry
breaking chain. The initial breaking of $E_8$ to $E_6$ occur at
the scale $M \sim 10^{18}$ GeV, $E_6$ breaks to $SO(10)$ at the scale
$\frac{3}{2}M \sim (10^{14}$ GeV), $SO(10)$ breaks to $SU(5)$ at a scale $\frac{3}{2}M \sim (10^{10}$ GeV), $SU(5)$ breaks to $G_{\text{std}}$ at a scale $\frac{3}{2}M \sim (10^{6}$ GeV)
and finally $G_{\text{std}}$ and supersymmetry breaking occur at a scale
$\frac{3}{2}M \sim (10^{2}$ GeV). Here the breaking of Salam-Weinberg
symmetry and supersymmetry in visible sector are induced by
supergravity and in fact both break at the scale $\frac{3}{2}K^{-1} \sim M_\text{Pl}$,
which is of the order of $10^{2}$ GeV. Gravitino and the $W$ and $Z$-
bosons also get masses of this order. Supersymmetry in the
visible sector is exact, above the weak symmetry breaking
scale. So we conclude that the hierarchy introduced through
supergravity expansion parameter can cause successive gauge
symmetry breaking in a consistent manner.