8.1 INTRODUCTION

In Chapter 7, a stochastic inventory model with mixture of backorders involving reducible lead time and setup cost was developed. A problem we have in production and inventory is yield by production or procurement. Generally, the supply and the requirement may not be equal. Sometimes the manufacturers send lesser quantity than the order placed. This is maximum issue quantity restriction. Lot-sizing problems with random yields have been developed by many researchers such as Giri et. al., Silver, Yano and Lee [33, 104, 125]. A variable yield lot sizing problem with stochastic demand was put forward by Gerchak, Karlo and Gohil, Nadjib et. al., Noori and Keller, Parlar and Wang, Spence, Wang and Gerchak [29, 54, 73, 75, 87, 105, 116].

A producer has a chance to choose production process and machines. Modern production technology can improve the production process and yield. Gerchak and Parlar [30] suggested that through appropriate investment, yield variability can be reduced.

In the real market, some customers are ready to wait for familiar goods like fashionable commodities. If the quantity of shortages is accumulated to a degree that exceeds the waiting patience of customers, some may refuse the back orders, i.e. if the lead time is longer, then shortage accumulation is higher,
i.e. some customers will be ready to wait for their choice of products while
some may go for some other brand, consequently the proportion of waiting
patience of customers is smaller and hence the backorder rate will be reduced.
This assumption is very realistic. Here the lead time, setup cost and yield
variability are reduced through additional investments. To the best of my
knowledge there is no inventory model which incorporates all the above
concepts. Also we consider the concepts like partial backlogging and backorders
price discount. Lead time here is verily reduced. That is lead time is almost nil.
The chapter is arranged as follows. In section 2, problem description,
assumptions and notations are given. In section 3, a cost minimization model is
formulated. Numerical examples are given in section 4. We conclude this
chapter in section 5.

8.2 PROBLEM DESCRIPTION

In this chapter, lead time reduction by capital investment and the
assumption that the lead time demand’s probability distribution is unknown, are
considered. It is also assumed that backorder rate depends on the length of lead
time through the amount of shortages. The relationships between the reduction in
lead time, yield variability and setup cost with capital investment can be
described by logarithmic investment function. This approach is consistent with
JIT (Just-in-time) manufacturing philosophy. So this function was used by many
researchers like Porteus, Sarkar and Coates [89, 90, 96]. The nominal lead time,
yield standard deviation, setup cost, the reorder point, optimal order quantity,
optimal safety factor and the associated capital investments are taken as decision
variables. It is proved that expected annual cost function with capital investment
is convex. Optimal order quantity, reorder point, optimal values of lead time,
yield standard deviation and setup cost are found by using a mini-max
distribution free procedure. Significant cost savings is achieved in the model.

8.2.1 ASSUMPTIONS

1. The inventory system involves only one item.
2. The distribution of lead time demand \( x \), which is assumed to be
   independent of \( Y_Q \), is unknown.
3. The reorder point \( r = \) expected demand during lead time + safety stock,
   where safety stock = safety factor × standard deviation of lead time
demand = \( A \sigma \sqrt{L} \)
4. The quantity received is a random variable and depends upon the quantity
   ordered.
5. The bias factor \( \mu \) remains fixed when the yield standard deviation \( \sigma \), can
   be reduced through capital investments. While \( \mu \) is necessarily greater
   than zero, it need not be less than one, as the expected amount received
   may exceed the quantity ordered for reasons such as counting errors, a
   good production run leading to a quantity larger than the required
   quantity, built in allowances by the supplier, etc as assumed in Karlo and
   Gohil [54] and Gerchak and Palar [30].
6. The relationship between lead time reduction and capital investment can
   be described by using a logarithmic investment cost function. i.e. the lead
   time \( L \) and the capital investment in lead time reduction, \( \Phi_L \) can be
   stated as \( \Phi_L(L) = u \ln \left( \frac{L_0}{L} \right) \) for \( 0 < L \leq L_0 \). The relationship between the
yield standard deviation $\sigma_y$ and the capital investment in the yield standard deviation reduction $\Phi_{\sigma_y}$ is stated as

$$
\Phi_{\sigma_y}(\sigma_y) = v \ln \left( \frac{\sigma_y}{\sigma_y^0} \right) \text{ for } 0 < \sigma_y \leq \sigma_y^0.
$$

The relationship between the setup cost reduction and capital investment can be given by $\Phi_K(K) = w \ln \left( \frac{K_0}{K} \right) \text{ for } 0 < K \leq K_0$.

8. During the stock out period, the backorder rate $B$ is a variable and is a function of lead time through the amount of shortage. $B$ is a function of $L$ through $E[(x-r)^+]$ and

$$
B = \frac{1}{1 + \delta E[(x-r)^+]} \text{ where the backorder parameter } \delta \text{ is a positive constant.}
$$

8.2.2 NOTATIONS

- $Y_Q$: Quantity received given that $Q$ units are ordered, a random variable.
- $K_0$: Original setup cost per setup.
- $\sigma_y^0$: Original yield standard deviation.
- $L_0$: Original length of lead time.
- $u'$: The fraction of reduction in $L$ per dollar increase in $\Phi_L$ and $u = 1/u'$.
- $v'$: The fraction of reduction in $\sigma_y$ per dollar increase in $\Phi_{\sigma_y}$ and $v = 1/v'$.
$w'$  The fraction of reduction in $K$ per dollar increase in $w$, and $w = 1/w'$.

$i$  cost of capital per $ per year.

$\sigma$  Standard deviation of demand per unit time.

$T$  Expected length of the Replenishment cycle.

$\text{TAC}$  Total annual cost.

$\text{EAC}$  Expected Total Annual Cost.

$\mu$  bias factor $= E(Y_0)/Q$, proportion of expected quantity received to the quantity ordered, $\mu$ may be greater than 1.

$\sigma_y$  Nominal yield standard deviation.

$L$  Nominal length of lead time.

$\Phi_k$  Capital investment required to reduce setup cost from $k_0$ to $K$.

$\Phi_{\sigma_y}$  Capital investment required to reduce yield standard deviation from $\sigma_{y_0}$ to $\sigma_y$.

$\Phi_L$  Capital investment required to reduce lead time from $L_0$ to $L$.

$A$  Safety factor.

$\text{EAC}^*$  Least upper bound of expected total annual cost.

8.3 MODEL FORMULATION

In the proposed model, the inventory level of an item is reviewed continuously and when the inventory level drops to a reorder point $r$, a lot size $Q$ is ordered. Fig. 8a geometrically represents the behaviour of the inventory system. We have assumed that the lead time demand $x$ has p.d.f $f_x(x)$ with finite mean $RL$ and standard deviation $\sigma\sqrt{L}$ and the reorder point
\[ r = RL + A \sigma \sqrt{L} \]. Then the expected shortage at the end of the cycle is given by \( E[(x-r)^+] \). Thus the expected number of backorders per cycle is \( BE[(x-r)^+] \).

![Graph showing inventory level over time](image)

**Figure 8a. Behaviour of the inventory system**

Hence the expected demand lost per cycle is \( (1-B)E[(x-r)^+] \). Then the expected net inventory level just before the order arrives is \( r - RL + (1-B)E[(x-r)^+] \) and the expected net inventory level at the beginning of the cycle is

\[
\frac{Q(\sigma_y^2 + \mu^2)}{\mu} + r - RL + (1-B)E[(x-r)^+].
\]

\[ \therefore \] The expected holding cost per year is

120
\[
\begin{align*}
= h \left\{ \frac{Q(\sigma_y^2 + \mu^2)}{2\mu} + A \sigma \sqrt{L} + \left\{ \frac{\delta \mathbb{E}[x - r]^+}{1 + \delta \mathbb{E}[x - r]^+} \right\} \mathbb{E}[x - r]^+ \right\}
\end{align*}
\]

Now, the expected total demand per cycle is \( \mu Q + (1 - B) \mathbb{E}[x - r]^+ \).

The expected cycle length \( T = \frac{\mu Q + (1 - B) \mathbb{E}[x - r]^+}{R} \).

Ouyang and Chuang [80] used an approximate cycle length \( T \approx Q/R \). Lin and Hou [64] used a cycle length \( T = \mu Q/R \). Here we consider generalized expected cycle length in which we include safety stock and expected demand lost per cycle. This indicates that the model herein differs from the above models.

Annual setup cost

\[
\begin{align*}
= \frac{K}{T} = \frac{KR}{\mu Q + \frac{\delta \mathbb{E}[x - r]^+}{1 + \delta \mathbb{E}[x - r]^+} \mathbb{E}[x - r]^+}
\end{align*}
\]

Annual stockout cost

\[
\begin{align*}
= \frac{SB \mathbb{E}[x - r]^+ + (S + \pi_0)(1 - B) \mathbb{E}[x - r]^+}{T}
\end{align*}
\]

The expected total annual cost is given by

\[
\text{TAC} = \text{Holding cost} + \text{Setup cost} + \text{Stockout cost}.
\]

As it takes investment to reduce lead time, yield standard deviation and setup cost, we should include an amortized investment cost in our model.

Therefore, the expected total annual cost for the system

\[
\begin{align*}
EAC(L, \sigma_y, Q, K) = h \left\{ \frac{Q(\sigma_y^2 + \mu^2)}{2\mu} + A \sigma \sqrt{L} + \left\{ \frac{\delta \mathbb{E}[x - r]^+}{1 + \delta \mathbb{E}[x - r]^+} \right\} \mathbb{E}[x - r]^+ \right\}
\end{align*}
\]

\[
+ \frac{KR}{\mu Q + \frac{\delta \mathbb{E}[x - r]^+}{1 + \delta \mathbb{E}[x - r]^+} \mathbb{E}[x - r]^+}
\]

121
\[
RE[(x-r)^+] \left\{ S + \pi_0 \left[ \frac{\delta E[(x-r)^+]}{1 + \delta E[(x-r)^+]} \right] \right\} \\
+ \mu Q + \frac{\delta E[(x-r)^+]}{1 + \delta E[(x-r)^+]} \\
+ \Phi_L(L) + \Phi_{\sigma_y}(\sigma_y) + \Phi_K(K).
\]

It is assumed that the probability distribution of lead time demand \( x \) has given first two moments: i.e., the p.d.f \( f_x \) belongs to the class \( \Omega \) of p.d.f's with finite mean RL and standard deviation \( \sigma \sqrt{L} \). Since the probability distribution of \( x \) is unknown, we can not find the exact value of \( E[(x-r)^+] \). We propose to apply the minimax distribution free procedure for our problem. The minimax distribution free approach for this problem is to find the "most unfavorable" p.d.f. \( f_x \) in \( \Omega \) for each \( (L, \sigma_y, k, Q) \) and then minimize the total expected annual cost over \( (L, \sigma_y, k, Q) \);

i.e., we need to solve

\[
\text{Min}_{(L, \sigma_y, k, Q)} \text{Max}_{f_x \in \Omega} EAC(L, \sigma_y, k, Q)
\]

For this purpose, we need the following proposition which was asserted by Gallego and Moon [28] (also used earlier in Chapter 7, p. 1041 - 1042).

Hence our problem is to minimize \( EAC^w(L, \sigma_y, K, Q) \) which is the least upper bound of \( EAC(L, \sigma_y, K, Q) \).

\[
EAC^w(L, \sigma_y, K, Q) = h \left\{ \frac{Q(\sigma_y^2 + \mu^2)}{2\mu} + A\sigma \sqrt{L} \right\} + \frac{\sigma \sqrt{L}}{2} \left[ \sqrt{1 + A^2} - A \right] \left[ \frac{h\delta}{2} \frac{\sigma \sqrt{L}}{2} \sqrt{1 + A^2} - A \right]
\]

122
\[
\begin{align*}
R \left[ S + (S + \pi_0) S \frac{\sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) \right] + \\
\mu Q + \mu Q \frac{\delta \sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) + \frac{\delta \sigma L}{4} \left( \sqrt{1 + A^2} - A \right)^2 \\
+ \frac{K R \left[ 1 + \delta \frac{\sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) \right]}{} + \\
\mu Q + \mu Q \frac{\delta \sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) + \frac{\delta \sigma L}{4} \left( \sqrt{1 + A^2} - A \right)^2 \\
\frac{w}{v} \ln \left( \frac{L_0}{L} \right) + v \ln \left( \frac{\sigma y_0}{\sigma y} \right) + w \ln \left( \frac{K_0}{K} \right)
\end{align*}
\]

\(EAC^w(L, \sigma_y, K, Q)\) is convex with respect to \(K\) (Appendix 8.1).

By equating \(\frac{\partial EAC^w(L, \sigma_y, K, Q)}{\partial K}\) to zero, the value of \(K\) is obtained as:

\[
K = i w \mu + \mu Q \frac{\delta \sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) + \frac{\delta \sigma L}{4} \left( \sqrt{1 + A^2} - A \right)^2 \\
R \left[ 1 + \delta \frac{\sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) \right]
\]

(76)

Also,

\(EAC^w(L, \sigma_y, K, Q)\) is convex with respect to \(\sigma_y\) (Appendix 8.2). Equating the first order derivatives to zero, that is,

\[
\frac{\partial EAC^w(L, \sigma_y, K, Q)}{\partial \sigma_y} = 0 \Rightarrow \sigma_y = \frac{i v \mu}{w Q}
\]

(77)

Using equations (76) and (77) in \(EAC^w(L, \sigma_y, K, Q)\), we can find \(EAC^w(L, Q)\)

Now,

\[
i w = \frac{K \sigma \sqrt{L}}{\mu Q + \mu Q \frac{\delta \sigma \sqrt{L}}{2} \left( \sqrt{1 + A^2} - A \right) + \frac{\delta \sigma L}{4} \left( \sqrt{1 + A^2} - A \right)^2}
\]

123
and \( \frac{iv}{2} = \frac{hQ}{2} \sigma_y s^2 \)

where \( iv \) is the optimal annual setup cost and \( \frac{iv}{2} \) is the extra holding cost when the yield is random and is a portion of expected annual inventory holding cost. In general, this extra holding cost is usually not greater than the total annual setup cost in practical situations for inventory models.

\( i.e. \quad iv > \frac{iv}{2} \). Also \( EAC^w[L,Q] \) is convex with respect to \( Q \) and \( L \) (Appendix 8.3).

By using the following equations we can find the optimal value of the expected total cost.

\[
\frac{\partial EAC^w(L,Q)}{\partial Q} = 0
\]  \hspace{1cm} (78)

\[
\frac{\partial EAC^w(L,Q)}{\partial L} = 0
\]  \hspace{1cm} (79)

Now, \( A \in \left[ 0, \frac{1}{\sqrt{\frac{1}{q} - 1}} \right] \) as in Chapter 7, p. 102.

Since the distribution of lead time demand is unknown we establish the following algorithm to find a suitable safety factor \( A \) and hence \( Q, L \).

**Algorithm**

**Step 1.** For each \( q \) divide the interval \( \left[ 0, \frac{1}{\sqrt{\frac{1}{q} - 1}} \right] \) into \( N \) equal subintervals.

\[
A_0 = 0, \quad A_N = \frac{1}{\sqrt{q}} - 1.
\]

\[
A_\ell = A_{\ell-1} + \frac{A_N - A_0}{N}, \quad 1 = 1,2,\ldots,N-1
\]
Step 2.

a) For given \( A_l \in \{A_0, A_1, \ldots, A_N\}; \; l = 1, 2, \ldots, N \) put

\[
Q_{A_l} = \frac{2hw - iv}{v} \quad \text{in equation (79) to find } L_{A_l}.
\]

b) Put \( L_{A_l} \) in equation (78) to find \( Q_{A_l} \).

c) Put \( Q_{A_l} \) in equation (79) to find \( L_{A_l} \).

Repeat (b) and (c) until there is no change in \( Q_{A_l} \).

Step 3. Find \( EAC^w(Q_{A_l}, L_{A_l}) \)

Step 4. Find \( \min_{A_l \in \{A_0, A_1, \ldots, A_N\}} EAC^w(Q_{A_l}, L_{A_l}) \)

8.4 NUMERICAL EXAMPLES

Numerical examples are given to mention the workability of the proposed theory, model and to discuss the sensitivity analysis. Computational works are carried out using MATLAB 7.0. Some data input values were taken from the earlier published work [64].

8.4.1 WORKABILITY OF THE THEORY AND MODEL

Example 1

Consider the following data: \( R = 1000 \) units per year, \( h = $10 \) per unit per year, \( \delta = 10, \; \sigma = 3 \) units, \( \mu = 1.5, \; i = 0.15, \; u = 15, \; v = 25, \; w = 1600, \; S = $140 \) per unit short, \( \pi_0 = $100 \) per unit, \( N = 200 \) and \( q = 0.2 \). The initial value of \( Q \) is 31.75. All the above data are in-tact. Let \( \sigma_{y_0} = 9, \; L_0 = 10 \) and \( K_0 = $75 \) per setup.

We change one parameter among \( \sigma_{y_0}, \; L_0 \) and \( K_0 \) while the other two are kept fixed. When we increase one of these parameters we see that there will be an
increase in the expected total annual cost and the capital investment to reduce the corresponding parameter. The nominal lead time, yield standard deviation, setup cost, the reorder point, optimal order quantity, optimal safety factor and the associated capital investments are also found. The results are shown in Table 8.i, Figures 8b, 8c and 8d. Table 8.i shows that lead time, setup cost and yield variability are certainly reduced. Figure 8b gives us a clear idea that expected annual cost increases if setup cost increases. Figure 8c shows that if lead time increases then expected annual cost will also increase. Figure 8d shows the effects of yield standard deviation on expected annual cost. From all the above three figures we observe that expected annual cost can be minimized by reducing setup cost, lead time and yield variability.

**Example 2**

Let $\sigma_{y_0} = 3$, $L_0 = 15$, $K_0 = $150 per setup, $R = 1500$ units per year, $h = $10 per unit per year, $\delta = 15$, $\sigma = 10$ units, $\mu = 1.5$, $i = 0.15$, $u = 50$, $v = 100$, $w = 1800$, $S = $50 per unit short, $\pi_0 = $200 per unit, $N = 100$ and $q = 0.2$. The initial value of $Q = \frac{2hw - iv}{h\mu}$ is obtained as 35. The results are shown in Table 8.ii.

**Example 3**

Let $\sigma_{y_0} = 2$, $L_0 = 8$, $K_0 = $200 per setup, $R = 1600$ units per year, $h = $9 per unit per year, $\delta = 20$, $\sigma = 5$ units, $\mu = 1.5$, $i = 0.15$, $u = 40$, $v = 125$, $w = 1750$, $S = $75 per shortage, $\pi_0 = $150 per unit, $N = 50$ and $q = 0.2$. The initial value of $Q$ is 37.5. The calculated values are given in Table 8.ii. From Table 8.ii we observe that an increase in one parameter ($L_0$ or $K_0$ or $\sigma_{y_0}$) will increase the expected annual cost.
Table 8.1 Brief summary of the results of example 1

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<th>$K_0$</th>
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<th>$L^*$</th>
<th>$A^*$</th>
<th>$K^*$</th>
<th>$\sigma_{y^*}$</th>
<th>$r^*$</th>
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<th>$\phi_{\sigma_y}(\sigma_{y^*})$</th>
<th>$\phi_L(L^*)$</th>
<th>$EAC^W(K^<em>, \sigma_{y^</em>}, Q^<em>, L^</em>)$</th>
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<td>104.23</td>
<td>248.91</td>
<td>1196.5</td>
</tr>
</tbody>
</table>
Table 8.ii  Brief summary of the results of examples 2 and 3

<table>
<thead>
<tr>
<th>Example Number</th>
<th>$Q^*$</th>
<th>$L^*$</th>
<th>$A^*$</th>
<th>$K^*$</th>
<th>$\sigma_y^*$</th>
<th>$r^*$</th>
<th>$\phi_k(K^*)$</th>
<th>$\phi_{\sigma_y}(\sigma_y^*)$</th>
<th>$\phi_L(L^*)$</th>
<th>$EAC^w(K^<em>,\sigma_y^</em>,Q^<em>,L^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.9292</td>
<td>2.2929e-007</td>
<td>0.48</td>
<td>1.601</td>
<td>0.6159</td>
<td>0.0026</td>
<td>8171.91</td>
<td>158.32</td>
<td>899.80</td>
<td>1759.4</td>
</tr>
<tr>
<td>3</td>
<td>5.6337</td>
<td>2.0418e-007</td>
<td>0.36</td>
<td>1.4132</td>
<td>0.7448</td>
<td>0.0014</td>
<td>8666.19</td>
<td>123.45</td>
<td>699.34</td>
<td>1760.1</td>
</tr>
</tbody>
</table>
Figure 8b. Effect of Setup cost on Expected Annual Cost
Figure 8c. Effect of lead time on the expected annual cost
Figure 8d. Effect of yield standard deviation on expected annual cost
8.5 CONCLUSION

Reduction in setup cost, yield variability and lead time are important strategies in manufacturing. In this model we assumed that the distribution of lead time demand is unknown. We developed ordering policies when we could reduce setup cost, yield variability and lead time through capital investment, which is a single time investment but the profit is last longing. Here it is shown that the cost function is convex and an algorithm to determine the optimal order quantity and reorder point is developed. The optimal setup cost, yield standard deviation, lead time and the corresponding optimal capital investments are determined by using a minimax distribution free procedure. Numerical examples are given to elucidate the model. We reduced setup cost, yield standard deviation and lead time to a greater extent. In particular lead time is verily reduced. That is equal to zero. This approach is consistent with JIT economy, which calls for reducing setup cost, yield variability and lead time. Our model greatly differs from the model existing in the literature (the model by Lin and Hou [64]) in the following aspects: 1) In the above model, yield variability and setup cost were reduced through capital investment. In our model, we reduce yield variability, setup cost and also the lead time, which plays a vital role in any business. By reducing lead time we can improve the service level to the customer so as to increase the competitive edge in business. 2) In the model (the model by Lin and Hou [64]), it was assumed that lead time demand follows normal distribution. But in our model we take the distribution of lead time demand as distribution tree. That is, it can follow any distribution which is more general. 3) In the above model (the model by Lin and Hou [64]), shortages are completely backlogged. But we consider partial backlogging and take the backlogging rate as $0 \leq B < 1$. If we set backlogging rate $B=1$ we get the above model. That is, the above model is particular case of our model. 4) We also assumed that the backorder rate depends on the length of the lead time through the amount of shortages. If the lead time is longer then shortage
accumulation is higher. The patience of customers will result in failure in business since some customers may turn to some other supplier. Hence the backorder rate will be reduced. This assumption is very realistic.