Chapter 5

Isocurvature Fluctuations by Cosmic String Wakes.

5.1 Introduction

Here, we study another aspect of the QCD phase transition in the context of early universe if the transition is of 1st order in nature, that is, axion inhomogeneity generation during quark-hadron transition in the presence of cosmic string induced density fluctuations. We have mentioned in §3.3, the mechanism of axion trapping and generation of axionic isocurvature fluctuations, as was proposed by Hindamarsh [29]. We will present here, the phenomenon of axion trapping when cosmic strings are present at QCD epoch. We are already familiar with the dynamics of quark-hadron phase transition in the presence of cosmic string wakes. As we have seen that, the picture of slow combustion phase in our model is different from the standard scenario. For example, geometry of the collapsing interface in our case is of sheet like planar structure unlike spherical in the conventional case. Thus, the isocurvature fluctuations produced, as the axions leave the wake region, will spread in a sheet like regions in contrast to the spherical clumps as studied by Hindmarsh [29]. It will be interesting to study the subsequent evolution of large fluctuations with such geometrical structure and its astrophysical consequences.

Here, we should mention that, other than the trapping mechanism, axionic inhomogeneities can also be generated in the presence of cosmic string wakes in the following manner. We have already discussed the generation of density fluctuations produced by moving cosmic strings in §3.4.1. We have seen that, for collisionless particles, the magnitude of these density fluctuations can be of order one, with opening angle of the wake being equal to the deficit angle $8\pi G\mu$. The axions being very weakly interactive [29], they can be considered to be a fluid consisting of collisionless particles. Therefore, description of wake formation for collisionless particles can be operative for axions also and can lead to formation of axionic inhomogeneities. The magnitude of these inhomogeneities will be of order unity. These over dense axions could be concentrated within very thin sheet like region of thickness $8\pi G\mu \; d_H \sim 1$ cm ($d_H \sim$ 10 km being the horizon size at $T_c = 150$ MeV). (One has to properly account for the wavelength of axions.) However, we will see below that the magnitude of the isocurvature fluctuations produced through axion trapping by the collapsing interfaces will be very large compared to the above density fluctuation. Essentially, this thin sheet like region will be trapped initially inside the wake of larger thickness which is produced by the formation of shock by moving string through relativistic fluid (as discussed in §3.4.1). Ultimately, axions will also escape from this thin wake and contribute in producing overall isocurvature fluctuations.

5.2 Trapping of Axions and Baryons by Cosmic String Wakes

In the previous chapter, we have shown that, wake like overdensity regions form by cosmic strings, leading the trapping of the QGP region in between two planar interfaces. Collapse of these two interfaces towards each other leads to the concentration of baryons which was the subject of study in the previous chapter. However, collapsing interfaces not only trap the baryons, the low momentum axions are also expected to get trapped inside the wakes initially. As the interfaces further collapse, the axions gradually pick up momentum from the walls. The minimum momentum required for the axions to escape the wake is given in Eq.(3.6). These axions which are left behind as the interfaces collapse can form axionic fluctuations. The extra energy which is needed for the axions to escape from the QGP phase to the hadron phase comes from work done by the collapsing interfaces. The fluctuations must therefore be isocurvature [29] one. The time evolution of axion density inside the wakes depend on the momentum distribution of the axions and on the volume fraction f of the QGP region.

Now let us determine the effect of interface motion on axion momentum distribution and subsequently on the evolution of number density of axions inside the wake. Total number of axions which will remain initially inside the wake can be calculated by integrating Eq.(2.41). Thus, if $N(t_0)$ be the total number of axions trapped initially (Note, our initial time t_0 is set at the time when the wake like overdense regions has already been formed. The transition from the QGP phase to hadron phase further proceeds through collapsing interfaces. These collapsing interfaces will trap most of the axions inside the wakes) inside the wake, then,

$$N(t_0) = V_0 \int_{k_{min}}^{k_{max}} n_k dk (\frac{R(\tilde{t})}{R_0})^3. \tag{5.1}$$

Where the last factor $(\frac{R(\tilde{t})}{R_0})^3$ is due to the decrease of axion density from time $\tilde{t}$ (Eq.(2.40)) to t_0 causes by the expansion of the universe. n_k as given in Eq.(2.41) gives the number density of axions at time $\tilde{t}$. k_{min} and k_{max} are minimum and maximum comoving momentum, respectively. Since, the minimum comoving momentum which can be trapped initially inside the wake can not be smaller than $\frac{2\pi R_0}{z_0}$ (z_0 is the initial average thickness of the wake), we will take k_{min} as $\tilde{k}$ ($\tilde{k}$ is the comoving momentum of axions at time $\tilde{t}$, as was discussed in §2.3.2) or $2\pi R_0/z_0$ whichever is larger. To determine that, we take the ratio,

$$\frac{\tilde{k} z_0}{2\pi R_0} = \frac{R(\tilde{t})}{\tilde{t}} \frac{z_0}{2\pi R_0} = (\frac{1}{\tilde{t} t_0})^{1/2} \frac{z_0}{2\pi}. \tag{5.2}$$

Taking $\tilde{t}$ from Eq.(2.40), $t_0 \sim 10^{-5}$ sec and the initial thickness $z_0 \sim 1$ km or so, we get the ratio of $O(1)$. So, we take the minimum comoving momentum k_{min} as $\tilde{k}$. The maximum comoving momentum k_{max} of the axions which will remain inside the wake can be set as discussed below.

The momentum of the axions will keep increasing with the collapsing interfaces [29]. Thus, if $k(t)$ be the comoving momentum of an axion whose initial momentum is k_i, then the momentum at any time t, when thickness of the wake becomes $z(t)$ can be obtained from the following relation [29],

$$k(t) = k_i \frac{z_0}{z(t)} \frac{R(t)}{R_0}. \tag{5.3}$$

The minimum physical momentum required for the axions to leave the interface is equal to Δm (see, Eq.(3.6)). Therefore, the axions which have comoving momentum less than $\Delta m R_0$ will remain inside the wake. Thus, we can put the upper limit of the integration in Eq.(5.1) as k^* or $\Delta m R_0$ whichever is smaller. To determine that let us again calculate the ratio,

$$\frac{k^*}{\Delta m R_0} = \frac{\frac{R(t^*)}{t^*}}{\Delta m R_0} = (\frac{1}{t^* t_0})^{1/2} \frac{1}{\Delta m}. \tag{5.4}$$

Putting the value of Δm as obtained from Eq.(2.37) and Eq.(2.38) and the value of t_* from Eq.(2.35) the ratio can be determined in terms of the formation scale of axionic strings as,

$$\frac{k^*}{\Delta m R_0} \simeq 7.9 (\frac{t_0}{\text{sec}})^{-1/2} (\frac{\eta_a}{10^{12}\ \text{GeV}})^3 \simeq 1.2 \times 10^3 (\frac{\eta_a}{10^{12}\ \text{GeV}})^3. \tag{5.5}$$

Thus, the above ratio depends on the formation scale η_a of the axionic string, which is constrained by the terrestrial and astrophysical experiments as well as from the cosmological considerations. The most stringent lower bound has been obtained from the SN 1987A [81] as $\eta_a \geq 10^{10}$ GeV and the upper bound [49] $\eta_a \leq 10^{12}$ GeV comes from the consideration that the axions should not overclose the universe. If we take $\eta_a < 10^{11}$ GeV, then the above ratio is always less than unity. In this case we can take the upper limit of integration in Eq.(5.1) as k^*. Having set the limits of integration we can now calculate total number of axions which will remain initially inside the wake and given by,

$$N(t_0) = V_0 F[\frac{1}{\tilde{k}} - \frac{1}{k^*}](\frac{R(\tilde{t})}{R_0})^3. \tag{5.6}$$

Where, $F = R(\tilde{t})G\eta_a^2 \ln(\eta_a \tilde{t})\rho_c \simeq \frac{3}{32\pi}\eta_a^2 \ln(\eta_a \tilde{t})(\tilde{t}t_0)^{-1/2}\frac{R_0}{\tilde{t}}$. Now as the interfaces move towards each other momentum of each axion will be modified according to Eq.(5.3). Let $n_{k'}dk'$ be the modified spectrum due to trapping of axions within the collapsing walls when thickness becomes $z(t)$. Unless the momentum of each axion becomes $\Delta m R(t)$ the axions will remain inside the wake. So upto certain time t_1, total number of axions $N(t)$ (for $t_0 < t < t_1$) will remain fixed inside the wake. Subsequently, number density will be increased due to decrease of volume fraction of QGP region f caused by the interface motion. So, total number of axions at certain time $t(< t_1)$ can be written as,

$$N(t) = fV(t)\int_{k'_{min}}^{k'_{max}} n'_k dk' = fV(t)\int_{k'_{min}}^{k'_{max}} F'\frac{1}{k'^2}dk' \quad (t_0 < t < t_1). \tag{5.7}$$

Where, F' is new coefficient will be determined in terms of F. This coefficient F' essentially will take care of the change of the coefficient F due to shifting of momentum of each axion as is obtained from Eq.(5.3) and the change in QGP volume causes by interface motion. Determining this coefficient in terms F will be particularly useful in determining the evolution of axion density when axions will start leaking out the overdensity regions. $k' = k\frac{z_0}{z(t)}\frac{R(t)}{R_0} \equiv kc(t)$ (say) and k'_{min}, k'_{max} have to be substituted by $\tilde{k}c(t)$ and $k^*c(t)$, respectively. Substituting all these quantities and integrating Eq.(5.7) we get the total number of axions at any intermediate time between t_0 to t_1. Equating this number to the initial total number of axions at t_0 as obtained from Eq.(5.6) one gets the coefficients F' as follows,

$$F' = \frac{V_0 F c^2(t)}{fV(t)}(\frac{R(\tilde{t})}{R_0})^3. \tag{5.8}$$

Putting back F' in Eq.(5.7) and divided by the QGP volume $fV(t)$ we get the number density of axions upto time t_1 and given by,

$$\rho(t) = \frac{N(t)}{fV(t)} = \frac{V_0 F}{fV(t)}(\frac{R(\tilde{t})}{R_0})^3[\frac{1}{\tilde{k}} - \frac{1}{k^*}] = \frac{3}{32\pi}\frac{V_0\eta_a^2}{fV(t)t_0^2}\ln(\eta_a\tilde{t})[(\tilde{t}t_0)^{1/2} - (t_* t_0)^{1/2}]. \tag{5.9}$$

Now, as we mentioned earlier that the density written above is applicable upto time t_1 below which no axions will leave the QGP regions due to insufficient momentum required to escape the region. The thickness of the wake at that time t_1 can be calculated using the fact that when the momentum of axion exceeds the value $\Delta m R(t)$ they will leave the wake region, which is first happens when $k^* c(t)$ becomes equal to $\Delta m R(t)$. Using this equality we get the corresponding thickness at t_1 and can be determined as,

$$k'_{max} \equiv c(t_1)k^* = \frac{z_0}{z(t)}\frac{R(t_1)}{R_0}\frac{R(t^*)}{t^*} = \Delta m R(t_1). \tag{5.10}$$

Or,

$$\frac{z(t_1)}{z_0} = \frac{1}{\Delta m}(t^* t_0)^{-1/2} \simeq 7.8(\frac{t_0}{\text{sec}})^{-1/2}(\frac{\eta_a}{10^{12}\ \text{GeV}})^3 \simeq 1.2 \times 10^{-3}. \tag{5.11}$$

Since, the initial thickness z_0 of the wake is ~ 1 km, and $t_0 \sim 10^{-5}$ sec, thickness of the wake at time t_1 comes out to be $z(t_1) \sim 2$ m for string formation scale $\eta_a \sim 10^{10}$ GeV. As soon as the thickness decreases down to ~ 2 m, the axions will start leaving the QGP region and finally all the axions will leave the wake and form a sheet like structure. To determine the evolution of density after t_1, upper and lower limit of the integration of Eq.(5.7) should be replaced by $\Delta m R(t)$ and $\tilde{k}c(t)$ respectively. Thus, the evolution of axion density within the wake after time t_1 is given by,

$$\rho(t) = \frac{V_0 F c(t)}{f V(t)} (\frac{R(\tilde{t})}{R(t_0)})^3 [\frac{1}{\tilde{k} c(t)} - \frac{1}{\Delta m R(t)}] \tag{5.12}$$

$$= \frac{3}{32\pi} \frac{V_0 \eta_a^2}{f V(t) t_0^2} \ln(\eta_a \tilde{t}) [(\tilde{t} t_0)^{1/2} - \frac{z_0}{z(t) \Delta m}], \quad (t_1 < t < t_f). \tag{5.13}$$

Where, Eq.(5.8) and Eq.(5.9) have been used. t_f is the final time at which all the axions will leave the QGP region. The corresponding thickness can be obtained from the following expression,

$$k'_{min} \equiv \tilde{k} c(t_f) = \frac{z_0}{z(t)} \frac{R(t_f)}{R_0} \frac{R(\tilde{t})}{\tilde{t}} = \Delta m R(t_f). \tag{5.14}$$

So, the ratio of the final thickness when no axion will remain inside to the initial thickness becomes,

$$\frac{z(t_f)}{z_0} = \frac{1}{\Delta m} (\tilde{t} t_0)^{(-1/2)} = 8.31 \times 10^{-9} (\frac{t_c}{\text{sec}})^{-1/3} (\frac{t_0}{\text{sec}})^{-1/2} (\frac{\eta_a}{10^{12}\ \text{GeV}})^{5/6}. \tag{5.15}$$

Since, the time scale (t_c) at the onset of QCD phase transition is of same order of magnitude as our initial time t_0, which is of the order of $\sim 10^{-5}$ sec we get the ratio of final thickness to the initial thickness as $\sim 7.7 \times 10^{-7}$ for string formation scale $\eta_a \sim 10^{10}$ GeV. Therefore when the wake inside which axions were trapped reduced to a size of about ~ 0.1 cm, the axions which were still remained inside the wake will all leave the wake and form a sheet like region. The release of the axions from the wake thus happens for the duration when wake thickness lies between 2 m and 0.1 cm. Below the thickness of 0.1 cm, no axion will remain inside the wakes. The axions thus left behind as the interfaces collapse will be concentrated in a sheet like regions. One can calculate the density profile of these axions as follows.

Let, $\rho_{pr}(z)$ be the density of axions which is left behind at position z as the interfaces collapse. Then we can write down the following expression which relates total number of axions at position z to the density which are left behind as follows,

$$N(z) - N(z - dz) = Adz\rho_{pr}(z), \tag{5.16}$$

where the time dependence of z is given in Eq.(4.34). So We get,

$$\frac{dN}{dz} = A\rho_{pr}(z). \tag{5.17}$$

Thus, we finally get the density profile of axions as a function of thickness with collapsing interfaces as,

$$\rho_{pr}(z) = V_0^{(\frac{-2}{3})}(\frac{R_0}{R(t)})^2(\frac{dN}{dz}). \tag{5.18}$$

However, here we should mention that in deriving the above equation we have considered that the axions will not disperse away immediately as it leaves the wake. Strictly speaking, the axions after leaving the wake will move with a velocity corresponding to the momentum they have in the hadronic phase, as they leave the wake. Since axions leave as soon as their momentum in the QGP phase equals to Δm (Eq.(3.6)), their kinetic energy may not be large in the hadronic phase. Hence they may not move very far from the wake (note, axions are very weakly interacting). In this case, the density will be somewhat decreased compare to the density as obtained from Eq.(5.18). However the order of magnitude may not be changed much. So for simplicity we will consider the case where the axions will not be homogenized immediately after leaving the wake. In the following section we will discuss the results as is obtained from Eq.(5.9), Eq.(5.13) and Eq.(5.18).

5.3 Profile of the Axionic Isocurvature Fluctuations

Eq.(4.7) and Eq.(4.9) are solved numerically to get the evolution of scale factor and volume fraction f which is occupied by the QGP phase. The solution thus obtained is fed into Eq.(5.9), and Eq.(5.13) to get the evolution of axion density within the wake as the transition from QGP to hadron phase proceeds.

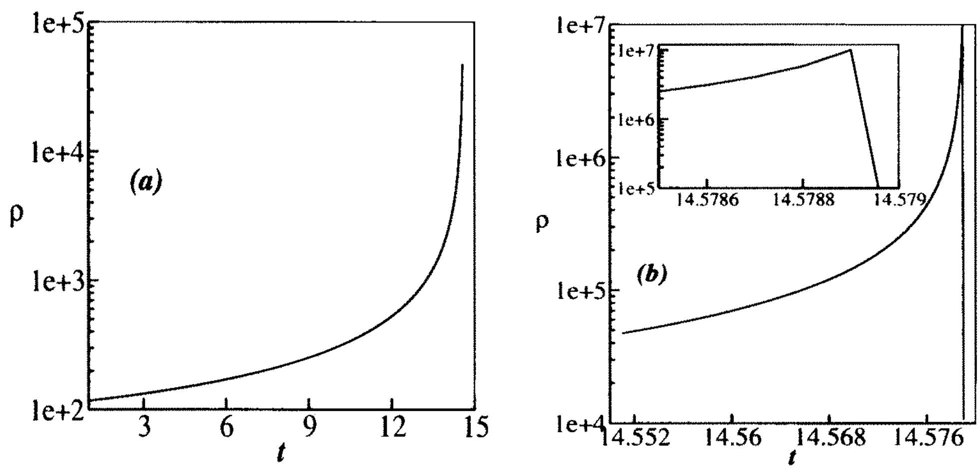

Figure 5.1: These figures show plot of evolution of axion density inside the wake as a function of time. The density ρ is given in unit of fm^{-3} and time t is in μsec. Figures (a) and (b) show the density plot for two time zones $t_0 \leq t \leq t_1$ and $t_1 \leq t \leq t_f$ respectively.

Fig.5.1 shows the evolution of axion density inside the wake for two time zones. Time axis is given in the unit of μsec, while density is in fm^{-3}. As is shown in the figure, most of the time during which transition from QGP phase to hadronic phase ($\Delta t_{trans} \simeq 15$ μsec) happens, the axions will remain inside the wake. The reason is obviously due to $\frac{1}{k^2}$ dependence of axions density spectrum. The number of axions with minimum momentum will be more and they will leave the over density region last. In Fig.5.1a, the density keeps increasing solely (for the time zone, $t_0 < t < t_1$)

due to decrease in volume of QGP phase. While for next time zone ($t_1 < t < t_f$), there are two competitive processes. Decrease in volume causes to increase the axion density, while leaking out of axions will cause the density to decrease. Most of the time during which transition from QGP phase to hadron phase proceeds, first effect will be dominated over the latter, hence there will be net increase in density which is evident from the Fig.5.1b. Finally, almost all the axion will acquire the momentum needed to escape from the QGP regions is achieved and density sharply decreases down towards zero. The inset in Fig.5.1b shows the detailed profile of decreasing part of the axion density plot. Fig.5.2 shows the evolution of density (in unit of fm^{-3})

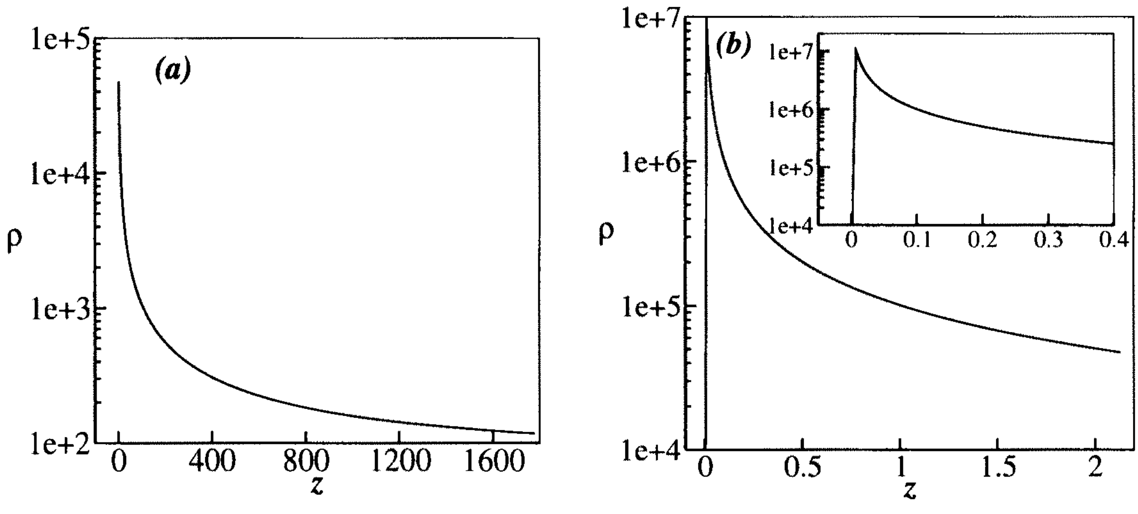

Figure 5.2: These figures show plot of evolution of axion density ρ (in unit of fm^{-3}) inside the wake as a function of thickness z (in meter) of the wake. Fig.(a) shows the plot upto thickness $z(t_1)$ and Fig.(b) is from $z(t_1)$ upto $z(t_f)$.

with the thickness $z(t)$ (in unit of meter). Fig.5.2a, shows the plot upto thickness

$z(t_1)$ until which density will keep on increasing due to the effect mentioned above. Fig.5.2b, gives the evolution of density from thickness $z(t_1)$ to $z(t_f)$. For convenience, the inset in Fig.5.2b. shows the expanded plot of the regions where density rapidly goes down towards zero. Fig.5.3 shows the plot of total number of axions inside the wake as a function of thickness for the time zones, $t_1 \leq t \leq t_f$.

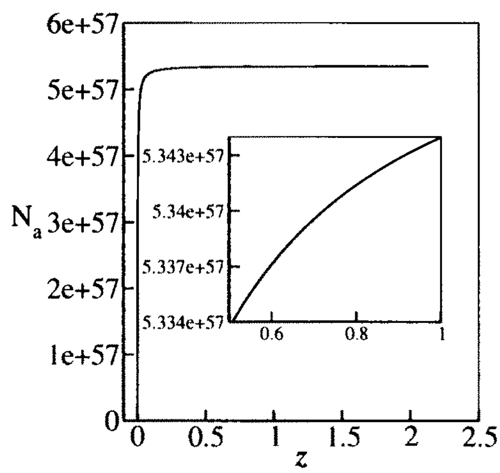

Figure 5.3: This figure shows the decrease of total number of axions during the time interval $t_1 < t < t_f$ as a function of thickness z (in meter). Small part of the apparently flat portion of the plot is expanded in the inset.

Here, volume of a wake is taken as the average thickness of the wake multiplying by the area $A(t)$ (taking 15 long strings per horizon [79] and assuming the sheets extend across the horizon, area of each planar sheet is given as $A(t) \sim \frac{(2t)^2}{15}$) as discussed earlier. The inset in Fig.5.3 shows the expanded plot within very small distance interval to illustrate the decrease of the total number of axions with the interface motions. Finally, the density profile of axions which are left behind as the interface collapse which is obtained from Eq.(5.18) is shown in Fig.5.4. Here, plot has been given for the relevant time interval ($t_1 < t < t_f$). We can take average axion density of the order of $\sim 10^2$ (in Fig.5.4), then we see that as the interfaces bordering

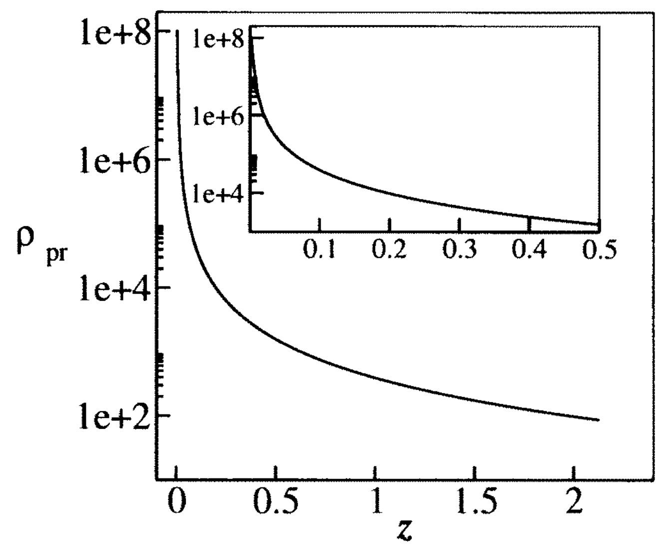

Figure 5.4: This figure shows the density profile of the axions which are left behind as the interfaces collapse. Density is given in unit of fm^{-3} and thickness z in meter.

the wake is reduced to about ~ 0.1 cm, then almost all the axions will leave the wake (see Fig.5.3) and the density will be increased by a factor of $\sim 10^5$. Since, the energy of the axions required to escape from the wake regions comes from the walls of the interfaces, these are kind of isocurvature fluctuations. One can also calculate the total mass (M_a) of the axions which are concentrated within this planar sheet region (formed by left behind axions). Taking total number of axions N_a of the order of $\simeq 10^{57}$ (See Fig.5.3), the mass of the axionic sheet can be obtained from Eq.(2.38) and given by,

$$M_q \simeq N_a\ m_{ah} \simeq 6.04 \times 10^{-6}\mathrm{eV}(\frac{10^{12}\ \mathrm{GeV}}{\eta_a})N_a \simeq 10^{19}(\frac{10^{12}\ \mathrm{GeV}}{\eta_a})\ \mathrm{gm}. \tag{5.19}$$

Taking $\eta_a \simeq 10^{10}$ GeV, the upper bound on mass of the axionic sheet will be order of 10^{21} gm ($\sim 10^{-12} M_\odot$ at QCD epoch). Since, the fluctuations produced by axion trapping are of isocurvature kind, they grow little [29, 82] before radiation-matter equality era, t_{eq}. After t_{eq}, they grow in proportional to the scale factor. Hogan and Rees [82] have studied the evolution of isocurvature fluctuations which were produced at the era of QCD phase transition. They have shown that isocurvature perturbations produced by axions at QCD epoch can lead to formation of axionic 'minicluster'. The amplitude of the isocurvature fluctuations produced in our model are very large and axions are concentrated within a very narrow sheet like regions of thickness of order 0.1 cm. This sheet will extend to a distance scale of order $\sqrt{A}(t)$. Where, $A(t) \simeq \frac{(2t)^2}{15}$ is the typical area of the wake as discussed above. This is about 2 km at QCD scale, which corresponds to comoving distance scale of order 10^{-7} Mpc today. One can study the evolution of such sheet like overdense region as it enters into the radiation-matter equality era. One can also study these overdensity at larger distance scales resulting from the large scale distribution of strings and their wakes. If these fluctuations survive until late stages, it will be interesting to study the effects of sheet like axionic clusters, especially whether they can have any effects on small scale CMBR anisotropies.

5.4 Conclusion and Discussion

In this chapter, we have presented our study on the isocurvature fluctuations in axion density at QCD phase transition epoch due to the presence of density fluctuations produced by moving cosmic strings. We have considered the axions which are produced from the radiation of the axionic strings which are formed at some scale η_a due to breaking $U(1)_{PQ}$ symmetry. If the mass of the axions is relatively higher in the hadron phase compared to QGP phase then the axions may get trapped initially inside the wake-like overdensity regions. As the transition from hadronic phase to

QGP phase proceeds with the motion of interfaces, these axions will acquire momentum due to collapsing interfaces and subsequently leave the wake. The axions thus left behind as the interfaces collapse may produce isocurvature fluctuations. We have estimated the detailed profile of the fluctuations. We have shown that, in our model the isocurvature fluctuations in the axion density will be of order $(\frac{\delta\rho}{\rho})_{axion} \sim 10^5$ and they will be concentrated within a planar sheet like region of thickness few cm. This sheet can extend upto a distance scale of order 2 km at QCD scale. This length scale corresponds to comoving length scale of order 10^{-7} Mpc today. It will be interesting to study the implications of such large fluctuations in the axion density especially on small scale CMBR fluctuations. Here, we should mention that, while discussing the trapping mechanism by the cosmic string wakes, we have taken the case for straight strings only. Since, the qualitative picture of our model remains same even for wiggly strings case, for simplicity, we have quoted the result of axionic inhomogeneities generation by straight string wakes only. Of course, in the treatment of shock formation by wiggly strings, one should take into account the non-uniform nature of fluid around the wiggles, as we have already mentioned in the previous chapter.

We have also mentioned the case where axion (being collisionless) directly (other than trapping mechanism) can give rise to density fluctuations. It turns out, that the value of these fluctuations are very small compared to the fluctuations produced from trapping of axions by cosmic string wakes.