Chapter 1

General Introduction

- In this thesis we study the dynamics and consequences of phase transitions in theory of quantum chromo dynamics (QCD), and in certain other field theory systems with explicit symmetry breaking. The study is focussed on formation of topological defects in phase transitions for systems with explicit symmetry breaking and on finding signatures of quark-hadron phase transitions in heavy-ion collision experiments.

  Topological defects are solutions of field equations which arise when order parameter space or the vacuum manifold of a theory has non-trivial topology, for example when there are closed loops, or closed surfaces in the vacuum manifold which can not be smoothly shrunk to a point. Examples of topological defects are cosmic strings, domain walls, monopoles, flux tubes, vortices etc. Topological defects arise in systems in particle physics as well as in condensed matter systems [1].

  Conventionally two types of processes are believed to be responsible for the defect formation. One of these is the thermal production of defects which leads to defect density suppressed by Boltzmann factor for low temperatures [2]. The other process is based on the formation of uncorrelated domains in physical space. These domains are regions in physical space where the order parameter field is nearly uniform while the order parameter varies randomly from one domain to the other. Defects are formed at the junction of these domains. This process is known as the Kibble Mechanism [3], which was originally proposed
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for studying defect formation in early Universe phase transitions. However, this mechanism as such has general applicability and has been experimentally verified for certain condensed matter systems [4]. Apart from this there have experimental study of scaling of defect network in condensed matter systems [5].

An important aspect of the Kibble mechanism is that it does not crucially depend on the dynamical details of the phase transition. The number of defects (per domain) produced via the Kibble mechanism depends only on the topology of the order parameter space and on the spatial dimensions. Dynamics plays a role here only in determining the domain size, affecting net number of defects produced in a given region. Still, the number of defects per domain is entirely independent of the dynamics. [Apart from some special situations, e.g. in a very slow first order transition, see ref. [6].]

One problem we have studied relates to the effect of the dynamics of order parameter field on the process of topological defect production. We have proposed a new mechanism for defect formation, which arises from the dynamics of the order parameter in the phase transition. We showed that whenever the field passes through zero magnitude, while oscillating in a region where the field is not uniform, it results in creation of a defect-antidefect pair [7]. Certain class of defects like textures, skyrmions may be produced singly via this mechanism. Large field oscillations which are essential for this mechanism depend on various dynamical details, such as nucleation rate for first order transition, quench time scale for second order transition case, and presence of damping in the problem. We find that for first order transition case defect production via this mechanism strongly varies as different parameters of the transition are changed. Also for systems with explicit symmetry breaking this mechanism completely dominates over the Kibble mechanism in defect production, leading to strong enhancement in defect production [8].

These results can be extended to baryon formation in a chiral phase transition. In the Skyrme picture baryon appears as a topological object. Their formation via the Kibble mechanism of defect formation has been discussed in literature [9]. In these models production of skyrmions (baryons) is studied using the linear sigma model where the chiral
symmetry of QCD is spontaneously broken with pions being the Goldstone bosons. However one knows that the chiral symmetry is explicitly broken as well which is responsible for non-zero pion mass. It has been argued in literature [10] that due to presence of explicit symmetry breaking there will be enhancement in the production of skyrmions. However the mechanism we have proposed for defect production may be the most dominant mechanism in this case, specifically due to presence of explicit symmetry breaking. Our results suggest that strong enhancement in production of baryons (at low $p_T$) could be a possible signature of chiral phase transition in relativistic heavy-ion collisions.

The second set of problems we have studied is related to the study of phase transition in QCD. QCD predicts that at extreme nuclear densities or at very large temperatures, strong coupling constant becomes very small [11]. This is known as asymptotic freedom. Under these conditions hadrons, which are the excitations of QCD at low energies, dissolve leading to the formation of a system of weakly interacting quarks and gluons, the Quark-Gluon Plasma. Lattice calculations show that this deconfinement phase transition may be of first order. Lattice calculations also show that the quark-antiquark condensate $<\bar{\psi}\psi>$ vanishes at high temperatures restoring (approximately) the chiral symmetry of the QCD Lagrangian. This is referred to as the chiral transition. For the two flavor case, the chiral transition is expected to be of second order. For a review on lattice approach to QCD phase transitions, see [12]. Experimental observation of such a phase of deconfined and/or chirally symmetric matter (of QCD) will be an important test of QCD.

One topic we have worked on in the context of relativistic heavy-ion collisions relates to formation of disoriented chiral condensate (DCC). DCC has been proposed in literature [16, 17, 18] as a signature for formation of an intermediate chirally symmetric phase of matter in heavy-ion collisions. In the linear sigma model in the limit of zero pion mass chiral symmetry is spontaneously broken below the critical temperature with the vacuum manifold being a 3-sphere $S^3$ (characterized by $\vec{\pi} \cdot \vec{\pi} + \sigma^2 = f_\pi^2$). As the system cools from high temperature symmetric phase, to a temperature below $T_c$, one particular point on $S^3$ will be chosen as the vacuum state in a given region of space, with all points on $S^3$ being
equally likely. This will lead to a sort of domain structure in the physical space where each domain will have the chiral field aligned in a given direction (specified by three angles characterizing the $S^3$), but the directions in different domains vary randomly. Even for non-zero pion mass one may expect that all points on $S^3$ are distributed in domains with roughly equal probability during the phase transition. After the phase transition, for non-zero pion mass, all the domains with non-zero chiral angles become unstable (called DCC domains), with chiral field rolling down towards the true vacuum. These DCC domains decay emitting low momentum coherent pions which can be detected by anomalous fluctuations in the ratio of neutral to all pions [18].

In an equilibrium second order phase transition expected DCC domain size is of order $1\text{fm} \sim m_\pi^{-1}$, pion being the lightest particle. However, it has been proposed that in a non-equilibrium phase transition involving a quench, large DCC domains may arise [19]. In such a transition, below the critical temperature, low momentum modes becomes unstable and grow exponentially till the order parameter (chiral condensate) rolls down the potential hill and crosses the point of inflection in the effective potential (this discussion is in the context of linear sigma model). This leads to growth of DCC domains. However due to strong coupling the chiral condensate crosses the spinodal point within a time scale of $\sim 1\text{fm}$, so domains grow only for a small time [20, 21]. Alternative scenarios have been proposed in literature [22] where the assumption of ideal quench has been relaxed. In such scenarios somewhat larger DCC domains can arise [22]. In such models one needs to study growth of domains for temperatures less than $T_c$. We have studied thermal fluctuations of regions of correlation size and its effect on the formation of DCC domains [23]. We made a rough estimate of the Ginzburg regime ($\sim 0.8T_c \leq T \leq T_c$ in the case of non-zero pion mass), and argued that in this regime, regions of correlation length size ($\sim m_\pi^{-1}$, say) can easily fluctuate to the chirally symmetric phase, making DCC domains ill defined. One implications of this results is that domains can form even in relatively lower energy heavy-ion collisions where the temperature of the system never rises to $T_c$, but to a temperature above the Ginzburg temperature $T_G \sim 0.8T_c$. Also, this implies that detection of DCC, though interesting
by itself, will not imply that a chirally symmetric phase of matter has been detected. A hot hadronic gas, in the spontaneously broken phase of chiral symmetry, can still lead to formation of DCC as long as its temperature reaches a value above $T_G$. We have argued that since the DCC domains are not defined above $T_G$ conventional calculations of DCC domain growth should be done only for temperatures below $T_G$.

The second topic we have worked on, in the context of relativistic heavy-ion collision, relates to the dynamics of quark-hadron transition. Ideas of first and second order phase transitions have been applied to find signatures like strangeness enhancement, entropy generation, enhancement in baryon production, large domains of DCC etc. Consequences of a first order transition in heavy ion collisions have been investigated by many people, for example, the dynamics of the transition and issues such as the duration of the transition etc. have been discussed by Csernai and Kapusta [13]. These studies have used homogeneous nucleation of hadronic bubbles in a uniform QGP background to study the dynamics of the quark-hadron transition. After a reasonable amount of supercooling, when the nucleation rate becomes large enough, bubbles of hadronic phase nucleate. These bubbles expand releasing latent heat which heats up the plasma to a temperature close to the critical temperature $T_c$. Since the nucleation rate is very small at temperatures close to $T_c$, further nucleation of bubbles shuts off and phase transition is completed by expansion and coalescing of bubbles which have been already nucleated. This results in delay in completion of the phase transition which leads to entropy generation [13].

In our study we discussed an alternative scenario for this first order quark-hadron transition focusing on the fact that QGP region produced in heavy-ion collisions is of finite size [14]. Outside this region one either has hadronic gas or simply the QCD vacuum in the confining phase. In both these cases there will be a boundary wall with non-zero surface tension between the QGP region and the region outside. Lattice simulations (with no dynamical quarks) suggest [15] a value of the surface tension of this wall $\sigma \leq 50 MeV/fm^2$. For simplicity we assume that the QGP region is surrounded by superheated hadron gas with temperature as that of QGP region. We show that as the temperature decreases due to
expansion, the QGP region becomes subcritical before $T_c$ is reached and the boundary starts shrinking. This shrinking of the boundary releases latent heat similar to the homogeneous nucleation case where expansion of bubbles releases latent heat. In both the cases QGP is being converted to Hadronic gas. This release in latent heat due to boundary wall collapse raises the temperature close to $T_c$, shutting off nucleation of hadronic bubbles in the inner region. Even if some bubbles are nucleated inside the QGP, for certain parameters of the model, bubbles of hadronic phase become subcritical after reheating and shrink away. So the phase transition is completed only via shrinking of the boundary. This picture of phase transition leads to interesting signatures like concentration of baryons in a thin pencil shaped region in the heavy-ion collisions, which can be detected by doing baryon-interferometry.

The organization of the thesis is as follows. In chapter two, we give a brief review of topological defects. Here we will discuss classification of defects and the Kibble mechanism of defect formation in phase transitions. Also we briefly describe numerical simulation of formation of defects via the Kibble mechanism in bubble collision for a first order phase transition. In chapter three we will discuss a new mechanism of defect formation which we have proposed for systems, where a continuous symmetry is spontaneously as well as explicitly broken. Here we will demonstrate formation of defects via this mechanism for a model with explicit (along with spontaneous) breaking of $U(1)$ symmetry through numerical simulation of bubble collisions. In Chapter four we will present our study of a full simulation of a first order phase transition in the presence of explicit symmetry breaking and discuss the relative importance of this mechanism in production of defects compared to the conventional Kibble mechanism.

A brief review of phase transitions in QCD will be given in Chapter five. Here we will discuss (DCC) as a signature proposed for detecting chirally symmetric phase of QCD expected to be formed in heavy-ion collision experiments. We will briefly discuss conventional scenario of formation of DCC during the chiral transition. At the end we will present our calculations of Ginzburg regime below the critical temperature $T_c$ and discuss the implications of these results for conventional DCC studies.
In chapter six we will discuss signatures proposed for deconfinement phase transition, i.e. formation of QGP, in heavy-ion collision experiments. We will describe in detail a new scenario for first order Quark-Hadron transition in heavy-ion collisions and discuss its implications.

Finally in chapter seven we will present the summary of our study of formation of topological defects in phase transitions and study of QCD phase transitions from the point of view of finding signatures of QGP formation in heavy-ion collision. Throughout, we will use the natural units with $\hbar = c = 1$. 