CHAPTER - 4

Fermion condensates and weak symmetry breaking

in a superstring based model.
4.1. Introduction:

In last two chapters, we have discussed about the supergravity theory and its phenomenological aspects. Here we shall discuss about a new kind of theory, namely the superstring theory\(^1\), whose low energy limit contains supergravity. There has been a tremendous amount of excitement for last three-four years over this so called superstring theories. There are indications that these theories may provide a consistent quantum theory of gravity. A superstring theory\(^1\) in ten dimensions may possibly be regarded as an ultimate theory which unifies all fundamental interactions. It has been discovered recently that \(d = 10\) superstring theories\(^2\) are anomaly-free if the gauge group is \(SO(32)\) or \(E_8 \times E_8\).\(^3\) Candelas, Horowitz, Strominger, and Witten\(^4\) have shown that an \(E_8 \times E_8\) ten dimensional Yang-Mills theory coupled to supergravity can undergo spontaneous compactification to \(M_4 \times K\), where \(M_4\) is the Minkowski space and \(K\) is a compact six-dimensional Calabi-Yau-manifold. They have shown that an unbroken \(N = 1\) local supersymmetry in \(d = 4\) obtained after compactification implies that \(K\) should be Kähler type with \(SU(3)\) holonomy. \(E_8\) has the maximal subgroup \(SU(3) \times E_6\), where \(SU(3)\) is identified as the \(SU(3)\) holonomy of the compact manifold \(K\). The effective theory after compactification would be an \(N = 1\) supergravity theory in \(d = 4\) with a symmetry group \(G\), which is a subgroup of \(E_6\). The second group \(E_8\) remains unbroken, forming the "hidden sector"\(^4\) of the theory.
4.2. Representation and structure of the gauge group:

Various subgroups of $E_6$ have been considered for grand-unification phenomenology. Usually one needs gaugino condensates in the hidden $E_8$ sector for supersymmetry breaking. We shall, however, consider another alternative where supersymmetry breaks in the visible sector through the condensate of chiral fermions. We illustrate this with the gauge group $G = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_N (\Xi G_{3211})$ as the relevant subgroup after compactification. We modify the model to take the conventional Higgs doublets as superheavy, and, as stated, weak symmetry breaks through the formation of a fermionic condensate, yielding in particular $G_{3211} \rightarrow SU(3)_C \times U(1)_Q (\Xi G_{LE})$, as well as supersymmetry breaking. Such a picture has direct experimental consequence in the sense that no light Higgs scalar will be observed, although the results of Salam and Weinberg at low energies will remain true. Our objective is to illustrate the feasibility of this concept with a super-string-inspired phenomenological model. Under the subgroup $G_{3211}$, the 27 matter field transforms as

\[
27 \rightarrow (3, 2, 0, 1/4)_q + (1, 2, 0, 1/4)_L \\
+ (1, 2, 1/2, -1/2)_H + (1, 2, -1/2, 1/2)_{\bar{H}} \\
+ (3, 1, 0, -1/2)_g + (\overline{3}, 1, 0, -1/2)_{g'}
\]
The subscripts show a specific particle content of the single representation. $H$ and $\overline{H}$ are Weinberg-Salam Higgs doublets. Each 27 family contains three quark like flavors, referred to as $u$, $d$, and $g$. Left-handed $u$ and $d$ quarks form weak doublets, while $g$ quarks are weak singlets, left or right, having electric charge $-\frac{1}{3}$. $N_1$ and $N_2$ are two singlets, where $N_1$ represents the left-handed antineutrino. $Q$ and $L$ represent ordinary quark and lepton doublets. While extremizing we give a large vacuum expectation value (VEV) to the singlet field $N_2$ and as a result the extra particles $g_1$, $g_{1c}$, and the Salam-Weinberg Higgs doublets $H$ and $\overline{H}$ get masses of the order of the Planck scale. Since the Higgs fields become superheavy, we integrate them out from the Lagrangian and replace them by the corresponding scalars with higher-dimensional fermionic operators, as will be illustrated below.

4.3. **Invariant superpotential:**

The form of the superpotential for the $G$ invariant theory is obtained by taking a $G$ invariant truncation of an $E_6$ invariant superpotential. If $\Phi$ represents 27 and $\overline{\Phi}$ represents
$27$ of $E_6$, then suppressing all the indices we may write the superpotential as

$$\mathcal{W}_1 (\Phi, \bar{\Phi}) = \lambda \Phi^3 + \lambda' \bar{\Phi}^3 \quad (4.1)$$

Since the relevant group is $G_{3211}$, we shall take $\mathcal{W}_1$ not an $E_6$ invariant, but an invariant with respect to $G_{3211}$. The most general superpotential which conserves the baryon and lepton numbers and which does not give a mass to the neutrinos is given by

$$\mathcal{W}(\Phi, \bar{\Phi}) = \lambda_u \bar{H} Q U_c + \lambda_d \bar{H} Q D_c + \lambda_e \bar{H} L E_e$$

$$+ \lambda_{\bar{H}} \bar{H} \bar{H} N_2 + \lambda_{\bar{q} \bar{g} \bar{q} c} \bar{N}_2 + \lambda'_{\bar{H}} \bar{H}' \bar{N}_2 \quad (4.2)$$

Here $\bar{N}_2, H, \bar{H}$ are the fields belonging to the $27$ representation and they have the quantum numbers of anti-$N_2$, anti-$H$, and anti-$\bar{H}$ fields, respectively. Out of these three fields, only $\bar{N}_2$ can take a nonzero VEV equal to that of the $N_2$ field and the other two fields have zero VEV. So as per our ansatz, the following fields have zero VEV's namely,

$$\langle \bar{H} \rangle = \left( \begin{array}{c} 0 \\ \langle \Phi \rangle \end{array} \right), \quad \langle N_2 \rangle = \langle \bar{N}_2 \rangle \neq 0.$$
4.4. Kähler potential:

Based on phenomenology arising from superstrings, we use a nonflat Kähler metric \( \mathcal{K} \) instead of a flat Kähler metric. This is obtained in terms of two chiral gauge-singlet fields \( S \) and \( T \), and chiral field \( C^A \) belonging to the 27-plet of \( E_6 \). We note that the \( C^A \) 's include quarks, leptons, and Higgs bosons, as well as some extra fields. The corresponding Kähler potential is defined as \(^8,9\)

\[
\mathcal{K} = M^2 \left[ -\ln \left( \frac{S + \bar{S}}{M} \right) - 3 \ln \left( \frac{T + \bar{T}}{M} - \frac{2 \bar{C}_A C^A}{M^2} \right) \right], \tag{4.3}
\]

where, \( M = M_{\text{Planck}} \sqrt{\frac{8}{3\pi}} = 2.4 \times 10^{18} \text{ GeV} \) and \( S, T, \) and \( C^A \) are complex conjugates of \( S, T, \) and \( C^A \), respectively. We also take in addition a Polonyi-type \(^{10}\) superpotential, given as

\[
\mathcal{W}_2(S) = m^2(S + B_0) \tag{4.4}
\]

where \( S \) is the singlet chiral superfield as defined before, \( B_0 \) is an arbitrary constant, and \( m \) is a mass parameter of the Polonyi potential. Supersymmetry breaking is then introduced through the \( \mathcal{W}_2(S) \) sector of the theory. We have now the total superpotential as
\[ W = W_1(\Phi, \overline{\Phi}) + W_2(S) \]  
\[ (4.5) \]

To calculate the tree-level potential energy, we define
\[ G = \frac{k}{M^2} + \ln \left( |W|^2 / M^6 \right) \]  
\[ (4.6) \]

The Kähler covariant derivatives and the Kähler metric are defined by
\[ D_{\bar{z}^i} W = W_i^j + \frac{K_i^j}{M^2} W \]  
\[ (4.7) \]

and
\[ g_{\bar{z}^i, \bar{z}^j} = K^{\bar{z}^i, \bar{z}^j} \]  
\[ (4.8) \]

respectively, and also
\[ G_{\bar{z}^i} = \frac{\partial G}{\partial \bar{z}^i} = \frac{1}{W} D_{\bar{z}^i} W \] \[ G^j_{\bar{z}^i} = \frac{\partial G}{\partial z^j} = \frac{1}{W^*} D_\bar{z}^j W^* \]  
\[ (4.9) \]

\[ G^{\bar{z}^i, \bar{z}^j} = \frac{1}{M^2} g_{\bar{z}^i, \bar{z}^j} \]  
\[ (4.10) \]
We have $z^1 = S$, $z^2 = T$, $z^3 = C^A$, and also we define

$$\hat{S} = S + \bar{S}, \quad \hat{T} = T + \bar{T}, \quad \hat{A} = (T + \bar{T}) - \frac{2e^A \bar{e}^A}{M}.$$ (4.11)

The bosonic part of the interaction Lagrangian is given by Cremmer et al. \cite{11}

$$e^{-4} V_B = M^4 e^G \left[ G'_{ij} \left( G''^{-1} \right)_j^i \cdot G'_{ij} - 3 \right]$$

$$+ M^4 g^2 \Re f^{-1}_{ab} \left[ G'_{ij} (T^a)^j_i \cdot Z^j \right] \left[ G'_{k} (T^b)^k_e \cdot Z^e \right],$$ (4.12)

where

$$f_{ab} = \delta_{ab} S/M,$$ (4.13)

and $T^a$ are the Lie-algebra generators of $G_{3211}$. Evaluation of (4.12) can be conveniently done where we note that the matrix $g$ of equation (4.8) and its inverse are given as

$$g = \begin{pmatrix}
\frac{M^2}{S^2} & 0 & 0 \\
0 & \frac{3M^2}{\bar{Q}^2} & -\frac{6MC^B}{\bar{Q}^2} \\
0 & -\frac{6MC^A}{\bar{Q}^2} & \frac{6M}{\bar{Q}^2} \left( \frac{B}{A} + \frac{2C^B}{MQ^2} \right)
\end{pmatrix},$$ (4.14)
The inverse Kähler metric is given by

\[ g^{-1} = \begin{bmatrix}
\frac{\hat{S}^2}{M^2} & 0 & 0 \\
0 & \frac{Q^T}{3M^2} & \frac{Q^B}{3M^2} \\
0 & \frac{\hat{Q}^A}{3M^2} & \frac{\hat{Q}^B}{6M}
\end{bmatrix} \quad (4.15) \]

By minimizing the potential given in equation (4.12), it has been found by Breit, Ovrut, and Segre that \( \langle c^A \rangle = 0 \), and the VEV of \( \langle T \rangle \) remains undetermined at the tree level. We make a departure from this procedure at this point. This is done with the introduction of two new features. First, we assume that the scalars formed from chiral fermions may attain nonzero VEV's. Such terms are, however, absent in the potential, although they are present in the Lagrangian. Hence, we further consider extremization of the Lagrangian (at a classical level) instead of considering that of the potential. We shall now extremize the "effective potential" which includes the negative of a part of the fermionic Lagrangian given as

\[ \hat{e}^{-1} \mathcal{V}_{\text{eff}} = \hat{e}^{-1} \mathcal{V}_B + \hat{e}^{-1} \mathcal{V}_{\text{FM}} \quad (4.16) \]

where
Here the composite operator $\overline{x}_k$, which is equivalent to a boson, is to be extremized.

4.5. **Extremization of the effective potential**:

In order to extremize the potential, first we shall convert all the fields, superpotential and potential, to the corresponding dimensionless quantities with the tilde notation, which will make the calculations a lot simpler, i.e.

$\tilde{W} = k^3 W$, $\tilde{V} = k^4 V$, and $k^2 m^2 = \tilde{g}^2$, such that $km^2 \simeq 300$ GeV and thus $\tilde{g} \ll 1$. We shall give nonzero VEV's to the following scalar fields: i.e., $\tilde{T}$, $\tilde{T'}$, $\tilde{N}_2$, $\tilde{N}_2'$, $\tilde{G}^{\alpha}$, and $\tilde{\phi}^0$, the neutral component of the $\tilde{H}$ Higgs field. We shall replace the field $\tilde{\phi}^0$ by the corresponding current (or equivalently chiral-fermion condensate) divided by the mass squared of the field and express the effective Lagrangian in terms of condensates where
the VEV's of $\tilde{\phi}$ and $(\tilde{\chi})$ are really interdependent. We shall assume that only the $(\tilde{f}_{\phi}, \tilde{f}_{\chi})$ condensate (where $f$ denotes the fermionic component) can take a nonzero VEV and all other composite operators have a zero VEV. Technically, we could achieve the same thing with a $(\tilde{u}_{\bar{1}}, \tilde{u}_{\bar{1}})$ condensate, but this would be unphysical, since the natural scale for quarks is of GeV order, and, as we shall see later, we need a VEV of much higher order. In our ansatz for extremization, we assume that fields $\tilde{T}$ and $\tilde{N}_2$ have VEV's of the order of the Planck scale, which is taken to be the same as the compactification scale. The condensate, superpotential, and the Higgs doublets (through condensates) have VEV's of the order of weak symmetry breaking. We first note that this yields the pure gauge-field term of the Lagrangian as

$$\mathcal{L} = \frac{1}{4} \text{Re} f_{ab} \left[ F_{\mu\nu}^a \right]^b + \ldots ,$$

(4.18)

where $f_{ab}$ is given by equation (4.13). Thus for us to have normal gauge theory, we shall take $\langle \tilde{S} \rangle = \epsilon_1$, which is of the Planck or compactification order. The conjecture is that the VEV arises from the dynamics of compactification, and thus is nonperturbative in ten dimensions. In the leading order, we take

$$\langle \tilde{T} \rangle = t_1 , \quad \langle \tilde{N}_2 \rangle = \epsilon_1' ,$$

(4.19a)
\[ \langle \tilde{\phi}^0 \rangle = \eta_1 \tilde{\phi}^{1/2}, \langle \tilde{\phi}^0 \rangle = \eta_1 \tilde{\phi}^{1/2}, \tilde{W} = b_1 \tilde{\phi}^{1/2}, (4.19b) \]

where \( t_1, \epsilon_4', \eta_1' \) and \( b_1 \) are unknown parameters of order one and \( \eta_1 \) is determined in terms of \( \eta_1' \). The arbitrary parameter \( b_1 \) in \( \tilde{W} \) is introduced to make the cosmological constant vanish. We shall determine these parameters through an extremization procedure.

From the above, the mass of \( \tilde{\phi}^0 \) (in units of \( k^{-1} \)) is given as

\[ M_{\tilde{\phi}^0} = \left[ \frac{\lambda_H^2 \epsilon_1^2}{48 \epsilon_1 (t_1 - \epsilon_1')^2} \right]^{1/2} k^{-1}. \quad (4.20) \]

Since this field is superheavy, we eliminate it in the Lagrangian in favor of

\[ \tilde{\phi}^0 = \frac{6 J \tilde{A} \tilde{A}^{\frac{1}{2}} \tilde{A}^{\frac{1}{2}}}{\lambda_H |\tilde{N}_2|^2} \left[ 1 + \frac{4 |\tilde{N}_2|^2}{\tilde{A}} \right], \quad (4.21) \]

where, \( J = \left( \tilde{f}_{2}^c \tilde{f}_2 \right) \), as derived from the interaction term. We note that the VEV of \( \tilde{\phi}^0 \) is given in terms of VEV's of \( J \). We use equation (4.21) to obtain the new effective potential in terms of the chiral fermions.
Then we consider the extremization of this effective potential in the $\tilde{V}_2$ channel. In the leading order $\tilde{V}_{N_2} = 0$ implies

$$4 + \frac{\theta_1^2}{\epsilon_1^2} - 4 \frac{\theta_1}{\epsilon_1} - \frac{16}{3} \left[ \frac{t_1}{\epsilon_1} - 2 \frac{\epsilon_1'}{\epsilon_1} \right] \eta_1^2 = 0 .$$

Next, considering the extremization of the potential in the leading order in the $\tilde{T}$ channel gives

$$\left[ \frac{t_1}{\epsilon_1} + \frac{\epsilon_1'}{\epsilon_1} \right]^2 \eta_1^2 + \frac{4}{(\epsilon_1' / \epsilon_1)} \left[ \frac{t_1}{\epsilon_1} + \frac{\epsilon_1'}{\epsilon_1} \right] \eta_1^2 = 0 .$$

(4.22)

Then we consider extremization with respect to $\langle j \rangle$. In the leading order, $\tilde{V}_J = 0$ implies

$$\frac{4}{8} + \frac{4}{32} \frac{\theta_1^2}{\epsilon_1^2} - \frac{\theta_1}{\epsilon_1} - \frac{4}{(\epsilon_1' / \epsilon_1)} \left[ \frac{4}{(\epsilon_1' / \epsilon_1)} \left( \frac{t_1^2}{\epsilon_1} \right) + \frac{4}{2} \frac{t_1^2}{\epsilon_1^2} \right] \eta_1^2 = 0 .$$

(4.23)

Then we consider extremization with respect to $\langle j \rangle$. In the leading order, $\tilde{V}_J = 0$ implies

$$\left[ \frac{t_1}{\epsilon_1} + \frac{\epsilon_1'}{\epsilon_1} \right]^2 \frac{\epsilon_1}{(\epsilon_1' / \epsilon_1)} - \left[ \frac{t_1}{\epsilon_1} - 2 \frac{\epsilon_1'}{\epsilon_1} \right] \eta_1^2 = 0 .$$

(4.24)
Since the cosmological constant in this model vanishes, we have another condition \( \tilde{\lambda} = 0 \). In the leading order, this gives

\[
\frac{1}{4} + \frac{1}{16} \frac{b_4}{\epsilon_1^2} - \frac{4}{4} \frac{b_4}{\epsilon_1} - 3 \left[ \frac{t_4}{\epsilon_1} + \frac{\epsilon_4^2}{\epsilon_1} \right] \eta_1^2
\]

\[- 3 \left[ \frac{t_4}{\epsilon_1} - 2 \frac{\epsilon_4^2}{\epsilon_1} \right] \eta_1^2 = 0 \quad (4.25)
\]

Now we have three extremization equations along with the condition \( \tilde{\lambda} = 0 \), characterized by equations (4.22), (4.23), (4.24) and (4.25) and four unknown quantities \( b_4/\epsilon_1, \epsilon_4^2/\epsilon_1, t_4/\epsilon_1 \) and \( \eta_1' \). For a given set of parameters, the above equations can be solved to give values of these quantities, and the VEV's of the scalar fields and condensate get determined selfconsistency at a tree level.

4.6. Mass of the scalar particles:

We now calculate the masses of the scalar particles in this model. As noted earlier, \( \phi \) has a mass of order \( K^{-1} \) through a corresponding VEV of the \( N_2 \) field. In fact both Higgs doublets get the same order of mass from symmetry. The mass of scalar g quark is also found to be
The weak scale is determined by

$$\mathcal{V} = \eta_1 \bar{s}^2 k^{-1} \left( = 175 \text{ GeV} \right)$$

(4.27)

such that the gauge-boson masses are $M_N = g_0 \sqrt{2}$, where $g$ is a

gauge coupling and $M_Z = M_W \cos \theta_W$. We now obtain the masses of

scalar $N_2$, scalar $u$ and $d$ quarks, and scalar electron and

scalar neutrino as

$$M_{\tilde{N}_2} = \left[ \frac{3 \epsilon_1}{4 (t_1 - \epsilon_1'^2)^4} + \frac{3 b_1}{16 \epsilon_1 (t_1 - \epsilon_1'^2)^4} - \frac{3 b_1}{4 (t_1 - \epsilon_1'^2)^4} + \frac{|\lambda \eta_1|^2}{48 \epsilon_1 (t_1 - \epsilon_1'^2)^2} \right]^{1/2} k^{-1/2},$$

(4.28)

$$M_{\tilde{u}, \tilde{d}_{ii}} = \left[ \frac{3 \epsilon_1}{4 (t_1 - \epsilon_1'^2)^4} + \frac{3 b_1}{16 \epsilon_1 (t_1 - \epsilon_1'^2)^4} - \frac{3 b_1}{4 (t_1 - \epsilon_1'^2)^4} + \frac{|\lambda u \eta_1|^2}{48 \epsilon_1 (t_1 - \epsilon_1'^2)^2} \right]^{1/2} x k^{-1},$$

(4.29)

$$M_{\tilde{e}, \tilde{e}_{ci}, \tilde{\nu}, \tilde{\nu}_{ci}} = \left[ \frac{3 \epsilon_1}{4 (t_1 - \epsilon_1'^2)^4} + \frac{3 b_1}{16 \epsilon_1 (t_1 - \epsilon_1'^2)^4} - \frac{3 b_1}{4 (t_1 - \epsilon_1'^2)^4} + \frac{|\lambda u | \eta_1|^2}{48 \epsilon_1 (t_1 - \epsilon_1'^2)^2} \right]^{1/2} x k^{-1},$$

(4.30)
Clearly we may expect these masses to be of the order of the weak scale.

4.7. Mass of the fermions:

The mass of the fermions can also be calculated from the Yukawa coupling present in the Lagrangian. The gravitino mass is found to be

\[ M_{3/2} = \frac{b_1}{4 \epsilon_1^{3/2} (t_1 - \epsilon_1'^2)^{3/2}} \epsilon_1' k^{-1}, \quad (4.31) \]

which sets the scale of supersymmetry breaking, which is of the weak scale. The \( u \)-quark mass is also found to be

\[ M_{u_i, u_{ci}} = \frac{\lambda_u \eta_1}{4 \epsilon_1^{3/2} (t_1 - \epsilon_1'^2)^{3/2}} \epsilon_1'^2 k^{-1} \]

\[ = \frac{\lambda_u \eta}{4 \epsilon_1^{3/2} (t_1 - \epsilon_1'^2)^{3/2}}. \quad (4.32) \]

As usual, it is obvious that \( \lambda_u \) has to be very small, so that hadron masses are much smaller than the weak scale. Masses of
the d quark and electron are found to be zero. The neutrino mass is also found to be identically zero. We are not tackling the general problem of fermion mass here since we consider a single generation of quarks and leptons. The mass of the fermionic partner of $N_2$, i.e. $\chi_{N_2}$, is found to be of weak scale, which is given by

$$M_{\chi_{N_2}} = \frac{q b_{\perp} \varepsilon'_1 \hat{3}^2}{\sqrt{\varepsilon'_1} (t_1 - \varepsilon'_1)^{7/2}} k^{-1}.$$  \hfill (4.33)

The masses of fermionic partners of the Higgs doublets $H, \tilde{H}$, and the $g$ quarks are also of the Planck scale, the same as that of the corresponding partners, as expected from supersymmetry.

4.8. Conclusions:

We may now note some of the salient features of our model. First of all, here chiral-fermion condensates play an important role in weak symmetry breaking, which was previously being done by the Higgs mechanism. Thus the VEV's of the scalar fields get determined even at the tree level. All the extra particles get a large mass so that they remain unobservable in the presently achieved energy limit. Since we start with the gauge group $G_{3211}$, where $(B - L)$ is already a broken symmetry, we have assumed that the particle $N_1$ has become
superheavy from the very beginning, whose nonzero VEV could have caused the $B - L$ symmetry breaking during compactification. In particular, it may be noted that the triple coupling with $N_1$, which could give a rapid proton decay, has been omitted.\textsuperscript{5}

We have presumed here as stated earlier that $E_6$ gets replaced by $G_{3211}$ at the compactification scale ("same" as the Planck scale), while it yields the effective superpotential and Kähler metric assumed here.

The scale for the condensates appears to be of the order of $10^{10-11}$ GeV and the dynamics which will occur at this scale and beyond has not been investigated in the present chapter and could be of some interest in itself. We obtain consistency in the extremization equations through condensates and superheavy Higgs doublets, a consequence of which is that no Higgs particle will be observed in the ongoing search for the same, although all other results of the Salam-Weinberg model will remain true.\textsuperscript{14} We have merely demonstrated the above possibility with the context of a superstring-motivated superpotential.