Chapter 2

Phase Transition

2.1 Introduction

In the introductory chapter, we have introduced several phase transitions which are predicted to have occurred in the early universe. However, the phenomenon of phase transition is very common even in our everyday life. The most familiar examples which we encounter in our day-to-day life are the melting of ice, boiling of water, etc. Several other phase transitions which are observed in the laboratory are the transition from para to ferro magnet below the curie temperature, $\lambda-$ transition of liquid He^4 from normal to superfluid phase etc.

Phase transition is usually defined in terms of singularity of the free energy or any thermodynamics quantities of the system (see, ref.[37] for more formal definition and detailed discussions on phase transition). The order of the phase transition is defined by the behavior of the first derivative of a thermodynamic observable of the system. It is called first order or of continuous type, if the above derivative diverges or continuous across the phase boundary, respectively. For a large class of systems, the phase transition occurs because of "ordering" (or, spontaneous symmetry breaking). As for example, para to ferro magnetic transition as mentioned above is associated with the ordering of the magnetic moments in the low temperature phase. Of course, "ordering" phenomenon is not necessarily a requirement for all phase transitions. For example, there is no symmetry breaking or ordering involved in liquid-vapor transition (both are isotropic). For former cases, it is sometimes more convenient to define the order of transition according to the behavior of the order parameter (or, scalar field known as Higgs field, in the context of particle physics) across the phase boundary. Generally, value of the order parameter (OP) is taken as zero in the less ordered phase (or symmetric phase) and (non-zero) finite in the ordered (or, symmetry breaking phase). The value of OP may change discontinuously or continuously (as shown in Fig.2.1) during phase transition as a function of the controlled parameter like temperature, pressure etc. The nature of the transition is classified as first order

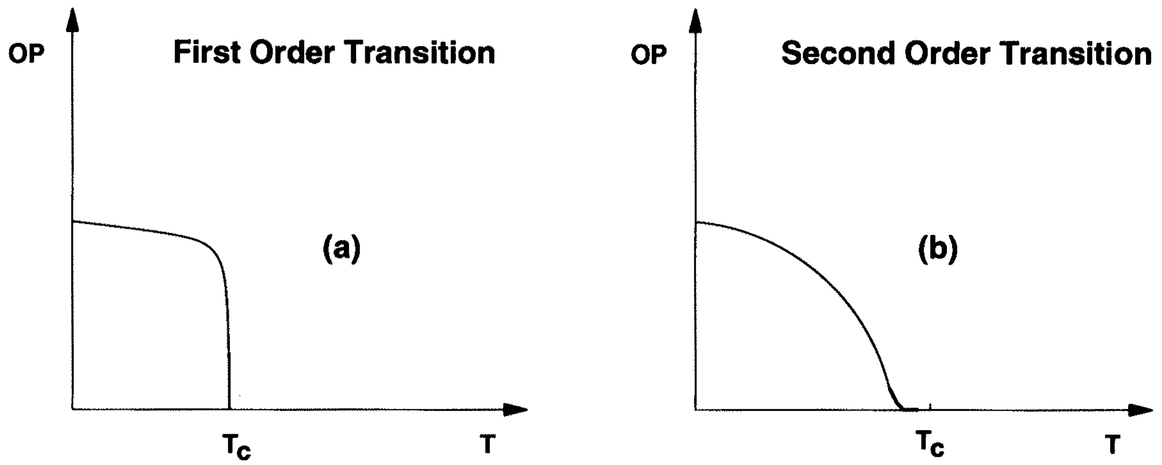

Figure 2.1: Typical behavior of the order parameter (OP) as a function of temperature for first order and second order phase transition, respectively.

in the former case or continuous in the later. Clearly, one needs to know the order parameter to determine the nature of the phase transition using this definition. As for example, magnetization (M) is considered to be an order parameter for para to ferro transition. Such transition is known to be of continuous type (in the absence of external magnetic field) in nature. Similarly, for liquid-vapor transition which is an example of first order transition, density is taken to be an order parameter. In contrast to the above examples, we come across such a transition like deconfinement-confinement transition in QCD, where no fundamental order parameter is known yet, though a good order parameter which appears to capture the basic physics of the transition is provided using the polyakov loop [38].

2.2 SSB and Phase Transition

In this section, we will focus on those phase transitions which are associated with the spontaneous symmetry breaking (SSB). Spontaneous symmetry breaking is an idea which was first originated in condensed matter physics [39]. Consider again the para to ferro-magnetic transition. The Hamiltonian of this system is rotationally invariant, but the ground state need not. All the magnetic moments point in the same direction in the ferromagnetic phase. This phenomenon is called spontaneous symmetry breaking. For a given theory, the symmetry is said to be spontaneously broken if the ground state is not invariant under the full symmetry of the Lagrangian.

The idea of spontaneous symmetry breaking plays a central role in unified theories in particle physics. For example electroweak theory, unifying weak & electromagnetic interactions is based on SSB. Similarly GUT models, which attempt to unify strong interactions with electroweak interactions are also based on SSB. The phenomenon of SSB is most often described by a scalar field, called Higgs field in the context of particle physics. In condensed matter system it is called order parameter as we have already introduced in the previous section. Underline picture of a unified theory is that, prior to the symmetry breaking, all vector bosons (which mediate the interactions) are massless, and there are no fundamental differences among the interactions. After symmetry breaking, however, some of the vector bosons acquire mass. The remaining symmetry corresponds to smaller gauge group leading to differences in various interactions.

The essential feature of the SSB can be understood by the following example which will also be useful while discussing the formation of topological defects [6] in the next section. Consider the Lagrangian [40, 41]

$$\mathcal{L} = \partial_\mu \Phi \partial^\mu \Phi^* - V(\Phi) \tag{2.1}$$

with

$$V(\Phi) = \frac{1}{4}\lambda(|\Phi|^2 - \eta^2)^2. \tag{2.2}$$

Here, Φ is a complex scalar field and λ, η are positive real constants. The Lagrangian is invariant under U(1) global transformation,

$$\Phi(x) \rightarrow e^{i\alpha}\Phi(x). \tag{2.3}$$

The minimum of the potential $V(\Phi)$ lie in the circle $|\Phi| = \eta$. Thus, the vacuum manifold $\mathcal{M}$ is a circle S^1. The ground states of the vacuum are characterized by

$$< 0|\phi|0 >= \eta e^{i\beta} \neq 0. \tag{2.4}$$

A phase transformation changes β into $\beta + \alpha$, hence the ground state is not invariant under the symmetry transformation given by Eq.(2.3). As a consequence of the spontaneous symmetry breaking of the theory, massless Goldstone bosons appear (Goldstones' theorem [42]). To determine the particle spectrum in this model, consider the fluctuation of the field Φ around a particular vacuum state. For convenient, we choose the vacuum state with vanishing phase, $< 0|\phi|0 >= \eta$. Expanding the field around this vacuum state yields,

$$\Phi(x) = (\eta + \frac{1}{\sqrt{2}}\rho(x))e^{i\theta(x)}. \tag{2.5}$$

Here, ρ and θ are the real fields. In terms of these fields, the Lagrangian, Eq.(2.1) can be written as [40],

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + (\partial_\mu \theta)^2 - \frac{1}{2}\lambda\eta^2\rho^2 + \mathcal{L}_{int}(\rho, \theta). \tag{2.6}$$

The interaction part of the Lagrangian can be obtained from Eq.(2.1) and Eq.(2.2). The form of the Lagrangian shows that the field ρ corresponding to the massive particle with mass, $m = \sqrt{(\lambda\eta^2)}$. While the field θ, representing the angular excitation becomes massless. Despite of it's simplicity, this model captures the essential physics of SSB. Whenever a continuous global symmetry is broken spontaneously, massless Goldstone bosons emerge. The number of Goldstone bosons will be equal to the dimension of the vacuum manifold (or, number of broken generators). In a phenomenological description, a well known example of Goldstone bosons are pions [31], which appear as a consequence of SSB of chiral symmetry in QCD in the massless limit of quarks. We will describe this in the next section.

In the above example, we have considered a model which is invariant under *global* U(1) symmetry. If the symmetry which is spontaneously broken is a gauge symmetry, then it leads to the phenomenon of Higgs mechanism [16], where some of the Goldstone bosons are absorbed by the gauge bosons leading to a theory with massive gauge bosons (in the broken phase). This is what happens in the electroweak theory. The simplest example of a local gauge symmetry is realized by the following Abelian Higgs model

$$\mathcal{L} = D_\mu \bar{\Phi} D^\mu \Phi - \frac{1}{4}\lambda(|\Phi|^2 - \eta^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2.7}$$

Where Φ is again a complex scalar field, $\mathcal{D}_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength. V(Φ) is the same potential as in Eq.(2.2). The Lagrangian, Eq.(2.7) is invariant under *local* U(1) symmetry

$$\Phi(x) \rightarrow e^{i\alpha(x)}\Phi(x); \quad A_\mu(x) \rightarrow A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x). \tag{2.8}$$

However, the ground state is not invariant under this symmetry. Thus, the symmetry is spontaneously broken and Higgs field acquires vacuum expectation value (VEV) $< 0|\phi|0 >= \eta$. One can observe the particle spectrum as before by fluctuating the field around the vacuum state $< 0|\phi|0 >= \eta$ and the Lagrangian in terms of the field ρ now becomes [40],

$$\mathcal{L} = (\partial_\mu \rho)^2 - \frac{1}{2}\lambda \eta^2 \rho^2 + \frac{1}{2} M A_\mu A^\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{int}. \tag{2.9}$$

Note that the field θ which appeared in global symmetry breaking case is no more a degree of freedom. This is *eaten up* by gauge boson A_μ, which becomes massive with mass $M = \sqrt{2}e\eta$. The mass of the ρ field corresponding to the radial excitation is same as before, with $m = \sqrt{\lambda}\eta$.

The model described above will be useful later in §2.3.1, while discussing the formation of local cosmic string and in finding out the static string solution. In the following section, we will give two specific purposeful examples of spontaneous symmetry breaking which are very relevant and will be useful later in chapter 5 and chapter 6 of this thesis.

2.2.1 An Example : Chiral Symmetry Breaking

In the introductory chapter, we have already introduced the series of phase transitions which are believed to have occurred in the early universe. QCD phase transition is one of them. There is a hope that the QCD transition can be probed in experiments involving heavy ion collisions. The main motivation of the ongoing experiment at RHIC [20] and upcoming LHC at CERN [17] is to recreate the quark-gluon plasma state which prevailed in the early universe until it was few microsecond old. These studies under controlled environment will help us to understand various aspects of the phase transitions in the early universe.

There are two main issues related to the QCD phase transition, namely, restoration of chiral symmetry (chiral phase transition [43]), and deconfinement of quarks and gluons to from quark-gluon plasma [44]. Although, lattice studies [30] indicates that both transition occur at the same temperature, there is no satisfactory answer yet, why this is so. These issues have been discussed elsewhere in the literatures (see ref.[45] and the references there in). Here, we will briefly describe the realization of chiral symmetry breaking in QCD and consequently, appearance of pions as Goldstone bosons. The chiral symmetry of the QCD can be understood from the QCD Lagrangian

$$\mathcal{L} = \sum_{f=1}^{N_f} \bar{q}_f(i\gamma^\mu D_\mu - m_f)q_f - \frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}. \tag{2.10}$$

The Lagrangian is invariant under the symmetry transformations corresponding to the gauge group $SU(3)_c$. m_f denotes the mass of a quark flavor q_f. In addition to the gauge symmetry, the Lagrangian also exhibits a global chiral symmetry in the massless limit of quarks. For simplicity, let us consider only the u and d quarks. Splitting the quark fields in term of left handed and right handed components, one can write down the above Lagrangian in the following form (ignoring the kinetic energy term of the gauge field, which is not relevant for the present purpose)

$$q_{L,R} = \frac{1}{2}(1 \pm \gamma_5)q \tag{2.11}$$

Then, the Lagrangian Eq.(2.10) can be written as

$$\mathcal{L} = \sum_{q=u,d} \bar{q}_L(i\gamma^\mu D_\mu - m)q_L + \bar{q}_R(i\gamma^\mu D_\mu - m)q_R + m(\bar{q}_L q_R + \bar{q}_R q_L). \tag{2.12}$$

From the Lagrangian, it is noticed that, if one puts the mass of u and d quarks to zero ($m_u = m_d = 0$), then the left and right components of the field are completely decoupled. The Lagrangian now becomes invariant under $SU(2)_L \times SU(2)_R$ global symmetry transformations. If the ground state is also invariant under this symmetry, then one would expect particles to form degenerate multiplets corresponding to the irreducible representations of the group $SU(2)_L \times SU(2)_R$. For example, triplet pseudo-scalar mesons (pions) should have been accompanied by their parity partner. However, it is well known that there is no such parity partner of pions. So, the chiral symmetry must be spontaneously broken to reproduce the correct particle spectrum.

A popular model which implements the ideas of chiral symmetry breaking is the linear sigma model [31]. The model was originally constructed to study the chiral symmetry in the pion-nucleon system. In the linear sigma model, the Lagrangian is constructed from the iso-triplet of pion $\pi = (\pi_1, \pi_2, \pi_3)$ fields and an iso-scalar σ-field. The Lagrangian (omitting the interaction term corresponding to the quark field with pion and sigma fields) in terms of these fields is given by [41]

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi)^2] - V(\sigma, \pi) \tag{2.13}$$

with the potential $V(\sigma, \pi)$ describing the self interaction of the scalars and is given by

$$V(\sigma, \pi) = \frac{\mu^2}{2}(\sigma^2 + \pi^2)^2 + \frac{\lambda}{24}(\sigma^2 + \pi^2)^4. \tag{2.14}$$

If the mass term μ^2 is negative, there is a minimum of the potential located at

$$\sigma^2 + \pi^2 = -\frac{6\mu^2}{\lambda}. \tag{2.15}$$

This defines a vacuum manifold $\mathcal{M}$ to be a 3-sphere, S^3 in the four dimensional field space. Each point on the 3-sphere is invariant under O(3) (which is locally isomorphic to SU(2)) rotations. Thus, the theory described by the Lagrangian Eq.(2.13) is spontaneously broken from O(4) down to O(3). Particles spectrum in this model can be determined by considering the fluctuation of the field around a vacuum state. For convenient we chose the particular ground state such as

$$< \sigma >= v \equiv -\frac{6\mu^2}{\lambda}, \quad \pi = 0. \tag{2.16}$$

Expanding the sigma field around the minimum as $\sigma = v + \sigma'$, the Lagrangian now becomes

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \sigma')^2 + (\partial_\mu \pi)^2] + \mu^2 \sigma'^2 - \frac{\lambda v}{6}\sigma'(\sigma'^2 + \pi^2) - \frac{\lambda}{24}(\sigma'^2 + \pi^2)^2. \tag{2.17}$$

We observe that, as a consequence of the spontaneous symmetry breaking three massless (number of broken generator is three) pions appear. While, the radial excitation corresponding to the sigma field is massive. Despite of it's simplicity, the phenomenological model described above displays many important features of QCD in the low energy regime. However, in this thesis, we will present interesting possibilities later in chapter 6, that incorporating a chemical potential term in the above Lagrangian can capture very important physics within Skyrmion picture of baryons.

2.2.2 A Special Example : $U(1)_{PQ}$ Symmetry Breaking

Here, we will give another example of SSB which is of crucial importance related to strong CP problem [46] in QCD. This problem arises from the non-perturbative instanton effects, leading to a term in the QCD Lagrangian which violates CP invariance of QCD. Including this term, the Lagrangian of QCD is written as

$$\mathcal{L} = \bar{q}_f(i\gamma^\mu D_\mu - m_f)q_f - \frac{1}{4}\text{Tr}F_{\mu\nu}F^{\mu\nu} + \theta\frac{g^2}{32\pi^2}\text{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{2.18}$$

The last term, so-called θ-term, contains the dual of the gluon field strength

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{2.19}$$

The θ-term is Lorentz invariant and gauge invariant. But, this term is proportional to $\vec{E}_c.\vec{B}_c$ and is CP-violating. The most important observational consequences of this term is that, it leads to electric dipole moment of neutron d_n [26]. The present experimental bound [46] on d_n is $d_n < 6.3 \times 10^{-26}$ e-cm, constraining θ to be less than 10^{-9}. Why the θ parameter is so small is so-called the "strong CP problem". One of the most elegant solutions of the problem was suggested by Peccei and Quinn [24] by introducing an axial symmetry $U(1)_{PQ}$. A prototype model constructed to solve the strong CP problem is given as [46, 47]

$$\mathcal{L} = \partial_\mu\Phi\partial^\mu\Phi^* - \frac{1}{2}\lambda(|\Phi|^2 - \eta_a^2)^2 + 2m^4(cos\theta - 1), \tag{2.20}$$

where

$$\Phi = |\Phi|e^{i\theta}. \tag{2.21}$$

In such a model, the approximate U(1) symmetry (approximate, because of the presence of last term in the above Lagrangian) of the Lagrangian, Eq.(2.20) is explicitly broken at the strong interaction mass scale. As the universe cools down to a temperature $T \simeq \eta_a$, the $U(1)_{PQ}$ symmetry is spontaneously broken and the field Φ acquires a non-zero VEV. At such temperature the last term in Eq.(2.20) is negligible and the model turns out to be same as described in §2.2 (see, Eq.(2.1) and Eq.(2.2)). As a consequence of SSB, massless (pseudo) Goldstone boson appears. These are called axions. However, as we will describe in the next section, because of non-trivial vacuum structure, cosmic string [47] (called, axionic string in this context) should also appear in such model. This string while oscillating back and forth can emit radiation in the form of massless axions. These axions can later on lead to the formation of isocurvature fluctuations during the QCD phase transition, as we will discuss in the next chapter and in chapter 5.

2.3 Formation of Topological Defects

A very fascinating object which can emerge as a consequence of phase transition is the topological defect. Formation of such defects during the phase transition is not necessarily restricted to a model of particle physics only. Topological defects may form in general, whenever symmetry breaking phase transition occurs. Common examples of such defects in condensed matter systems are the vortices [8] in liquid He^3, flux tubes [7] in type II superconductor, or line defects [9, 10] in liquid crystals. It usually occurs after symmetry breaking transition because of random choice of the order parameter (or, Higgs field) among the many possible ground states in different region of space. As the system further evolves, in some parts of the system, the order parameter may get 'locked' in symmetric state, while the remaining system

Topological Defects	Dimension	Classification
Domain Wall	2	$\pi_0(\mathcal{M})$
Cosmic String	1	$\pi_1(\mathcal{M})$
Monopole	0	$\pi_2(\mathcal{M})$
Texture/Skyrmion	-	$\pi_3(\mathcal{M})$

Table 2.1: Topological classification [6] of defects with homopotopy group $\pi_n(\mathcal{M})$

will be in the symmetry breaking phase. These locked regions are in general called defects. When these defects arise because of the topology of the vacuum manifold, they are known as topological defects. The vacuum manifold is defined as the set of distinct ground states and is denoted by $\mathcal{M}$. The importance in the study of vacuum manifold lies in the fact that it is precisely the topology of $\mathcal{M}$ which determines the type of defects which may arise during symmetry breaking transition. The necessary conditions of the formation of such defects depends on the existence of non-trivial mapping from physical space to the vacuum manifold. A theory may have symmetry breaking pattern with vacuum manifold consisting of two or more disconnected pieces. Such vacuum manifold results into the zeroth homotopy group being non-trivial, i.e., $\pi_0(\mathcal{M}) \neq \mathbf{1}$. In such a model, domain walls can form. Domain walls are two-dimensional thin surfaces appearing at the junction of field values belonging to different disconnected sectors of $\mathcal{M}$. Similarly, other topological defects like cosmic strings, monopoles may arise when the first and second homotopy group of $\mathcal{M}$ are non-trivial, respectively. We have listed the different class of homotopy groups [6, 39] and the type of topological defects with their characteristic dimension in table 2.1.

In the next section we will describe how the defects can arise in a particular class of models. We limit our discussion to cosmic string and Skyrmion only. Static solution for cosmic string will be also presented, which will be useful later, while discussing the generation of density fluctuations by cosmic string wakes.

2.3.1 Cosmic String

Cosmic strings without any doubt are the most fascinating and throughly studied topological defects both in cosmology [48] and in condensed matter system (as vortices, flux tubes) [8]. String defects arise when the vacuum manifold contains loop which can not be shrunk smoothly to a point. A model which can describe the formation of such objects is given as (the model we have have already used to describe the mechanism of SSB in §2.2)

$$\mathcal{L} = \partial_\mu \Phi \partial^\mu \Phi^* - \frac{1}{4}\lambda(|\Phi|^2 - \eta^2)^2. \tag{2.22}$$

Φ is a complex scalar field and can be written as $\Phi = \phi e^{i\theta}$, with ϕ is the magnitude of Φ and θ is the phase. The Lagrangian is invariant under global U(1) symmetry which is spontaneously broken and the field acquires vacuum expectation value (VEV) η. However, the value of θ can be arbitrary within the range 0 to 2π. Thus, the vacuum manifold which can be written as $\Phi = \eta e^{i\theta}$, is a circle, i.e., $\mathcal{M} = S^1$. According to the homotopy theory, the first homotopy group of the vacuum manifold is non-trivial ($i.e., \pi_1(S^1) = \mathcal{Z}$, a set of integers) and gives rise to vortices in two dimensional space, or string defects in three dimensions.

The static solution corresponding to the string defect can be found by solving the equation of motion

$$\Box\phi - \frac{\partial V}{\partial \phi} = 0. \tag{2.23}$$

One can use the following cylindrical symmetric ansatz [6] (string lying along the z-axis)

$$\phi(x) = \eta f_s(\rho\eta) exp(in\chi). \tag{2.24}$$

With $\rho = \sqrt{x^2 + y^2}$ and $tan\chi = y/x$, χ is the polar angle. n is the winding number of the string. The field equation then reduces to ordinary differential equation

$$f_s'' + \frac{f_s'}{v} - \frac{n^2}{v^2} f_s - \frac{\lambda}{2} f_s (f_s^2 - 1) = 0, \tag{2.25}$$

with $v = \rho\eta$. The differential equation can now be solved numerically by putting the boundary conditions $f_s(0) = 0$ and $f_s(v \to \infty) = 1$. The behavior then turns out to be like

$$f_s \sim 1 - \mathcal{O}(\frac{1}{v^2}) \text{ for } \sqrt{\lambda}\rho\eta >> 1; \tag{2.26}$$

$$\sim \mathcal{O}(v^n) \text{ for } \sqrt{\lambda}\rho\eta << 1. \tag{2.27}$$

The energy momentum tensor $T_{\mu\nu}$ also can be determined and is given as

$$T_0^0 = T_z^z = -\frac{\lambda\eta^4}{2}[f'^2 - \frac{1}{2}(f^2-1)^2 + \frac{n^2}{\lambda\eta^2\rho^2} f^2 \tag{2.28}$$

$$T_\nu^\mu = 0 \text{ for all other components.} \tag{2.29}$$

The mass per unit length can be obtained by integrating T_0^0 component and given as

$$\mu(R) = 2\pi \int_0^R T_0^0 \rho d\rho \sim \pi\eta^2 ln(\sqrt{\lambda}\eta R). \tag{2.30}$$

The mass per unit length of the global string thus diverges logarithmically with the radius R. This arises because of angular dependence of ϕ giving gradient energy term which decays like $1/\rho^2$ as in Eq.(2.28). However, this divergence does not introduce any difficulties in the context of cosmology. There is a natural cut-off provided by the curvature radius of the string, or the typical distance between a string and anti-string to make the energy of the string finite. We will see below, for the case of local string the energy per unit length of the string is always finite.

Consider local cosmic strings described by the Abelian Higgs model (relativistic version of the model which describes the phenomenon of superconductivity). The Lagrangian is similar to the model as described above with ordinary derivative ∂_μ

should be replaced by covariant derivative $\mathcal{D}_\mu$ and adding the kinetic energy for the gauge fields A_μ as given in Eq.(2.7). Following the same technique as described above for the global string case, one can determine the static solution for local string also using the following boundary conditions,

$$\phi \rightarrow \eta e^{in\chi}; \quad A_\mu \rightarrow (n/e)\partial_\mu \chi \;\; \text{for} \;\; \rho \rightarrow \infty. \tag{2.31}$$

These boundary conditions now make the gradient energy $\mathcal{D}_\mu \chi$ zero at large distances ($\rho \rightarrow \infty$). Therefore, unlike the global string case, the mass per unit length of local string becomes finite, $\mu \simeq 2\pi\eta^2$. On length scale much larger than the thickness of the string, the energy momentum tensor can be approximated as

$$T_0^0 = \mu\delta(x)\text{diag}(1,0,0,1). \tag{2.32}$$

Using this energy-momentum tensor, one can show that the Newtonian gravitational potential because of the static string is zero. Thus, static string does not exert any gravitational force (unlike a one dimensional object with non-relativistic linear distribution of matters) on surrounding matter. However, the situation is not the same for a moving string. As we will see later in the next chapter, moving cosmic string can affect surrounding matter and can produce planar structure of density fluctuations. As we will discuss later, there are important implications of these density fluctuation produced by moving cosmic string, namely, it can alter the dynamics of the QCD phase transition in crucial ways and subsequently can produce baryon inhomogeneities.

2.3.2 Axionic String

The Lagrangian of the so-called axion model, Eq.(2.20) possess an axial $U(1)_{PQ}$ symmetry which is believed to have been broken spontaneously at very high temperature in the early universe. One can notice, that the symmetry pattern of the Lagrangian Eq.(2.20) is the same as the Lagrangian described above for the formation of global string (ignoring the last term of Eq.(2.20) which arises at strong energy scale due to instanton effects and can be neglected at very high temperature [47]). So, it is quite natural to expect string configuration [47] during the spontaneous breaking of the global $U(1)_{PQ}$ symmetry at temperature well above the QCD scale. The expression of the string energy will be same as for the global string and hence, only depends on the symmetry breaking scale η_a. The string formation scale η_a is constrained by the cosmological and astrophysical observation and current acceptable limit [27, 28] is given as, $10^{10}\text{GeV} < \eta_a < 10^{12}\text{GeV}$. At low temperatures set by strong interaction scale, the axion mass "switches on" due to instanton effects [25]. Then the last term in Eq.(2.20) will become important and domain walls are expected to form [47] which are attached to the strings.

The axionic strings are thought to evolve roughly in the same way as the standard global strings. After formation at a temperature $\sim \eta_a$ at corresponding time $t_a \simeq (G\eta_a^2)^{-1}t_{Pl}$, the strings are stuck in the plasma and are stretched by the Hubble expansion. During this time the dominant mechanism of dissipating energy is via heat due to the large frictional force exerted by the background plasma. However, with time, the plasma becomes dilute and below a temperature T_* which is given by [27],

$$T_* = 2 \times 10^7\text{GeV}(\frac{\eta_a}{10^{12}\text{GeV}})^2 \tag{2.33}$$

the strings move freely. The corresponding time t_*, can be obtained from the following time-temperature relation,

$$T^2 \simeq \frac{0.3}{\sqrt{g_*}}\frac{m_{pl}}{t} \tag{2.34}$$

and is given by,

$$t_* = \frac{0.3}{\sqrt{g_*}}\frac{m_{pl}}{T_*^2} \simeq 1.8 \times 10^{-21}\text{sec}(\frac{10^{12}\text{GeV}}{\eta_a})^4. \tag{2.35}$$

Here $g_* \simeq 106$ is the number of degrees of freedom relevant at temperature T_* (taking all the species in the standard model). After t_*, the string will lose energy through radiation of (pseudo) Goldstone bosons which are called axions. The axion is not truly a Goldstone Bosons. It will acquire mass due to the axial anomaly [25] once the instanton effect turns on. In the high temperature phase ($T > T_c$), the mass of axions arises from the θ-dependent part of the free energy density which have been calculated by Preskill et al. [49] in the dilute-instanton-gas approximations. For three quark flavors appropriate for T not too high compared to T_c, the authors in ref. [49] have determined the mass as,

$$m_{aq}(T) \simeq 2 \times 10^{-2}[\frac{\Lambda^2}{\eta_a}][\frac{m_u m_d m_s}{\Lambda^3}]^{1/2}[\frac{\Lambda}{\pi T}]^4[9\ln\frac{\pi T}{\Lambda}]^3. \tag{2.36}$$

Where, m_u, m_d, m_s are the current quark masses of u, d and s quark, respectively, and $\Lambda \equiv \Lambda_{QCD} \sim 200$ MeV is the QCD scale. Taking critical temperature $T_c \sim 150$ MeV and putting the values of u, d and s quark masses (see ref.[29] for details), we get the axion mass at the onset of QCD transition and can be written in terms of axionic string formation scale as,

$$m_{aq}(T_c) = 5.3 \times 10^{-7}\text{eV}(\frac{10^{12}\text{GeV}}{\eta_a}). \tag{2.37}$$

At low temperature in the hadron phase, the axion mass is calculated from the current-algebra technique which gives the value [49],

$$m_{ah} = \frac{(m_u m_d)^{1/2}}{m_u + m_d}\frac{m_\pi f_\pi}{\eta_a} = 6.04 \times 10^{-6}\text{eV}(\frac{10^{12}\text{GeV}}{\eta_a}). \tag{2.38}$$

Here, m_π and f_π are the mass and decay constant of pions, respectively. The axion mass is temperature dependent at high temperature phase as in Eq.(2.36). Axion is very light and ultra-relativistic at very high temperature. It becomes dynamically significant [29] at the temperature at which compton wave length of the axions fall within the horizon. This is the time ($\tilde{t}$), at which axion string will not be able to oscillate sufficiently to radiate axions due to damping of domain walls attached to each string. From the time temperature relation Eq.(2.34), one can get the time $\tilde{t}$, from the following relation (neglecting the logarithmic dependence on T in Eq.(2.36)),

$$m_{aq}(\tilde{t}) = m_{aq}(t_c)(\frac{\tilde{t}}{t_c})^2 = d_h(\tilde{t}) = (2\tilde{t})^{-1}. \tag{2.39}$$

Where, $m_{aq}(t_c)$ is the mass of the axion at the onset of QCD phase transition obtained from Eq.(2.36). Thus, one gets the time $\tilde{t}$ as,

$$\tilde{t} = \frac{1}{2}[m_{aq}(t_c)t_c]^{-1/3}t_c \simeq 1.6 \times 10^{-3}\text{sec}(\frac{\eta_a}{10^{12}\text{GeV}})^{1/3}(\frac{t_c}{\text{sec}})^{2/3}. \tag{2.40}$$

During the time $t_* < t < \tilde{t}$, the string network is expected to enter into the scaling regime [6, 29, 50] and the spectrum of axions in the comoving momentum range k^* and $\tilde{k}$ can be considered to be flat [29]. Then, the number density distribution of axions resulting from axionic string decay in the comoving momentum range $k^*(= \frac{(R(t_*)}{t_*})$ and $\tilde{k}(= \frac{R(\tilde{t})}{\tilde{t}})$ can be written as [29],

$$n_k dk = R(\tilde{t})G\eta_a^2 ln(\eta_a\tilde{t})\rho_c dk/k^2. \tag{2.41}$$

Where, $R(\tilde{t})$ is the scale factor at time $\tilde{t}$, and ρ_c is the corresponding critical density of the universe.

2.3.3 Texture/Skyrmion

Production of domain walls, cosmic strings, and monopoles in the early universe has been extensively discussed in the frame work of Kibble [48] mechanism. As we mentioned, these defects may arise during phase transition, if the vacuum manifold of the Higgs field has, respectively, zeroth, first, or second homotopy group which are non-trivial. In the introductory chapter, we have already introduced another kind of topological defects under the name of Skyrmion, or texture in the cosmological context. The texture, or Skyrmion may arise, when the vacuum manifold has non-trivial third homotopy group. In a cosmological setting, Davis [51] has discussed the formation of such objects during phase transition, considering the universe to be topologically S^3. The study of textures has also been done by Turok [52], considering the case, when the textures may form in a given region enclosing certain horizon volumes. He essentially studied the evolution of a texture with asymptotically fixed boundary conditions. However, to have a topological meaning of a texture, and to estimate the texture production applying Kibble mechanism, an appropriate boundary conditions is very important, as was first pointed out by Srivastava [34]. When one applied the Kibble mechanism to estimate the productions of strings or monopoles, one essentially considers respectively the circle S^1, or two-sphere S^2 in the physical space R^3 and constructs various non-trivial winding number maps associated with the corresponding homotopy groups of the vacuum manifold. The situation is somewhat different, when one is interested in constructing non-trivial field configuration corresponding to the third homotopy group of the vacuum manifold. A three-sphere S^3, can not be embedded in our physical space R^3. The only way to construct such non-trivial mapping is to view R^3 as a stereographic projection of an S^3, which is only meaningful by implementing appropriate boundary conditions [34]. In the study of ref.[34], the question of this boundary conditions was addressed and the case of texture formation was discussed, when the horizon size is much smaller than the size of the universe.

In the Skyrmion picture, baryon is viewed as a soliton in the pion condensate field valued in the group SU(2) for the two flavor case. Using this Skyrmion picture of baryons, Kibble mechanism has also been used to study the baryon formation during chiral symmetry breaking transition in relativistic heavy ion collisions [33, 34, 35, 53]. It has been mentioned in ref.[34], that in the model of topological production of baryons, in the interior of the jets, appropriate boundary condition leads to strong suppression (compare to the case, when one relaxes the boundary condition) of the Skyrmion production rate.

In all these discussions of formation of topological defects, defects and anti-defects form with equal probabilities. However, there are many physical situations where formation of defects is favored over the anti-defects (or vice-versa). For example, in discussing the formation of flux tubes in type II superconductors in the presence of external magnetic field there will be a finite density of flux tubes, all oriented along the direction of external field (which we call as strings, the opposite orientation being associated with anti-strings). In addition there will be random formation of strings and anti-strings. Thus the basic mechanism of defect formation here should be able to account for the bias in the formation of strings over anti-strings.

A similar situation arises in the context of Skyrmion picture of baryon formation. When one wants to study baryon formation during chiral phase transition [33, 34, 35, 53] in relativistic heavy-ion collisions, then one has to deal with the situation of non-zero baryon excess over antibaryons. This is certainly true for energies up to SPS. Even for RHIC, the baryon chemical potential is about 50 MeV for the central rapidity region [36]. This requires that the basic mechanism of Skyrmion formation should be able to incorporate an intrinsic bias in favor of Skyrmions over anti-Skyrmions (depending on the sign of the chemical potential). Such a bias will certainly affect the estimates of net density of Skyrmions and anti-Skyrmions.

In chapter 6, we will present our study in the direction, namely, the effects of a bias in the theory of defect formation via the Kibble mechanism. As we will see that, the introduction of a chemical potential term in the effective potential for the linear sigma model §2.2.1 will lead to enhancement of net Skyrmion production rate (or, of net anti-Skyrmion rate, depending on the sign of the chemical potential term).