Chapter 1

Introduction

It is now accepted that the string theory holds the promise of unifying the four fundamental forces of Nature. The Electro-Weak Unification was proposed almost four decades ago. Subsequently, the theory of strong interactions, Quantum Chromodynamics (QCD) came into existence. These two theories were constructed on the principle of local gauge invariance. The ‘standard model’ comprising of the Electro-Weak theory and the QCD has been subjected to experimental tests and existing data do not contradict the main predictions of the standard model. However, unification of the gravity with rest of the forces still remained as a fundamental challenging problem within the framework of conventional quantum field theory since the higher order diagrams in the usual perturbation theory were inflicted with severe ultraviolet divergences.

If we examine the origin of string theory in the historical perspective, the Veneziano model was the forerunner of string theory[1, 2]. The construction of N-point dual amplitudes led to discovery of the bosonic string theory[2]. The worldsheet fermions were introduced by Neveu and Schwarz and Ramond[3]. Therefore, the early developments of string theory were intended to provide an understanding of the dynamics of strongly interacting particles. Since the closed string massless spectrum naturally contained a spin-2 particle, it was natural to identify it with the graviton. The Scherk-Schwarz proposal [4] was a bold step towards unification of fundamental forces, including gravity when they argued that the string theory should describe the fundamental interactions. However, with the success of the standard model the developments in string theory were rather slow. It was shown by Green and Schwarz
that the acceptable gauge groups to satisfy the requirements of anomaly cancelations were limited to two only, $E_8 \times E_8$ or $SO(32)$ [5]. Furthermore, the construction of the heterotic string theory was a major step towards unification of the four fundamental interactions [6]. The heterotic string in ten dimensions is endowed with $N = 1$ supergravity multiplet together with a Yang-Mills multiplet where the choice of the gauge group was no longer arbitrary.

Since the spacetime dimension is four, the theory has to be defined in the physical dimensions. In this context the ideas of Kaluza and Klein played a very important role. Indeed the importance of Kaluza-Klein theory was recognized during the developments of supergravity theories since it was possible to construct supergravity theories in higher spacetime dimensions. In fact it was shown that there is a unique supergravity theory in $D = 11$ [7]. The compactification of string theories to lower dimensions resulted in a rich dividend. Since string is a one-dimensional object, there were special symmetries which were only attributes of the string.

If we consider compactification of one of the spatial dimensions of a string theory, on a circle of radius $R$, then the spectrum can be derived in a straightforward manner. We note that there will be contribution to the mass squared term from the Kaluza-Klein mechanism since the compactification on the circle will quantize the momenta along that direction. Furthermore, string being a one-dimensional object, it can wind around that direction and as it stretches it will cost energy. So mass operator will have two types of terms; one proportional to $1/R$, coming from the Kaluza-Klein part and the other directly proportional to $R$, due to the winding of the string. Moreover each term will be multiplied by an arbitrary integer. The mass squared operator will get usual additional contribution from the oscillators. Thus, if one interchanges the Kaluza-Klein modes and the winding modes (the two arbitrary integers) together with the inversion of the radius of compactification, the spectrum can remain invariant. This symmetry is special to the string theory and known as T-duality [8, 9, 10, 11, 12]. Furthermore, there is analog of strong-weak duality in field theory [13, 14, 15, 16], called S-duality which connects strong and weak coupling phases of the string theory [17, 18, 19, 20, 21]; in certain cases strong coupling phase of a theory is related to weak coupling phase of another theory.

In ten dimensions, there are five consistent string theories: type IIA and type IIB theory, the heterotic string theory with gauge groups $E_8 \times E_8$ and $SO(32)$ and
type I theory with gauge group $SO(32)$. Recent progresses in string theory have related these theories in various dimensions through the web of duality relations\cite{22, 23, 24, 25, 26}. One of the important outcome of the study of dualities has been to understand the nonperturbative aspects of string theories and it is believed that there might be an underlying unique theory in a higher dimension such that all the consistent ten dimensional string theories appear as different phases of the fundamental theory \cite{23, 24}. It is also a striking result from the string theory that the Beckenstein-Hawking entropy formula for extremal and near extremal black holes can be derived from a microscopic theory \cite{27, 28, 29, 30}.

1.1 Sigma-model and the effective action

One of the simplest way to study the string dynamics is to consider the evolution of the string in the first quantized frame work. If we look at the evolution of the one-dimensional object in flat spacetime, it traces a two dimensional surface just as a point particle traces a line in its temporal evolution. The classical action for the string evolution, in the curved space can be written as\cite{2}

$$S_1 = -\frac{T}{2} \int d^2\sigma \sqrt{h} \tilde{h}^{\alpha\beta} g_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu,$$

(1.1)

where $\sigma^0 = t$ is time-like and $\sigma^1$ is spatial coordinate on 2-dimensional world-sheet, $h_{\alpha\beta}$ describes the intrinsic geometry of the world-sheet manifold, the functions $X^\mu(\sigma)(0 \leq \mu \leq D)$ are D-dimensional target space coordinates which give a map of the world-trajectory onto the physical D-dimensional spacetime manifold described by the metric $g_{\mu\nu}$. Action (1.1) describes the motion of a string embedded in a D-dimensional spacetime. The parameter $T = \frac{1}{2\pi\alpha'}$ is usually called tension (energy per unit world-volume), where $\alpha'$ is the slope parameter of the Regge trajectories. The string-like objects have a characteristic length scale $l = \sqrt{2\alpha'} \sim 10^{-33} cm$ which is the Planck length. Therefore, the string excitations will have masses in Planck units $m_P = \sqrt{\frac{1}{G_N}}$, where $G_N$ is Newtonian gravitational constant. We shall be working in natural units ($\hbar = c = 1$) in this text.

Let us discuss the symmetries of the above action. Note that action (1.1) has 2-dimensional reparametrization invariance since $\sqrt{h} d^2\sigma$ is a reparametrization invariant
volume element and $h^{\alpha \beta} g_{\mu \nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu$ has properly contracted world indices. The string action also possesses an invariance under the following Weyl rescaling of the world-sheet metric

$$h_{\alpha \beta} \rightarrow \Omega(\sigma) h_{\alpha \beta} \quad (1.2)$$

Note that the variation of the action (1.1) with respects to the world-sheet metric yields the two dimensional stress energy momentum tensor. In fact, the equation of motion for $h_{\alpha \beta}$ implies that the energy momentum tensor must be zero (addition of an Einstein-Hilbert type term in two dimensions amounts to having a topological term which does not contribute to the equation of motion). Thus,

$$T_{\alpha \beta} = -\frac{2}{T} \frac{\delta S_1}{\delta h_{\alpha \beta}} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2} h_{\alpha \beta} (\partial X)^2 \quad (1.3)$$

vanishes. It can also be seen from the above that, as an immediate consequence of the Weyl invariant world-sheet action, the energy-momentum tensor is traceless, $h^{\alpha \beta} T_{\alpha \beta} = 0$. The reparametrization invariance and Weyl invariance together form the “conformal symmetry” of the string action[31]. The conformal invariance becomes the guiding principle of string theory. The conformal invariance of the theory in flat target space implies that the theory is ought to live in critical dimensions if we want it to be anomaly free. Therefore, for the bosonic string, the critical dimension is $D = 26$ and it is ten for the superstrings.

The requirement of conformal invariance of the string world-sheet theory restricts the choice of other terms as well. For a closed bosonic string, there is another massless excitation in the spectrum: the antisymmetric tensor field. It couples to the string as follows,

$$S_B = -\frac{T}{2} \int d^2 \sigma \epsilon^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu \nu}(x) \quad (1.4)$$

where $\epsilon^{\alpha \beta} (\epsilon^{01} = 1)$ is the usual two dimensional antisymmetric tensor density. $B_{\mu \nu}$ is the antisymmetric source field which couples to the string.

The third massless state is the scalar, dilaton, and it couples to the two dimensional curvature scalar,

$$S_\Phi = \frac{T}{2} \alpha' \int d^2 \sigma \sqrt{h} R^{(2)} \Phi(x) \quad (1.5)$$

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Note that this interaction term is of next higher order in $\alpha'$ compared to $S_1$ and $S_B$. At the classical level, this piece of the action is not Weyl invariant. We shall see below that the quantum conformal invariance leads to interesting consequences when we look for consistency of the string theory in the presence of its massless backgrounds. Thus, the first quantised worldsheet action for a bosonic string is given by

$$S = S_1 + S_B + S_\Phi. \quad (1.6)$$

Action (1.6) includes the background fields; graviton $g_{\mu\nu}$, antisymmetric tensor $B_{\mu\nu}$, and the dilaton $\Phi$ in the form of the sigma model couplings. The quantum fluctuations of these background fields would constitute the low lying massless excitations of the string.

Now if we demand the theory to respect conformal invariance, then there are severe constraints on the background field configurations. The conformal anomaly is related to the trace of the stress energy momentum tensor which in turn is proportional to the corresponding $\beta$ functions of the underlying $\sigma$-model. Thus, the absence of conformal anomaly is guaranteed by requiring that the $\beta$ functions vanish. Note that the $\beta$ functions are determined perturbatively from the $\sigma$-model. Therefore, one arrives at the following constraint equations for the backgrounds[32, 33],

$$\beta_\Phi = 0 = \frac{(D - 26)}{3\alpha'} - R + 4 \nabla_\mu \Phi \nabla^\mu \Phi - 4 \nabla_\mu \nabla^\mu \Phi + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + O(\alpha'),$$

$$\beta_g = 0 = R_{\mu\nu} - \frac{1}{4} H_{\mu\lambda\rho} H^{\lambda\rho} + 2 \nabla_\mu \nabla_\nu \Phi + O(\alpha'),$$

$$\beta_B = 0 = \nabla_\rho H^\rho_{\mu\nu} - 2 H^\rho_{\mu\nu} \nabla_\rho \Phi + O(\alpha'). \quad (1.7)$$

where $H_{\mu\nu\rho} = (\partial_\mu B_{\nu\rho} + \text{cyclic permutations of indices})$, $R^\rho_{\lambda\mu\nu} = \partial_\mu \Gamma^\rho_{\lambda\nu} + \cdots$, and the Ricci tensor is defined as $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$.

Another remarkable consequence of the string perturbation analysis is that the $\beta$-eqs. in (1.7) can be viewed as the dynamical field equations derived from 26-dimensional field theory

$$S = \int d^{26} x \sqrt{-g} e^{-2\Phi} \left( R + 4 g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2.3!} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right), \quad (1.8)$$

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which describes some low-energy (long wavelength) physics for the massless background degrees of freedom. Action (1.8) is called the effective action for bosonic string moving in 26-dimensional target spacetime.

When we consider supersymmetric theories, the critical dimension is ten. The effective action given above can be viewed (for superstrings) as the bosonic part of the corresponding supergravity theory and the full effective action can be derived through the standard procedure of supersymmetrization.

### 1.2 T-duality

We have seen in the last section that string world-sheet action possesses conformal invariance along with spacetime symmetries like general coordinate invariance and gauge invariance. Now if we compactify one of the spacetime direction on a circle further new symmetry emerges. This symmetry is the invariance of the string spectrum under the inversion of the radius of the circle on which the string is moving [8, 9].

Let us first make an convenient choice of the gauge. The reparametrization and scaling invariance can be utilised to choose $h_{\alpha \beta} = \eta_{\alpha \beta}$ so that the string action is

$$S = \frac{-T}{2} \int d^2 \sigma \eta^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X_\mu, \quad (1.9)$$

where we have also taken the spacetime metric to be flat Minkowskian. The Equation of motion for $X^\mu$ is

$$\partial_\alpha \partial^\alpha X^\mu = 0. \quad (1.10)$$

For a closed string the coordinates $X^\mu$ satisfy the boundary condition

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + 2\pi) \quad (1.11)$$

which is just the periodicity condition along $\sigma$. The general solution of (1.10) which is consistent with the condition (1.11) is

$$X^\mu = X^\mu_R(\tau - \sigma) + X^\mu_L(\tau + \sigma),$$

$$X^\mu_R = x^\mu_R + \alpha' p^\mu_R(\tau - \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\alpha'^2}{n} e^{-in(\tau - \sigma)},$$

$$X^\mu_L = x^\mu_L + \alpha' p^\mu_L(\tau + \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\alpha'^2}{n} e^{-in(\tau + \sigma)}, \quad (1.12)$$
where \( x^\mu = x^\mu_L + x^\mu_R \) describe center of mass coordinates of the string having total momentum \( P^\mu = \int_0^{2\pi} d\sigma P^\mu(\sigma) = p^\mu_L + p^\mu_R \). Here \( P^\mu(\sigma) \) describe the momentum conjugate to \( X_\mu \),

\[
P^\mu(\sigma) = \frac{1}{2\pi} \left[ p^\mu_L + p^\mu_R + \frac{1}{\sqrt{2\alpha'}} \sum_{n \neq 0} (\alpha_n^\mu e^{-i(n-\tau)} + \tilde{\alpha}_n^\mu e^{-i(n+\tau)}) \right]. \tag{1.13}
\]

Various commutation relations are:

\[
[x^\mu_L, p^\mu_L] = [x^\mu_R, p^\mu_R] = \frac{i}{2} \eta^{\mu\nu}, \quad [\alpha^\mu_n, \tilde{\alpha}^\mu_n] = 0
\]

\[
[\alpha^\mu_n, \alpha^\mu_{n'}] = [\tilde{\alpha}_n^\mu, \tilde{\alpha}_n^\mu'] = m \delta_{m+n,0}. \tag{1.14}
\]

The boundary condition (1.11) implies that \( p^\mu_L = p^\mu_R = p^\mu \) and the commutation relations then imply that \( p^\mu_{L,R} \) can have the following representation

\[
p^\mu = -\frac{i}{2} \frac{\partial}{\partial x^\mu}. \tag{1.15}
\]

Now, let us come to the main issue of compactifying the string on to a circle. For a string configuration where \( X^9 \) is compactified on a circle of radius \( R \), the closed string boundary condition along the compactified direction modifies to

\[
X^9(\tau, \sigma) = X^9(\tau, \sigma + 2\pi) + 2\pi R m, \quad m \in \mathbb{Z}, \tag{1.16}
\]

where the string winds \( m \) number of times around the circle. This suggests that

\[
p^9_L - p^9_R = \frac{mR}{\alpha'}. \tag{1.17}
\]

The commutation relations then imply

\[
p^9_L = -\frac{i}{2} \frac{\partial}{\partial x^9} + \frac{mR}{2\alpha'}, \quad p^9_R = -\frac{i}{2} \frac{\partial}{\partial x^9} - \frac{mR}{2\alpha'}. \tag{1.18}
\]

The single valuedness of the the string wavefunction, \( \Psi(x) = \Psi_0 e^{i m x^9} \), then gives

\[
p^9_L = \frac{n}{2R} + \frac{mR}{2\alpha'}, \quad p^9_R = \frac{n}{2R} - \frac{mR}{2\alpha'}. \tag{1.19}
\]

It is interesting to note that a momentum vector defined as \( \frac{1}{\sqrt{2\alpha'}} p^9_L; \frac{1}{\sqrt{2\alpha'}} p^9_R \) covers two-dimensional Narain lattice, \( R^{1,1} \) (an even, self-dual Lorentzian lattice)[34], defined by the inner product

\[
\left( \frac{1}{\sqrt{2\alpha'}} p^9_L; \frac{1}{\sqrt{2\alpha'}} p^9_R \right)^2 = \left( \frac{1}{\sqrt{2\alpha'}} p^9_L \right)^2 - \left( \frac{1}{\sqrt{2\alpha'}} p^9_R \right)^2 = 2mn. \tag{1.20}
\]
The lattice remain invariant under the transformation

\[ R \rightarrow \frac{\alpha'}{R}, \quad m \leftrightarrow n \]  

(1.21)

Under (1.21), \( p_L^0 \rightarrow p_L^0 \) while \( p_R^0 \rightarrow -p_R^0 \). Similarly the Hamiltonian of the system

\[ H = \alpha'(p_L^0)^2 + p_R^0)^2 + \text{oscillator terms} \]  

(1.22)

remains invariant if oscillators transform appropriately. This implies that the spectrum of the theory remains invariant under (1.21). Since \( R \rightarrow \frac{\alpha'}{R} \)-duality is the symmetry under the inversion of the radius of the compactified target space dimension, it is also called “T-duality”. In essence T-duality is purely stringy in nature.

When generalising above situation to a compactification on a \( d \)-dimensional torus [11] for constant background configuration, the closed string boundary conditions along \( d \) compactified directions become

\[ X_\alpha(2\pi, \tau) = X_\alpha(0, \tau) + 2\pi m_\alpha, \quad \alpha, \beta = 1, \ldots, d \]  

(1.23)

where the integers \( m_\alpha \) are called winding numbers. It follows from the single-valuedness of the wave function on the torus that the zero modes of the canonical momentum, \( P_\alpha = G_{\alpha\beta}\partial_\tau X_\beta + B_{\alpha\beta}\partial_{\tau}X^\beta \), are also integers \( n_\alpha \). Therefore the zero modes of \( X_\alpha \) are given by (for \( 2\pi\alpha' = 1 \))

\[ X_0^\alpha = x^\alpha + m^\alpha \sigma + G^{\alpha\beta}(n_\beta - B_{\beta\gamma}n^\gamma)\tau, \]  

(1.24)

where \( G^{\alpha\beta} \) is the inverse of \( G_{\alpha\beta} \) as before. The Hamiltonian is given by

\[ \mathcal{H} = \frac{1}{2}G_{\alpha\beta}(\dot{X}^\alpha \dot{X}^\beta + \dot{X}^\alpha X'^\beta), \]  

(1.25)

where \( \dot{X}^\alpha \) and \( X'^\beta \) are derivatives with respect to \( \tau \) and \( \sigma \), respectively.

Since \( X^\alpha(\sigma, \tau) \) satisfies the free wave equation, we can decompose it as the sum of left- and right-moving pieces. The zero mode of \( P^\alpha = G^{\alpha\beta}P_\beta \) is given by \( p_L^0 + p_R^0 \) where

\[ p_L^\alpha = \frac{1}{2}[m^\alpha + G^{\alpha\beta}(n_\beta - B_{\beta\gamma}m^\gamma)] \quad \text{and} \quad p_R^\alpha = \frac{1}{2}[-m^\alpha + G^{\alpha\beta}(n_\beta - B_{\beta\gamma}m^\gamma)] \]  

(1.26)
The mass-squared operator, which corresponds to the zero mode of \( \mathcal{H} \), is given (aside from a constant) by

\[
(mass)^2 = G_{\alpha\beta}(p_L^\alpha p_L^\beta + p_R^\alpha p_R^\beta) + \sum_{m=1}^{\infty} \sum_{i=1}^{d}(\alpha_{-m}^i \alpha_m^i + \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i),
\]

(1.27)

As usual, \( \{\alpha_m\} \) and \( \{\tilde{\alpha}_m\} \) denote oscillators associated with right- and left-moving coordinates, respectively. Substituting the expressions for \( p_L \) and \( p_R \), the mass squared can be rewritten as

\[
(mass)^2 = \frac{1}{2} G_{\alpha\beta} m^\alpha m^\beta + \frac{1}{2} G_{\alpha\beta} (n_\alpha - B_{\alpha\gamma} m^\gamma)(\eta_\beta - B_{\beta\delta} m^\delta) + \sum (\alpha_{-m}^i \alpha_m^i + \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^i).
\]

(1.28)

It is significant that the zero mode portion of (1.28) can be expressed in the form

\[
(M_0)^2 = \frac{1}{2} (m \cdot n) M^{-1} \begin{pmatrix} n \\ n \end{pmatrix},
\]

(1.29)

where \( M \) is the \( 2d \times 2d \) matrix given by

\[
M = \begin{pmatrix} G^{-1} & -G^{-1} B \\ B G^{-1} & G - B G^{-1} B \end{pmatrix}.
\]

(1.30)

In order to satisfy \( \sigma \)-translation symmetry, the contributions of left- and right-moving sectors to the mass squared must agree (\( L_0 = \tilde{L}_0 \) in the usual notation). The zero mode contribution to their difference is

\[
G_{\alpha\beta}(p_L^\alpha p_L^\beta - p_R^\alpha p_R^\beta) = m^\alpha n_\alpha.
\]

(1.31)

Since this is an integer, it always can be compensated by oscillator contributions, which are also integers.

Equation (1.31) is invariant under interchange of the winding numbers \( m^\alpha \) and the discrete momenta \( n_\alpha \). Indeed, the entire spectrum remains invariant if we interchange \( m^\alpha \leftrightarrow n_\alpha \) and simultaneously let

\[
(G - B G^{-1} B) \leftrightarrow G^{-1} \quad \text{and} \quad B G^{-1} \leftrightarrow -G^{-1} B.
\]

(1.32)

These interchanges precisely correspond to inverting the \( 2d \times 2d \) matrix \( M \). This is the spacetime duality transformation generalizing the well-known duality \( R \leftrightarrow \alpha'/R \).
in the $d = 1$ case. The general duality symmetry implies that the $2d$-dimensional Lorentzian lattice spanned by the vectors $\sqrt{2}(p_L^\alpha, p_R^\alpha)$ with inner product

$$\sqrt{2} \,(p_L, p_R) \cdot \sqrt{2} \,(p'_L, p'_R) \equiv 2G_{\alpha\beta}(p^\alpha_L p^\beta_L - p^\alpha_R p^\beta_R) = (m^\alpha n'_\alpha + m'^\alpha n_\alpha),$$  \hspace{1cm} (1.33)$$
is even and self-dual [34].

The moduli space parametrized by $G_{\alpha\beta}$ and $B_{\alpha\beta}$ is locally the coset $O(d, d)/O(d) \times O(d)$ [34]. The global geometry requires also modding out the group of discrete symmetries generated by $B_{\alpha\beta} \rightarrow B_{\alpha\beta} + N_{\alpha\beta}$ and $G + B \rightarrow (G + B)^{-1}$. These symmetries generate the $O(d, d, Z)$ subgroup of $O(d, d)$. An $O(d, d, Z)$ transformation is given by a $2d \times 2d$ matrix $A$ having integral entries and satisfying $A^T \eta A = \eta$, where $\eta$ consists of off-diagonal unit matrices as before. Under an $O(d, d, Z)$ transformation

$$\begin{pmatrix} m \\ n \end{pmatrix} \rightarrow \begin{pmatrix} m' \\ n' \end{pmatrix} = A \begin{pmatrix} m \\ n \end{pmatrix} \quad \text{and} \quad M \rightarrow A M A^T .$$

It is evident that

$$m \cdot n = \frac{1}{2} (m \cdot n) \eta \begin{pmatrix} m \\ n \end{pmatrix},$$

which appears in eq. (1.31), and $M_0^2$ in eq. (1.29) are preserved under these transformations. The crucial fact, already evident from the spectrum, is that toroidally compactified string theory certainly does not share the full $O(d, d)$ symmetry of the low energy effective theory. It is at most invariant under the discrete $O(d, d, Z)$ subgroup. However, as emphasized by Sen [12], if the $X$ coordinates are not compactified, but still flat, so that $K = R^d$, there is a continuous $G(d) \times O(d)$ symmetry (the compact part of $O(d, d)$) corresponding to independent rotations of $X_L$ and $X_R$. The diagonal subgroup describes ordinary rotations of $K$.

Thus, T-duality manifests into a bigger symmetry when string theory is compactified on $d$-dimensional torus $T^d$. The compactified lower dimensional theory thus obtained acquires $O(d, d)$-symmetry under which the background fields transform in specific ways. The world-sheet manifestations of this symmetry requires the introduction of extra auxiliary(non-dynamical) world-sheet coordinates $Y^i$, $(1 \leq i \leq d)$[35, 11]. However, $O(d, d)$ symmetry manifests automatically in the corresponding low energy effective action. In the next section we shall be dealing with this symmetry of the effective action.
In the Hamiltonian formulation of the world-sheet theory, one can identify those 'canonical transformations' which give rise to T-duality transformation of the background fields. This programme has been well understood when backgrounds have Abelian isometries only\cite{36, 37}. However, when non-Abelian isometries are present the T-duality and corresponding canonical structure is not well understood\cite{38}.

1.3 \textbf{O(d,d) symmetry in the string effective action}

In what follows, we discuss dimensional reduction of the string effective action on a $d$-dimensional torus \cite{39}. We shall follow the notational convention of the ref.\cite{11}. In toroidal compactification the background fields are taken to be independent of the directions which are compactified. First, to learn the general procedure of dimensional reduction let us start with the following action in $D$ spacetime dimensions,

$$S = \int d^D x \sqrt{-\hat{g}} e^{-2\Phi} \left( \hat{R}_\flat + 4\hat{g}^{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}} \hat{\Phi} \partial_{\hat{\nu}} \hat{\Phi} - \frac{1}{12} \hat{H}_{\hat{\mu}\hat{\nu}\hat{\lambda}} \hat{H}^{\hat{\mu}\hat{\nu}\hat{\lambda}} \right)$$ \hspace{1cm} (1.34)

where we have attributed $D$-dimensional fields with hats over them and have adopted hatted indices $\hat{\mu}, \hat{\nu}, \cdots$ for $D$-dimensional indices. Here $\hat{R}_\flat$ is the Ricci scalar for the spacetime metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ whose determinant is represented by $\hat{g}$. $\hat{\Phi}$ is the dilaton field and the 3-rank tensor defines antisymmetric field strength

$$\hat{H}_{\hat{\mu}\hat{\nu}\hat{\lambda}} = \partial_{[\hat{\mu}} \hat{B}_{\hat{\nu}\hat{\lambda}]}$$ \hspace{1cm} (1.35)

where notation $[\hat{\mu}\hat{\nu}\cdots]$ stands for the antisymmetrisation of the indices. For the dimensional reduction scheme on $T^d$, we shall break the coordinates $x^\hat{\mu}$ into two patches, viz., "spacetime" coordinates $x^\mu$ with $\mu = 0, 1, \cdots, (D - d - 1)$ and $x^\alpha$ with $(\alpha = D - d, \cdots, D)$ representing "internal" coordinates tangent to the torus. Let us denote lower dimensional fields with out hats. We adopt the parameterization of the D-dimensional vielbein as $e_\alpha^\mu = \begin{pmatrix} e_\mu^\alpha A^{(1)}_{\mu\nu} E_\nu^\beta \\ 0 & E_\alpha^\beta \end{pmatrix}$ such that the spacetime metric is $g_{\mu\nu} = e_\mu^\alpha \eta_{\nu\sigma} e_\nu^\sigma$ and the internal metric is $G_{\alpha\beta} = E_\alpha^\sigma \delta_{\sigma\beta} E_\beta^\beta$. This gives

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} + A_\mu^{(1)\alpha} A_\nu^{(1)} A_\alpha^{(1)} & A_{\mu\beta}^{(1)} \\ A_{\nu\alpha}^{(1)} & G_{\alpha\beta} \end{pmatrix}.$$ \hspace{1cm} (1.36)
Similarly, for the antisymmetric tensor field, coming from the NS-NS sector, the decomposition is

\[ A^{(2)}_{\mu\alpha} = \dot{B}_{\mu\alpha} - \dot{A}^{(1)\beta}_{\mu} B_{\alpha\beta}, \quad b_{\alpha\beta} = \dot{B}_{\alpha\beta}, \]

\[ B_{\mu\nu} = \dot{B}_{\mu\nu} - \frac{1}{2} \dot{A}^{(1)\alpha}_{[\mu} A^{(2)}_{\nu]\alpha} - \dot{A}^{(1)\alpha}_{\mu} A^{(1)\beta}_{\nu} b_{\alpha\beta}. \]  

(1.37)

Now recall that the scalar densities are constructed in the \( D \)-dimensional theory by contracting the hat indices of various tensors. In order to obtain tensors with unhatted indices we can adopt the following prescription:

\[ \mathcal{H}_{\mu
u\alpha\beta..} = O^\mu_\mu O^\nu_\nu \cdots \hat{\mathcal{H}}_{\mu\nu\alpha\beta..} \]  

(1.38)

where \( O^\mu_\mu = e^\mu_r \delta_r^\mu \) and \( \hat{\mathcal{H}}_{\mu\nu\alpha\beta..} \) is a tensor in \( D \) dimensions. Thus scalars constructed out of contraction of \( D \)-dimensional indices, as is the case with kinetic energy terms in the action, can be expressed in the following form in terms of scalars constructed out of various tensors in \( D \)-dimensions, obtained through the dimensional reduction procedure,

\[ \hat{\mathcal{H}}_{\mu\nu..} \hat{\mathcal{H}}_{\mu\nu..} = \mathcal{H}_{\mu\nu..} \mathcal{H}_{\mu\nu..} + n \mathcal{H}_{\mu\nu..} \mathcal{H}^{\mu\nu..} + \frac{n(n-1)}{2!} \mathcal{H}_{\mu\nu..} \mathcal{H}^{\mu\nu..} \mathcal{H}_{\mu\nu..} + \cdots + \mathcal{H}_{\alpha\beta..} \mathcal{H}^{\alpha\beta..}, \]  

(1.39)

for an \( n \)-form field strength. Following (1.38) NS-NS field strengths are obtained as below,

\[ H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} - F^{(1)\delta}_{\mu\nu} A^{(2)}_{\rho\delta}, \]

\[ H_{\mu\nu\alpha} = F^{(2)}_{\mu\nu\alpha} - F^{(1)\delta}_{\mu\nu} b_{\alpha\delta}, \]

\[ H_{\mu\alpha\beta} = \partial_{\mu} b_{\alpha\beta}. \]  

(1.40)

where \( F^{(\alpha)}_{\mu\nu} = \partial_{[\mu} A^{(\alpha)}_{\nu]} \).

After the substitutions, we get the following theory in \((D - d)\)-dimensions

\[ S = \int d^{D-d}X \sqrt{-\det g_{\mu\nu}} e^{-2\Phi} \left[ R_g + 4g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} \text{Tr} \partial_{\mu} M^{-1} \partial^\mu M - \frac{1}{4} F^i_{\mu\nu} (M^{-1})_{ij} F_j^{\mu\nu} - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right] \]  

(1.41)

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where index $i = 1, \cdots, 2d$ runs over $O(d,d)$ indices,

$$
2\Phi = 2\hat{\Phi} - \frac{1}{2} \ln \text{det} G_{\alpha\beta}
$$

$$
M = \begin{pmatrix}
G^{-1} & -G^{-1}B \\
BG^{-1} & G - BG^{-1}B
\end{pmatrix}, \\
\mathcal{F}_{\mu\nu} \equiv \begin{pmatrix}
F_{\mu\nu}^{(1)} \\
F_{\mu\nu}^{(2)}
\end{pmatrix}.
$$

Note that the matrix valued field $M$ satisfies $M \mathcal{L} M^T \mathcal{L} = 1$, where $\mathcal{L}$ is the $O(d,d;R)$ metric

$$
\mathcal{L} = \begin{pmatrix} 0 & I_d \\
I_d & 0 \end{pmatrix}, \\
\Omega^T \mathcal{L} \Omega = \mathcal{L}.
$$

Here $I_d$ is d-dimensional identity matrix and $\Omega$ is an element of the group $O(d,d;R)$.

Now it is straightforward to check that the action (1.41) is manifestly invariant under the following noncompact continuous global transformations

$$
M \rightarrow \Omega M \Omega^T \\
\Phi \rightarrow \Phi, \ g_{\mu\nu} \rightarrow g_{\mu\nu}, \ H_{\mu\nu\lambda} \rightarrow H_{\mu\nu\lambda}, \\
A_{\mu}^i \rightarrow \Omega_j^i A_{\mu}^j
$$

which form a noncompact group $O(d,d;R)$. The total number of $O(d,d)$ generators is $2d^2 - d$ out of which the $\frac{d(d-1)}{2}$ generators of the form $\Omega_\Theta = \begin{pmatrix} I & \Theta \\
0 & I \end{pmatrix}$, $\Theta_{ij} = -\Theta_{ji}$, correspond to the constant shifts of the antisymmetric tensor field $B_{\alpha\beta}$ which already is a gauge symmetry. The transformations of the form $\Omega_A = \begin{pmatrix} A & 0 \\
0 & (A^T)^{-1} \end{pmatrix}$, $A \in GL(d,R)$, do correspond to the general coordinate transformations in the internal space. These form $\frac{d(d+1)}{2}$ of the generators. The left over $d(d-1)$ generators can be written in the form

$$
\Omega_{O(d) \times O(d)} = \frac{1}{4} \begin{pmatrix} R+S & R-S \\
R-S & R+S \end{pmatrix},
$$

where $R,S \in O(d,R)$, and do belong to the compact subgroup $O(d) \times O(d)$. These exhaust all the possible transformations of $O(d,d)$. In contrast to $\Omega_\Theta$ and $\Omega_A$, the transformations $\Omega_{O(d) \times O(d)}$ are less obvious symmetry of the spectrum. It relates the states in the string spectrum which are not related through general coordinate transformations of $G_{\alpha\beta}$ and gauge transformations of $B_{\alpha\beta}$ [40]. Therefore, new string backgrounds can be generated by the action of the transformations $O(d) \times O(d)$ on 15
the known background solutions of the string equations of motion which are nothing but $\beta$-equations. We shall be dealing with this aspect of $O(d, d)$ symmetry in second chapter. The dimensionality of the coset space $\frac{O(d,d;\mathbb{R})}{O(d) \times O(d)}$ matches exactly with the dimensions of the moduli space, $d^2$, which is parametrized by the matrix $M$. The transformations $O(d) \times O(d)$ take the theory from one point in the moduli space to another.

The continuous symmetry group is broken down to the discrete subgroup $O(d, d; \mathbb{Z})$ if spacetime instanton corrections are taken into account in the effective action. A world-sheet analysis of the $O(d, d)$ symmetry shows that closed string boundary conditions only respect the discrete group $O(d, d; \mathbb{Z})$ as a symmetry of the spectrum [11]. It is argued that $O(d, d; \mathbb{Z})$ is the exact duality symmetry of full quantum string theory and holds good in string perturbation theory[40, 41]. The transformations of the form

$$\Omega_i = \begin{pmatrix} I_d - e_i & e_i \\ e_i & I_d - e_i \end{pmatrix}$$

where $e_i$ is a $d \times d$-matrix whose non-vanishing entry is the element $e_{ii}$, inverts the radius of the $i$-th circle for the background metric which is a direct product of the $i$-th circle and $(d - 1)$-dimensional background. This particular element gives T-duality and belongs to $O(d, d; \mathbb{Z})$ group.

1.4 Strong-weak duality

The symmetry which maps strong coupling to the weak coupling phases in the supersymmetric gauge theories[14, 15, 16] has been a focus of much attention in string theory[17, 18, 20, 19, 21] also. The symmetrized Maxwell equations (i.e., in the presence of monopole charges)

$$\nabla \cdot E = 4\pi e, \quad \nabla \times E + \frac{\partial B}{\partial t} = k$$

$$\nabla \cdot B = 4\pi g, \quad \nabla \times B - \frac{\partial E}{\partial t} = j$$

and the Dirac quantization condition $e g = 2\pi n, \; n \in \mathbb{Z}$, between monopole charge $g$ and electric charge $e$ remain invariant under

$$E \rightarrow B, \; B \rightarrow -E, \; e \rightarrow g, \; g \rightarrow -e, \; j \rightarrow k, \; k \rightarrow -j.$$
Followed by this, Montonin and Olive[13] were led to formulate a conjecture for electric-magnetic duality in gauge theories such that in strong coupling regime of the gauge theory a monopole should become point like while the fundamental particle must become fat. In string theory the vacuum expectation value of $e^\Phi$ plays the role of the coupling. Thus under

$$\Phi \rightarrow -\Phi \quad (1.48)$$

the string theory will go from weak coupling regime to strong coupling regime. This exchange duality has been recently of very much interest in string theory. The existence of strong-weak symmetry in string theory would strongly constraint the vast vacuum structure of the theory and could also help in determining the unique vacuum structure.

Let us demonstrate the duality in (1.48), conventionally called S-duality[20], in the framework of string effective action. We write down a four-dimensional string effective action which can be obtained from the dimensional reduction of the action in ten dimensions,

$$S = \int d^4x \sqrt{-\det g_{\mu\nu}} \ e^{-2\Phi} \left[ R_g + 4g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} \text{Tr} \partial_\mu M^{-1} \partial^\mu M \
- \frac{1}{4} \mathcal{F}_\mu^i (M^{-1})_{ij} \mathcal{F}_\nu^{j\mu
\nu} - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right] \quad (1.49)$$

Let us perform the scaling transformation $g_{\mu\nu} \rightarrow e^{-2\Phi} g_{\mu\nu}$, and introduce axion field, $\chi$, as dual of a 3-rank tensor field strength in four dimensions, $\partial_\tau \chi = (\eta^2/6) \sqrt{-g} \epsilon_{\mu\nu\lambda\sigma} H^{\mu\nu\lambda}$ where we have defined $\eta = e^{-2\Phi}$. Then (1.49) can be reexpressed as

$$S = \int d^4x \sqrt{-g} \left( R - \frac{1}{2} \eta^2 g^{\mu\nu} \partial_\mu \Psi \partial_\nu \bar{\Psi} + \frac{1}{8} \text{Tr} (\partial_\mu M^{-1} \partial^\mu M) \right. \
- \frac{1}{4} \eta \mathcal{F}_\mu^i M^{-1}_{ij} \mathcal{F}_\nu^{j\mu\nu} + \frac{1}{4} \chi \mathcal{F}_\mu^i \mathcal{L}_{ij} \mathcal{F}^{j\mu\nu} \right), \quad (1.50)$$

where $\Psi = \chi + i \eta$ is a complex scalar field and

$$\mathcal{F}_\mu^i = \frac{1}{2} \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} \mathcal{F}^{i\nu\rho\sigma}. \quad (1.51)$$
The equations of motion corresponding to $\Psi$, $g_{\mu\nu}$ and $A_\mu$ derived from the action (1.50) are

$$\nabla_\mu \nabla^\mu \Psi \frac{i}{\eta^2} + i \frac{\nabla_\mu \Psi \nabla^\mu \Psi}{\eta^3} - \frac{i}{4} \mathcal{F}^i_{\mu\nu} M^{-1}_{ij} \mathcal{F}^j_{\mu\nu} + \frac{1}{4} \mathcal{F}^i_{\mu\nu} \mathcal{L}_{ij} \mathcal{F}^j_{\mu\nu} = 0,$$

$$R_{\mu\nu} - \frac{\nabla_\mu \Psi \nabla_\nu \Psi}{2 \eta^2} + \frac{1}{8} \text{Tr}(\partial_\mu M^{-1} \partial_\nu M)$$

$$- \frac{\eta}{2} \mathcal{F}^i_{\mu\lambda} M^{-1}_{ij} \mathcal{F}^j_{\nu\lambda} + g_{\mu\nu} \left( \frac{\eta}{8} \mathcal{F}^i_{\mu\nu} M^{-1}_{ij} \mathcal{F}^j_{\mu\nu} \right) = 0,$$

$$\nabla_\mu \left( \eta (M \mathcal{L})_{ij} \mathcal{F}^j_{\mu\nu} - \chi \mathcal{F}^i_{\mu\nu} \right) = 0; \quad (1.52)$$

and the Bianchi identity is

$$\nabla_\mu \mathcal{F}^i_{\mu\nu} = 0. \quad (1.53)$$

Now let us note down the S-duality transformations [20]

$$\Psi \rightarrow \frac{a \Psi + b}{c \Psi + d}, \quad a d - b c = 1, \quad a, b, \cdots \in \mathbb{R},$$

$$\mathcal{F}^i_{\mu\nu} \rightarrow c \eta (M \mathcal{L})_{ij} \mathcal{F}^j_{\mu\nu} + (c \chi + d) \mathcal{F}^i_{\mu\nu} \quad (1.54)$$

while the metric $g_{\mu\nu}$ and moduli $M$ remain invariant.

Explicit calculations show that under $SL(2, \mathbb{R})$ transformations in (1.54) eqs. (1.52) and (1.53) remain invariant. However, the action (1.50) as such does not remain invariant under the transformations in (1.54) because the Maxwell field strength gets modified. It is not surprising; the Maxwell action does not remain invariant under $E \rightarrow B, B \rightarrow -E$ in electrodynamics though the source-free Maxwell equations do so. In fact the symmetrized (in the presence of both electric and magnetic charges) Maxwell equations (1.47) do get interchanged under duality, leaving behind the dynamical laws intact. Let us come back to string theory. We have seen that string effective action does not remain invariant under $SL(2, \mathbb{R})$-duality transformations though the corresponding equations of motion make a duality invariant set of dynamical laws. Thus, if we find a solution to these equations of motion, we can generate new solutions through implementation of the $SL(2, \mathbb{R})$ transformations.

It has been conjectured that the discrete subgroup $SL(2, \mathbb{Z})$, or S-duality group, can in fact be the exact symmetry of the string theory. But the check of a non-perturbative symmetry is a nontrivial task. However, string theory possesses large
amount of supersymmetry also. A test of S-duality can be done by involving BPS-saturated states. Due to supersymmetric nonrenormalisation theorems a BPS state remain BPS even quantum machenically because firmicnic and bosonic loop-corrections to their masses and charges cancel due to supersymmetry. In other words a BPS state is also a solution of full quantum theory. If S-duality is exact, the BPS fundamental state at weak coupling must correspond to a BPS solitonic state in strong coupling regime and viceversa. This equivalence of string BPS spectrum must hold at all couplings because the number of states cannot change as the coupling is made strong slowly.

1.5 Consequences of dualities

There have been rapid developments in our understanding of string dynamics in last couple of years. We may recall that T-duality can be tested in a perturbative frame work. On the other hand, since the S-duality relates strong and weak coupling domains, this can be tested only in the nonperturbative manner.

One of the most important result was to discover that the strong coupling limit of the type IIA theory is related to the eleven-dimensional supergravity theory. If one starts from the $D = 11$ supergravity action\textsuperscript{[7]}, the bosonic sector consists of the graviton and the three index antisymmetric tensor potential. The action consists of the Einstein-Hilbert part for gravity and the kinetic energy term for the antisymmetric tensor field as

\[ S_{11} = \int d^{11}x \sqrt{-\mathcal{g}^{(11)}} \left[ \mathcal{R} - \frac{1}{2 \cdot 4!} \mathcal{G}_{\mu\nu\lambda\rho} \mathcal{G}^{\mu\nu\lambda\rho} \right] + \text{Chern – Simon term} \tag{1.55} \]

where $\mathcal{G}_{\mu\nu\lambda\rho} = \partial_{[\mu} \mathcal{A}_{\nu\lambda\rho]}^{(3)}$ is the field strength of a three rank antisymmetric tensor field $\mathcal{A}_{\nu\lambda\rho}^{(3)}$. If this theory is compactified on a circle, say of radius $e^\gamma$, then the standard Kaluza-Klein reduction tells us that the resulting ten-dimensional theory now has graviton, a three index antisymmetric tensor; furthermore, there arises a gauge field and a scalar from the metric. Also, we get a two index antisymmetric tensor field due to reduction of the three index antisymmetric tensor of the $D = 11$ theory. These are exactly the fields which appear in type IIA string theory in ten dimensions. However, the action obtained from dimensional reduction, at the first sight, does not look like
an action which has any connection with the type IIA effective action. However, if
the spacetime metric is Weyl rescaled with a suitable choice (it is related to the scalar
field) then, one immediately recovers the type IIA effective action \[ \text{(1.56)} \]

\[
S_{\text{IIA}} = \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} \left( R_g + 4 \partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{2 \cdot 3!} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) - \frac{1}{2 \cdot 2!} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2 \cdot 4!} G_{\mu\nu\lambda\rho} G^{\mu\nu\lambda\rho} \right] + C - \text{S term}.
\]

Under this identification the radius of compactification of the $D = 11$ supergravity
gets related to the coupling constant (i.e. exponential of dilaton) of type IIA, as $e^{-2\Phi} = e^{-3\gamma}$.

In ten dimensions, the type IIB theory is conjectured to possess $SL(2, R)$ S-duality
group. It was shown by Schwarz that the presence of this symmetry leads to many
interesting consequences [42]. Furthermore, when the theory is compactified on a
circle to nine dimensions, one can relate it to the type IIA theory through T-duality.
We also know that type IIA theory is intimately connected to the $D = 11$ supergravity
theory. Thus in $D = 9$, the type IIB theory must have some connection with the eleven
dimensional supergravity. Indeed, it was found that the $D = 9$ type IIB theory, is
related to the $D = 11$ theory compactified on $T^2$.

In view of these evidences, it was proposed by Schwarz [24] that, there must be
an underlying fundamental theory such that all the five consistent string theories can
be viewed as different phases of this unique theory. Although, one does not know
this fundamental theory, the M-theory, it is expected that the low energy limit of this
theory should correspond to the $D = 11$ membrane theory. Subsequently, there has
been tremendous progress in this field to relate the dynamics of string theories in
various dimensions.

It is worthwhile to recall that the idea of string-string duality [22, 23, 43] played
a very important role in these developments. In four dimensions, the dual of the
two index field strength of the vector potential is also a two index object and thus
the dual of a charged particle is the monopole which couples to the dual tensor.
In six dimensions, the dual of a three index antisymmetric tensor is a three index
antisymmetric tensor too. Thus, the dual of a string is another string. One can look
for a fundamental string in six dimensions and then its solitonic counterpart; just like if we have charged particles in four dimensions then the monopoles are the corresponding solitons. Indeed, the string-string duality has been tested where it has been found that the heterotic string compactified on $T^4$ is S-dual to type IIA compactified on $K_3[43]$. From M-theory perspective, we can visualise the situation as follows [24]. A type IIA string compactified on $K_3$ is equivalent to M-theory compactified on $K_3 \times S^1$. Thus it can be inferred that heterotic compactified on $T^3 \times S^1$ must also be related to M-theory, i.e.,

$$M \text{- theory on } K_3 \equiv \text{Heterotic on } T^3,$$

which will require the wrapping of M-theory five-brane on four-cycles of $K_3$ to get the heterotic string. Similarly, type IIB compactification on $K_3$ also has dual interpretation in terms of M-theory on the orbifold $T^5/Z_2$. However, this orbifolding of $T^5$ compactification by $Z_2$ introduces 32 orbifold points each charged with $-\frac{1}{2}$ unit of magnetic charge. To cancel this charge one needs to introduce 16 M-theory 5-branes each carrying $+1$ unit of magnetic charge. It is to note that the M-theory 5-brane is Poincare-dual to M-theory 2-brane and, therefore, carries magnetic charge.

### 1.6 Plan of the thesis

The purpose of this thesis is to study symmetries of effective action and obtain classical solutions of the corresponding equations of motion. In string theory, we know that the equations of motion derived from the effective action correspond to the condition of the vanishing of the $\beta$-functions. Therefore, each solution set of solutions can be interpreted as vacuum configurations of the string theory. Furthermore, under the symmetry transformations one can go from one vacuum configuration to another. In several cases, a T-duality transformation relates two sets of backgrounds. Furthermore, S-duality will relate strong and weak coupling domains. The text in this thesis is organized as follows:

In chapter 2 we shall study about various duality symmetries of the string theory in the presence of the cosmological constant term. We derive both Minkowskian and Euclidean backgrounds and then by exploiting the $O(d,d)$ symmetry generate new background solutions of the string equations. We also study the consequences
of strong-weak duality in detail, along with specific examples. It will be argued that since the equations of motion of string effective action do not remain invariant under S-duality transformation in the presence of cosmological constant, one can use 't Hooft's naturalness arguments and claim that cosmological constant should be a small number.

In the recent past, there was considerable attention to work out the duality symmetries in string theory in Hamiltonian formulation. The aim was to understand T-duality on the world-sheet as a canonical symmetry of the corresponding Hamiltonian. This programme has been well understood for the backgrounds having Abelian isometries.

In Chapter III, we study another aspect of S-duality, the $SL(2, \mathbb{R})$ symmetry, for the string theory. The usual form of string effective action is not invariant under S-duality whereas the equations of motion are. However, it was shown by Schwarz and Sen that one can write a modified form of the effective action by introducing auxiliary fields such that the resulting action is invariant under S-duality and the field transform linearly under S-duality. Thus we can derive the conserved currents associated with the transformation and then visualise S-duality transformation, at least classically, as canonical transformation in the underlying Hamiltonian phase space.

Chapter IV contains a study of type-IIA superstrings in presence of ten-form field strength. In ten dimensions dual of ten-form field strength is a cosmological constant itself. Therefore, it is interesting to study various symmetries in lower dimensional compactification of this theory. This theory allows maximally symmetric black hole solutions.

Chapter V contains author's study on spacetime wormholes and derivation of vertex operators for various fields in the wormhole background. Wormholes are interesting Euclidean objects which are inevitable in a theory of quantum gravity. They carry definite amount of charge and other quantum numbers. The branching off of the wormholes from our universe might lead to the loss of certain global quantum numbers in the universe. Some of these effects caused due to wormholes were amongst the interesting topics in recent past.

We summarise the main results of our thesis in Chapter VI.