Chapter 1

General Introduction

Standard model of cosmology [1, 2] tells us that the universe starts with a "big bang", the term coined by F. Hoyle to describe the origin of the universe. According to the so-called hot big-bang model (the origin of the Big Bang theory can be credited to Edwin Hubble [3]), the universe is expanding since it's birth sometimes about 15 billions years ago. The theory was subsequently supported by the observation made by E. Hubble [3], that the velocities of the galaxies are proportional to the distances. The theory also predicts the existence of cosmic thermal radiation with a black body spectrum in the microwave frequency range. This thermal radiation is the leftover glow from the early hot stages of the universe. As the expansion of the universe is accompanied by subsequent cooling, the wavelength of the cosmic background radiations were red-shifted and is expected to be observed in the sky at the present stage as a black body radiation with a temperature of about 2.73K. The big bang theory received it's one of the strongest confirmation from the dramatic discovery of this background radiation in 1965 by Penzias and Wilson [4].

The standard model of cosmology, when marrying with particle physics gives birth to the belief that the universe, as it cooled down, underwent a number of successive phase transitions. Such phase transitions have several exciting cosmological consequences and thus provide an important link between particle physics and cosmology.

There is a conjecture that a grand unified theory (GUT) phase transition has taken place at a temperature of about 10^{15} GeV, when the universe was only about 10^{-35} second old. Once this temperature was reached, symmetry of the grand unified theory [5], described by some general gauge group G (e.g., SU(5) or SO(10)) was spontaneously broken down resulting into the standard model, described by gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Unfortunately, the energy scale of this phase transition is too high to be accessible directly in the laboratory. Thus, it is very difficult to fix the model conclusively. However, the crucial aspect of this phase transition is that, it leaves several imprints on the universe, which could subsequently affect the evolution of the universe in crucial ways. By studying these effects theoretically and by comparing with experimental observations, one can put several constraints on the parameter space of a particular unified model. For instances, it is believed that GUT phase transition was followed by the formation of topological defects [6], viz., cosmic strings, domain walls, monopoles etc. The presence of such defects can be probed indirectly by studying their all possible effects on the evolution of the early universe.

The topological defects are essentially solitonic solutions of the field equations which arise when the underlying symmetry of a theory is broken down spontaneously and gives rise to a non-trivial topology (hence, the name topological defects) of the vacuum manifold ($\mathcal{M}$). Since, it is the topology of the vacuum manifold which determines the possibility of appearance of certain kind of defects, formation of such objects is a universal phenomenon, irrespective of the symmetry breaking scale or, the detailed dynamics of the phase transition. The best known examples of defect formation in condensed matter systems are the flux tubes [7] in Type II superconductor, vortex line [8] in a liquid Helium, or line defects [9, 10] in liquid crystals.

Depending on the specific details of the unified models, the phase transition may lead to the formation of domain walls, strings or monopoles. These defects arise during phase transition when the vacuum manifold $\mathcal{M}$ has (two or more) disconnected ground states or it has non-contractible loops or non-contractible surfaces in it, respectively. Among these defects, cosmic strings are the most fascinating and most throughly studied defects because of it's various important cosmological and astrophysical consequences. It can be mentioned here, that the superstring theories also predict such type of string defects with acceptable parameters (see ref.[11]). It therefore becomes important to investigate how such cosmic strings can affect the universe. There has been extensive study of density fluctuations generated by cosmic strings from the point of view of structure formation [6]. Although, it is clear by now from latest WMAP data [12] that contributions of cosmic strings and other topological defects to structure formation is insignificant [13]. However, even with present models of cosmic string network evolution, it is not ruled out that cosmic strings may contribute to some part in the structure formation in the universe. Also, there can be several other physical phenomenon in the early universe which can be affected by the presence of cosmic strings. For example, density fluctuations produced by moving cosmic string can alter the dynamics of the phase transition in crucial ways [14, 15]. If so, the outcome of the phase transitions will be different from the standard scenario. In this thesis, we would like to discuss some of these issues in the context of QCD phase transition.

The next phase transition, namely, electroweak phase transition took place when the symmetry of the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ breaks down spontaneously to $SU(3)_c \times U(1)_{em}$ at a temperature of about 100 GeV, when the age of the universe is about 10^{-11} sec. This electroweak phase transition is believed to be responsible for generating the masses for all the elementary particles through Higgs mechanism [16]. Of course, it is still a matter of debate whether Higgs field, which plays the central role in symmetry breaking transition, is really a fundamental field. Hopefully, these issues will be settled down very soon by tracking down the Higgs particle, if it exits, through the upcoming Large Hadron Collider (LHC) at CERN [17]. Another very important process, which might have occurred during electroweak phase transition is the synthesis of overall net baryon number. One of the long standing problems in cosmology is to understand baryogenesis [18], the process which can explain how the matter dominated universe had been evolved from the initial baryonic symmetric condition. The observed abundance of baryons today implies that by the time of the nucleosynthesis, when the universe had a temperature of order MeV, the ratio of net baryon to photon density η was about 10^{-9}. This number though very small, is sufficient to confront the observational results of primordial abundances of light elements (e.g., D, He^3, He^4, Li^7) with the values calculated based on homogeneous big-bang nucleosynthesis theory [19]. There are enough ingredients in the electroweak sector of the standard theory of particles physics to believe that baryogenesis might have occurred during the electroweak phase transition [18]. However, due to the small value of CP violation in the standard model and requirement of strong first order electroweak phase transition, it seems unlikely that required magnitude of baryon asymmetry can be generated within standard electroweak theory.

Another very important and the last phase transition, namely, the quark-hadron phase transition is believed to have occurred at an energy scale of about 200 MeV, when the universe was only few microsecond old. The strong interaction of quarks and gluons are best described by quantum choromodynamics, or QCD, in short. The theory predicts that at very high temperature/density, the color objects quarks and gluons move almost freely because of asymptotic freedom. This situation is achieved in the early universe when the temperature was much higher than 200 MeV. Once the universe reached at this temperature, confinement sets in and the color singlet hadrons are formed. Although the observational consequences of this phase transition from cosmology are indirect, there are current and future accelerator experiments which can shed light on different aspects of this phase transition. The main objects of the ongoing program of relativistic heavy ion collision at BNL [20] and upcoming LHC at CERN [17] is aimed at creating the quark-gluon plasma state, and studying its evolution and subsequent phase transition to the hadronic matter. If the properties of the phase transition is known through these experiments, then it can shed light on different aspects of phase transition in the early universe as well.

One of the important cosmological consequences of QCD phase transition is the possibility of formation of baryonic lumps with high baryon concentration or, even formation of quark nuggets [21]. Like any typical first order phase transition, quark-hadron transition also proceeds through the nucleation of hadronic bubbles in the background of QGP phases. These bubbles will then grow, coalesce, and eventually convert the QGP phase to the hadronic phase. Because of larger mass of baryons in the hadronic phase, baryons may get concentrated inside the shrinking quark phase and may lead to the above mentioned baryonic lumps. The separation scale of these lumps can be determined by the typical distance between the bubbles. Generation of baryon inhomogeneities during quark-hadron transition is an important issue due to its consequences for the nucleosynthesis as discussed below.

Standard big bang nucleosynthesis [19] (SBBN) has emerged as one of the cornerstones of the big bang model, joining with Hubble expansion and observation of cosmic background radiations as discussed earlier. Essentially, the physics of nucleosynthesis is all about to understand the formation of light elements, e.g., D, He^3, He^4, Li^7 etc. The event started taking place typically at a temperature of about 0.1 MeV. In SBBN, it assumes that the baryon density is distributed homogeneously at the onset of nucleosynthesis. Based on this assumption, the primordial light elemental abundances had been calculated, and can be expressed in terms of a single cosmological parameter η $(= n_b/n_\gamma)$, which has been mentioned earlier. Though it is quite remarkable that the parameters required for the calculations of SBBN are reasonably consistent with recent measurements of the angular power spectrum of the cosmic microwave background radiation (CMBR) by WMAP experiment [12], several modifications to SBBN are still being considered to better account for the abundances of the light elements. Large number of alternative models have been proposed for that. Inhomogeneous big bang nucleosynthesis [22, 23] (IBBN) is one of them, where nucleosynthesis takes place in the presence of an inhomogeneous baryon density distribution. Therefore, it is interesting to study how the baryons were distributed in the early universe and study it's possible effect on nucleosynthesis calculation.

In this thesis, we will discuss how the presence of density fluctuations produced by cosmic string wakes at QCD phase transition can alter the baryon distribution resulting in inhomogeneous distribution of baryon density at the end of the phase transition. We also discuss the survival probability of the baryon inhomogeneities upto nucleosynthesis epoch and subsequently, the effect on the outcome of primordial abundances of light elements.

In this thesis, we will also explore another consequence of QCD phase transition in the context of early universe, namely, generation of axionic isocurvature fluctuation through the axion trapping by cosmic string wakes. So far in the discussion of several phase transitions in the context of early universe, we have skipped one energy scale, which can be very important in solving the "strong CP problem" of QCD, as was suggested by Peccei and Quinn [24] in the late seventies. The strong CP problem is related to the CP violation term in the QCD Lagrangian, usually called θ- term in the literatures. It was not known, until the discovery of the instanton [25] effects of a non-abelian gauge theory. This θ- term violates the CP invariance and thus leads to electric dipole moment of neutron d_n [26]. The value of d_n is constrained by the experiments and turns out to be $d_n < 6.3 \times 10^{-26}$ e-cm. This value in turn, translates to the upper limit on the value of θ to a tiny region $\theta < 10^{-9}$. The small value of this parameter (sometimes called the vacuum angle) in strong interactions is called the "strong CP problem". One of the most elegant solutions of the problem was suggested by Peccei and Quinn [24] by introducing an axial symmetry $U(1)_{PQ}$. Axion arises as a (pseudo) Goldstone boson when the $U(1)_{PQ}$ symmetry is spontaneously broken at some scale η_a. The symmetry breaking scale, η_a is strictly constrained by considerations of accelerators and various astrophysical observations. The permitted region [27, 28] is given by $10^9\text{GeV} \leq \eta_a \leq 10^{12}\text{GeV}$. However, since $U(1)_{PQ}$ is a global symmetry, global strings are also produced as a consequence of this symmetry breaking at the scale η_a. These strings are called axionic strings [6]. For the early stages, the properties of these strings are of similar kind as of standard global cosmic string. After formation, the strings are stuck in the plasma and are stretched by the Hubble expansion. During this time the dominant mechanism of dissipating energy is via heat because of large frictional force exerted by the background plasma. However, with time the plasma becomes dilute and the strings move freely. The string will then lose it's energy through the radiation of (pseudo) Goldstone bosons which are called axions. However, the axion is not truly a Goldstone boson. It acquires mass, once the instanton effects [25] turn on. The mass of the axions depends on temperature and is larger in hadronic phase than in the QGP phase. Using this mass difference of the axions across the quark-hadron phase boundary, Hindmarsh [29] had shown that, axion can be trapped initially inside the shrinking quark phases. These axions thus trapped may eventually escape from the quark phases, once the required momentum is achieved from the collapsing walls [29] and can lead to formation of isocurvature fluctuations. We will discuss in this thesis, the trapping mechanism of axions and the productions of isocurvature fluctuations, when the cosmic string wakes are present.

The quark-hadron phase transition which we have discussed above is usually called deconfinement-confinement (D-C) transition, where the color degree of freedom is confined to form color singlet objects. However, in the massless limit of the quarks, the QCD theory is invariant under chiral symmetry transformation. Spontaneous break down of this symmetry is associated with the phase transition called chiral phase transition. Lattice studies [30] show that the D-C phase transition and the chiral phase transition happen at the same temperature (There is no satisfactory answer yet, why these two phase transitions should occur at the same temperature.). A popular model which implements the idea of SSB of chiral symmetry in QCD is the linear sigma model. It was originally constructed to study the chiral symmetry in the pion-nucleon system [31]. Despite of the simplicity, this phenomenological model displays many important features of QCD in the low energy regime. However, in addition to the massless pions (real mass of pions is recovered by adding a mass term to the Lagrangian), this model also contains a σ-meson, mass of which can be large. To describe only the dynamics of pions, one introduces the minimal non-linear sigma model, in which the role of σ-meson is eliminated by integrating it out from the theory.

It was long back suggested by Skyrme [32] that the non-linear sigma model has a topological conserved charge, which can be identified with baryons. The topological charge is associated with the non-trivial topological field configurations called Skyrmions (which are similar to textures in cosmological context). The Skyrmion is a kind of topological defect which can arise because of non-trivial mapping from the compactified 3-dimensional space to the target space S^3 of the vacuum manifold. (For the sake of uniform terminology, we are calling them topological defects. Though, the properties of Skyrmions are somewhat different from other defects, like strings, monopoles etc. In case of Skyrmion, the field never leaves the vacuum manifold unlike the case for other defects.) There are extensive discussions in the literatures regarding the formation and the implications of such defects in cosmology. The standard theory of defect formation has also been used to study the baryon formation during chiral symmetry breaking transition in relativistic heavy ion collisions [33, 34, 35] within the context of Skyrmion picture of baryons. In all these studies of defect formation, the defects and anti-defects form with equal probabilities, resulting in the average net defects density being zero. However, there are many physical situations which demand non-zero defect density. As for example, in heavy ion collision experiments, when one wants to study baryon formation during chiral phase transition, one has to deal with non-zero baryon density. This is certainly true upto SPS [17] energies. Even at RHIC [20], the baryon chemical potential is about 50 MeV in the central rapidity region [36]. This requires a mechanism which can produce more Skyrmions over anti-Skyrmions. In this thesis, we will introduce such an interesting possibility, where formation of defects can be enhanced over anti-defects (or, vice versa) within the framework of linear sigma model. Our studies show that, the formation of Skyrmion can be enhanced by incorporating a positive chemical potential term in the effective potential of linear sigma model. In this thesis, however, we will mainly focus on 1+1 dimension case and discuss the effect of chemical potential term on Skyrmion formation.

After this brief introduction, let us outline the contents of the thesis. In **chapter 2**, we will briefly overview the mechanism of spontaneous symmetry breaking and the phase transition in general. Special emphasis will be given to chiral symmetry breaking phase transition and the spontaneous symmetry breaking of $U(1)_{pq}$. Here, we will describe how the topological defects can be formed during phase transition in certain class of models.

In **chapter 3**, we will be focusing on a particular phase transition, viz., deconfinement-confinement phase transition. The dynamics of this phase transition and it's consequences in the context of early universe will be reviewed. In this chapter, we will also review earlier work, where the quark-hadron transition dynamics was studied in the presence of cosmic string induced density fluctuations.

In **chapter 4, 5 and 6**, we will describe our studies on which this thesis is based upon. In **chapter 4**, we will discuss how baryon inhomogeneities can be generated at the end of quark-hadron phase transition in the presence of density fluctuations produced by cosmic string wakes. Possible effects of these density inhomogeneities on nucleosynthesis will also be discussed in this chapter.

In **chapter 5**, the mechanism through which axions can be trapped inside the cosmic string wakes will be discussed in details. In this scenario, large axionic fluctuations can be generated at the end of QCD phase transition. Possible astrophysical consequences will also be discussed.

In **chapter 6**, possible biased formation of defects over anti-defects will be discussed within the context of linear sigma models. It will be shown that with chemical potential term in the chiral Lagrangian, there is an enhancement of Skyrmion over anti-Skyrmion production rate (or vice verse), depending on the sign of the chemical the potential term. Here, we will produce our result based on our studies in 1+1 dimensional case. An interesting possible implication of our results with QCD phase diagram will be also presented in this chapter.

In **chapter 7**, we summarize our results.

Throughout this thesis, we will use natural units, i.e., $\hbar = c = k = 1$.