CHAPTER 7

DEVELOPMENT OF STOCHASTIC INVENTORY MODEL WITH MIXTURE OF BACKORDERS INVOLVING REDUCIBLE LEAD TIME AND SETUP COST

7.1 INTRODUCTION

In Chapters 3, 4, 5 and 6, we have discussed periodic review policy. In reality some cases may need continuous review policy. So in this chapter we adopt continuous review policy. In Chapter 6, a stochastic inventory model in which demand depends on initial net inventory level, auto-correlated and price sensitive. Time and cost are the most important competitive factors in business. Although lead time can be constant or variable, it is often treated as a prescribed parameter in most of the inventory models and consequently not controllable. Under cost considerations, a firm can apply a variety of means to reduce the lead time to satisfy customer’s demands. Lead time plays a vital role in several models [6, 16, 21, 26, 41, 49, 51, 58]. Usually, lead time consists of the following components namely (as mentioned by Tersine [108]) order preparation, order transit, supplier lead time, delivery time and setup time. The concept of Just-in-time delivery has grown popular in the last two decades. In certain cases, such as the automobile or computer assembly industries, most suppliers have to ship materials to third-party warehouses and then to ship from the warehouses to the manufacturing assembler in a just-in-time pattern. Several researchers like Hariga and Ben-Daya, Moon and Choi, Ouyang and Chuang,
Ouyang et al., [41, 70, 78, 81] have presented various inventory models with lead time reductions.

Setup cost has the following components: cost of tooling up, administrative cost, record keeping, purchasing raw materials in bulk, purchase cost, requisition, follow-up, receiving the goods, quality control. The benefits of reduced setups are well documented by many researchers like Billington, Hall, Kim et al., Nori and Sarkar, Paknejad et al., Porteus, Sarkar and Coates, Trevino et al., [8, 40, 57, 76, 84, 89, 96, 110].

In classical economic order quantity model, dealing with the problem of shortages, it is observed that while shortages occur, some customers may prefer their demands to be backordered and some may refuse backorders. During shortage period many factors affect customer’s acceptance of backorders. Price discount from the supplier is the prime factor. By offering price discounts the supplier can secure more backorders through negotiation. The higher the price discount from the supplier, a large number of back order ratio could be fetched by the supplier. Pan and Hsiao [85] presented inventory model with backorder discount with variable lead time. Existing research articles in the literature focused either setup cost reduction or lead time reduction. But in the proposed model both reductions are considered with distribution free lead time demand. Besides this we also propose partial backlogging and price discounts for backorders. Compared to the earlier models significant cost savings are achieved in the proposed model. The chapter is arranged as follows: In section 2, problem description, assumptions and notations are given. In section 3, a cost
minimization model is formulated. Numerical examples are given in section 4. In section 5, we conclude with our results.

7.2 PROBLEM DESCRIPTION

In this chapter the concept of capital investment allocated to reduce lead time and setup cost is considered. It is assumed that probability distribution of lead time demand is unknown but its first two moments are known. Back order price discount and order quantity are taken as decision variables in this model. Mini-max distribution free procedure is applied to minimize expected annual cost. By reducing lead time and setup cost we can achieve significant savings and it is made known through numerical examples.

7.2.1 ASSUMPTIONS

1. The reorder point \( r = \text{expected demand during lead time} + \text{safety stock} \) where safety stock = \( A \times (\text{standard deviation of lead time demand}) \) \( ie., r = RL + A\sigma\sqrt{L} \) where \( A \) is the safety factor and satisfies \( P[x > r] = q \), \( q \) represents the allowable stockout probability during \( L \) and it is given.

2. Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point \( r \).

3. The lead time \( L \) has \( n \) mutually independent components. The \( i^{th} \) component has a normal duration \( b_i \) and minimum duration \( a_i \) and a crashing cost \( c_i \) when the normal duration \( b_i \) reduces to the duration \( a_i \). That is, there is a discontinuous relationship between the crashing cost and lead time reduction.
4. Assume \( \frac{c_1}{b_1 - a_1} \leq \frac{c_2}{b_2 - a_2} \leq \ldots \leq \frac{c_n}{b_n - a_n} \), the components of lead time are crashed one at a time starting with the component of least \( \frac{c_i}{b_i - a_i} \) and so on.

5. Let \( L_0 = \sum_{j=1}^{n} b_j \) and \( L_i \) be the length of lead time with components 1, 2, \ldots, \( i \) crashed to their minimum duration, then \( L_i \) can be expressed as \( L_i = L_0 - \sum_{j=1}^{i} (b_j - a_j) \), \( i = 1, 2, \ldots, n \) and the lead time crashing cost per cycle \( C(L_i) \) is given by \( C(L_i) = \sum_{j=1}^{i} c_j \).

6. The setup cost \( K \) has \( m \) mutually independent components. The \( j^{th} \) component has a normal cost \( e_j \) and minimum cost \( d_j \) and a crashing cost \( f_j \) when the normal cost \( e_j \) reduces to minimum cost \( d_j \). That is, there is a discontinuous relationship between the crashing cost and setup cost reduction.

7. Assume \( \frac{f_1}{e_1 - d_1} \leq \frac{f_2}{e_2 - d_2} \leq \ldots \leq \frac{f_m}{e_m - d_m} \), the components of setup cost are crashed one at a time starting with the component of least \( \frac{f_i}{e_i - d_i} \) and so on.

8. Let \( K_0 = \sum_{i=1}^{m} e_i \) and \( K_j \) be the setup cost with components 1, 2, 3, \ldots, \( j \) crashed to their minimum cost, then \( K_j \) can be expressed as
\[ K_j = K_0 - \sum_{i=1}^{j} (e_i - d_i), \ j = 1,2,\ldots,m \] and setup crashing cost per cycle \( C(k_j) \) is given by \( C(k_j) = \sum_{i=1}^{j} f_i \).

9. During the stockout period, the backorder ratio \( B \) is variable and is in proportion to the price discount \( \pi_x \) offered by the supplier per unit. Thus
\[
B = \frac{B_0 \pi_x}{\pi_0} \quad \text{where} \ 0 \leq B_0 < 1 \ \text{and} \ 0 \leq \pi_x \leq \pi_0.
\]

7.2.2 NOTATIONS

- \( h \)  
  Holding Cost per unit per year.

- \( T \)  
  Expected length of the Replenishment cycle.

- \( E(\cdot) \)  
  Mathematical expectation.

- \( \text{EAC} \)  
  Expected Annual cost.

- \( \text{EAC}^w \)  
  Least upper bound of expected annual cost.

- \( x \)  
  The lead time demand which has a p.d.f \( f_x(x) \) with finite mean RL and standard deviation \( \sigma \sqrt{L} (> 0) \) where \( \sigma \) denotes the standard deviation of the demand per unit time.

7.3 MODEL FORMULATION

In the proposed model, the inventory level of an item is reviewed continuously and when the inventory level drops to a reorder point \( r \), a lot size \( Q \) is ordered. As mentioned earlier, we have assumed that the lead time demand \( x \) has p.d.f \( f_x(x) \) with finite mean RL and standard deviation \( \sigma \sqrt{L} \) and the reorder point \( r = RL + A\sigma \sqrt{L} \). Then the expected demand during the shortage period at
the end of the cycle is given by $E[(x-r)^+]$. Thus the expected number of backorders per cycle is $BE[(x-r)^+]$. Hence the expected demand lost per cycle is $(1-B)E[(x-r)^+]$.

Since the expected total demand per cycle is $Q + (1-B)E[(x-r)^+]$, the expected cycle length $T$ is given as $T = \frac{Q + (1-B)E[(x-r)^+]}{R}$.

Also, we assume that backorder ratio $B$ depends on the back order price discount $\pi_x$, i.e., $B = \frac{B_0\pi_x}{\pi_0}$.

$$Q + \left[1 - \frac{B_0\pi_x}{\pi_0}\right]E[(x-r)^+]$$

Now, $T = \frac{Q + \left[1 - \frac{B_0\pi_x}{\pi_0}\right]E[(x-r)^+]}{R}$.

This indicates that the model herein differs from that of Ouyang et al., [81] and Chaung et al., [10] since they used an approximate cycle length.

Therefore, average annual cost of placing orders is given by, Ordering cost = $\frac{K}{T}$

i.e., Ordering cost $= \frac{KR}{Q + \left[1 - \frac{B_0\pi_x}{\pi_0}\right]E[(x-r)^+]$.

When the setup cost $K = K_j$, the annual setup crashing cost is

$$\frac{\sum_{i=1}^{j} f_i}{T} = \frac{R \sum_{i=1}^{j} f_i}{Q + \left[1 - \frac{B_0\pi_x}{\pi_0}\right]E[(x-r)^+]$$

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The average annual stock out cost is

\[
\pi_x B \frac{E \left[ (x-r)^+ \right]}{T} + \left( S + \pi_0 \right) (1-B) E \left[ (x-r)^+ \right] = \left[ \frac{B_0 \pi_x^2}{\pi_0} + S + \pi_0 - \frac{SB_0 \pi_x}{\pi_0} - B_0 \pi_x \right] \frac{E \left[ (x-r)^+ \right]}{T}
\]

\[
RE \left[ (x-r)^+ \right] = \frac{\left[ \frac{B_0 \pi_x^2}{\pi_0} + S + \pi_0 - \frac{SB_0 \pi_x}{\pi_0} - B_0 \pi_x \right]}{Q + \left[ 1 - \frac{B_0 \pi_x}{\pi_0} \right]} E \left[ (x-r)^+ \right]
\]

The expected net inventory level just before the order arrives is \( r - RL + (1-B) E[(x-r)^+] \) and the expected net inventory level at the beginning of the cycle is \( Q + r - RL + (1-B) E[(x-r)^+] \).

Therefore, the expected holding cost per year is

\[
h \left[ \frac{Q}{2} + r - RL + (1-B) E[(x-r)^+] \right].
\]

When the lead time \( L = L_i \), the annual lead time crashing cost is

\[
= \frac{\sum_{j=1}^{i} c_j}{T} = \frac{R \sum_{j=1}^{i} c_j}{Q + \left[ 1 - \frac{B_0 \pi_x}{\pi_0} \right] E[(x-r)^+]}.
\]
The objective of the problem is to minimize the total expected annual cost, which is the sum of the ordering cost, setup crashing cost, stock out cost, holding cost, and lead time crashing cost.

That is, our problem is to minimize

\[
EAC(Q, \pi_x, L) = \frac{R}{Q + \left[1 - \frac{B_0 \pi_x}{\pi_0}\right] E(x - r)^+} \left\{K + E[(x - r)^+]\right\}
\]

\[
\times \left\{\frac{B_0 \pi_x^2}{\pi_0} + S + \pi_0 - \frac{SB_0 \pi_x}{\pi_0} - B_0 \pi_x \right\} + \sum_{i=1}^{j} f_i + \sum_{j=1}^{c} c_j
\]

\[
+h \left[\frac{Q}{2} + A_{\sigma} \sqrt{L} + \left[1 - \frac{B_0 \pi_x}{\pi_0}\right] E[(x - r)^+]\right].
\]

It is assumed that the first two moments of the probability distribution of lead time demand \(x\) are given. That is the p.d.f \(f_x\) belongs to the class \(\Omega\) of p.d.f's with finite mean \(RL\) and standard deviation \(\sigma \sqrt{L}\). Since the probability distribution of \(x\) is unknown, we cannot find the exact value of \(E(x - r)^+\). We propose to apply the minimax distribution free procedure for our problem. The minimax distribution free approach for this problem is to find the “most unfavorable” p.d.f. \(f_x\) in \(\Omega\) for each \((Q, \pi_x, L)\) and then to minimize the total expected annual cost over \((Q, \pi_x, L)\); more exactly we need to solve

\[
\text{Min}_{(Q, \pi_x, L)} \text{Max}_{f_x \in \Omega} EAC(Q, \pi_x, L).
\]

For this purpose, we need the following proposition which was asserted by Gallego and Moon [28].
Proposition: For any \( f_x \in \Omega, \ E[(x-r)^+] \leq \frac{1}{2} \left\{ \sqrt{\sigma^2 L + (r - RL)^2} - (r - RL) \right\}.

Proof.

Given that \( r = RL + A\sigma\sqrt{L} \) and for any probability distribution of the lead time demand \( x \), the above inequality always holds.

\[
E[(x-r)^+] \leq \frac{1}{2} \left\{ \sqrt{\sigma^2 L + A^2 \sigma^2 L - A\sigma\sqrt{L}} - \frac{\sigma\sqrt{L}}{2} \left\{ \sqrt{1+A^2 - A} \right\} \right\}
\]

Hence our problem is to minimize

\[
EAC^w(Q, \pi_x, L) = \frac{R}{Q + \left[ 1 - \frac{B_0 \pi_x}{\pi_0} \right]} \frac{\sigma\sqrt{L}}{2} \left[ \sqrt{1+A^2 - A} \right] + K + \frac{\sigma\sqrt{L}}{2} \left[ \sqrt{1+A^2 - A} \right]
\]

\[
\times \left\{ \frac{B_0 \pi_x^2}{\pi_0} + S + \pi_0 - \frac{SB_0 \pi_x}{\pi_0} - B_0 \pi_x \right\} + \sum_{i=1}^{\ell} f_i + \sum_{j=1}^{r} c_j
\]

\[
+ h \left( \frac{Q}{2} + A \sigma\sqrt{L} + \left[ 1 - \frac{B_0 \pi_x}{\pi_0} \right] \frac{\sigma\sqrt{L}}{2} \left[ \sqrt{1+A^2 - A} \right] \right)
\]

(72)

where \( EAC^w(Q, \pi_x, L) \) is the least upper bound of \( EAC(Q, \pi_x, L) \). It is clear that, for any given \( A \), we have \( \sqrt{1+A^2 - A} > 0 \).

Hence for fixed \( (Q, \pi_x) \), \( EAC^w(Q, \pi_x, L) \) is concave in \( L \in (L_i, L_{i-1}) \) (Appendix 7.1).

Therefore, for fixed \( (Q, \pi_x) \), the minimum total expected annual cost will occur at the end points of the interval \( [L_i, L_{i-1}] \). On the other hand, for a given value of \( L \in [L_i, L_{i-1}] \), it can be shown that \( EAC^w(Q, \pi_x, L) \) is convex in
\((Q, \pi_x)\) (Appendix 7.2). Thus for fixed \(L \in (L, L_{x})\), the minimum value of \(EAC^w(Q, \pi_x, L)\) will occur at the point \((Q, \pi_x)\) which satisfies

\[
\frac{\partial EAC^w(Q, \pi_x, L)}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial EAC^w(Q, \pi_x, L)}{\partial \pi_x} = 0.
\]

\[
\frac{\partial EAC^w(Q, \pi_x, L)}{\partial Q} = 0 \Rightarrow
\frac{h}{2} = R \left\{ K + \frac{\sigma \sqrt{L}}{2} \left[ \sqrt{1 + A^2} - A \right] \left[ \frac{B_0 \pi_x^2}{\pi_0} + S + \pi_0 - \frac{SB_0 \pi_x}{\pi_0} - B_0 \pi_x \right] + \sum c_j + \sum f_i \right\}
\]

\[
Q = \left\{ \frac{2R}{h} \left[ K + \frac{\sigma \sqrt{L}}{2} \left[ \sqrt{1 + A^2} - A \right] \left[ \frac{B_0 \pi_x^2}{\pi_0} + S + \pi_0 - \frac{SB_0 \pi_x}{\pi_0} - B_0 \pi_x \right] + \sum c_j + \sum f_i \right] \right\}^{1/2} - \frac{\sigma \sqrt{L}}{2} \left( 1 - \frac{B_0 \pi_x}{\pi_0} \right) \left[ \sqrt{1 + A^2} - A \right]
\]

\[
\frac{\partial EAC^w(Q, \pi_x, L)}{\partial \pi_x} = 0 \Rightarrow
\frac{B_0 \sigma \sqrt{L}}{2\pi_0} \left( \sqrt{1 + A^2} - A \right) \left\{ Q + \left( 1 - \frac{B_0 \pi_x}{\pi_0} \right) \frac{\sigma \sqrt{L}}{2} \left[ \sqrt{1 + A^2} - A \right] \right\} - \frac{B_0 \sigma \sqrt{L}}{2\pi_0} \left[ \sqrt{1 + A^2} - A \right] = 0
\]

\[
Q - \frac{B_0 \sigma \sqrt{L}}{2\pi_0} \left( \sqrt{1 + A^2} - A \right)
\]

Divide by \(\frac{B_0 \sigma \sqrt{L}}{2\pi_0} \left( \sqrt{1 + A^2} - A \right)\), and using (73) we get
\[
\pi_x = \frac{\pi_0 \left\{ 2hQ + h\sigma \sqrt{L} \left[ \sqrt{1 + A^2} - A \right] + 4RS + 4R\pi_0 \right\}}{8\pi_0 R + hB_0 \sigma \sqrt{L} \left[ \sqrt{1 + A^2} - A \right]}
\]

(75)

From (75), we can find \( \pi_x \). From (74) we note that \( Q > 0 \). Theoretically, for given \( K, R, h, \pi_0, B_0, \sigma, A \) which depends on the allowable stockout probability \( q \) and the p.d.f. \( f_x(x) \), and each \( L_i (i = 0, 1, 2, \ldots, n) \), from equations (74) and (75) we can obtain optimal values of \( Q \) and \( \pi_x \) and the corresponding total expected annual cost \( EAC^w(Q, \pi_x, L_i) \), \( i = 0, 1, 2, \ldots, n \).

Thus, the minimum total expected annual cost can be obtained. However, in practice, since the p.d.f. \( f_x(x) \) is unknown, even if the value of \( q \) is given, we cannot get the exact value of \( A \). Therefore, in order to find the value of \( A \), we need the proposition (Appendix 7.3).

Furthermore, since it is assumed that the allowable stockout probability \( q \) during lead time is known i.e., \( q = P[x > r] \), then we get \( q \leq \frac{1}{1 + A^2} \).

\[
\therefore A \in \left[ 0, \frac{1}{\sqrt{q} - 1} \right].
\]

We propose the following algorithm to obtain the suitable \( A \) and hence optimal \( Q, \pi_x, L \).
Algorithm

Step 1. For each q divide the interval \( \left[ 0, \frac{1}{\sqrt{q}} - 1 \right] \) into N equal subintervals.

Let \( A_0 = 0 \), \( A_N = \frac{1}{\sqrt{q}} - 1 \), \( A_l = A_{l-1} + \frac{A_N - A_0}{N} \), \( l = 1, 2, \ldots, N - 1 \).

Step 2. For each \( L_i (i = 0, 1, 2, \ldots, n) \) perform steps 3 and 4.

Step 3.

(a) For given \( A_i \in \{A_0, A_1, \ldots, A_N\} \), \( l = 0, 1, 2, \ldots, N \), we can use \( \sqrt{\frac{2RK}{h}} \) as \( Q_{i,A_i} \). Since \( Q^b \geq Q^w \) for backorders and lost sales cases.

(b) Substitute \( Q_{i,A_i} \) into equation (75) and compute \( \pi_{x_i,A_i} \).

(c) Use \( \pi_{x_i,A_i} \) in equation (74) to find \( Q_{i,A_i} \).

Repeat (b) and (c) until there is no change in the values of \( \pi_{x_i,A_i} \).

(d) Compare \( \pi_{x_i,A_i} \) and \( \pi_0 \). If \( \pi_{x_i,A_i} \leq \pi_0 \), then \( \pi_{x_i,A_i} \) is feasible go to step 4. If \( \pi_{x_i,A_i} > \pi_0 \), Set \( \pi_{x_i,A_i} = \pi_0 \) and calculate \( Q_{i,A_i} \) from equation (74), go to step 4.

Step 4.

(a) Calculate \( EAC^w\left(Q_{i,A_i}, \pi_{x_i,A_i}, L_i\right) \).

(b) Find \( \min_{A_i \in \{A_0, A_1, \ldots, A_N\}} EAC^w\left(Q_{i,A_i(i)}, \pi_{x_i, A_i(i)}, L_i\right) \).

Let this minimum be \( EAC^w\left(Q_{i,A_i(i)}, \pi_{x_i, A_{i0}}, L_i\right) \).
Step 5. Finding $\min_{A_i \in \{A_0, A_1, \ldots, A_N\}} EAC^w \left( Q_i, At(i), \pi_{x_i}, At(i), L_i \right)$

Let $EAC^w \left( Q^*, \pi_{x^*}, L^* \right)$ be the minimum.

Then

(1) $\left( Q^*, \pi_{x^*} \right)$ is the optimal solution.

(2) $A_i$ is the optimal safety factor and we denote by $A^*$.

(3) Optimal reorder point $r^* = RL^* + A^* \sigma \sqrt{L^*}$

(4) Optimal backorder ratio is $B^* = \frac{B_0 \pi_{x^*}}{\pi_0}$

(5) $K^*$ is the optimal setup crashing cost.

7.4 NUMERICAL EXAMPLE

Numerical example is given to mention the workability of the proposed theory, model and to discuss the sensitivity analysis. Computational works are carried out using MATLAB 7.0. Some data input values were taken from earlier works [10, 111].

Example

Let $R = 600$ units per year, $K = $200 per order, $S = $50 per unit short, $h = $ 20 per unit per year, $\pi_0 = $ 150 per unit, $\sigma = 7$ units per week. The lead time has three components with data shown in Table 7.i. and the setup cost has three components with data shown in Table 7.ii. We solve the cases when the upper bounds of the backorder ratio $B_0 = 0.2, 0.35, 0.5, 0.65, 0.8$ and 0.95 and $q = 0.2$
(in this situation we have $A_0 = 0$ and $A_N = 2$). Let $A_l = A_{l-1} + \frac{A_N - A_0}{N}$, 
$l = 1, 2, \ldots, N - 1$ and take $N = 500$.
Applying the proposed algorithm, the results are tabulated in Table 7.iii.

From Table 7.iii, we observe that the minimum total expected annual cost $EAC^*(Q^*, \pi^*, L')$ decreases as $B_0$ increases.

In Table 7.iv, we tabulate optimal safety factor $A'$, optimal reorder point $r'$ and the optimal backorder ratio $B'$ for different values of $B_0$. We compare the results obtained by the above solution procedure to the results of Chaung et al., [10] model. In our model we developed, the total expected annual cost per unit is less than the same total annual cost per unit obtained by Chaung et al., [10] if setup crashing cost is added to reduce the setup cost. A comparison is presented in Table 7.v. [cost savings per unit are shown in the last column].

We also analyze about the cost incurred in offering the price discounts for backorders by the supplier to the customers. Cheng et al., [111] did not consider backorder price discount. So we compare our model to the model by Cheng et al., [111]. The results are shown in Tables 7.vi and 7.vii. By offering the price discount a supplier can fetch a large number of backorders with no loss and in fact with less cost. Cost savings per unit increases as $B_0$ increases.
### Table 7.i Components of lead time with data

<table>
<thead>
<tr>
<th>Lead time Component</th>
<th>Normal duration (days), $b_i$</th>
<th>Minimum duration (days), $a_i$</th>
<th>Crashing cost as reduced to minimum duration, $c_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>6</td>
<td>5.6</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>6</td>
<td>16.8</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>9</td>
<td>35</td>
</tr>
</tbody>
</table>

### Table 7.ii Components of setup cost with data

<table>
<thead>
<tr>
<th>Setup cost Component</th>
<th>Normal cost $c_i$</th>
<th>Minimum cost $d_i$</th>
<th>Crashing cost as reduced to minimum Cost $f_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>20</td>
<td>56</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>20</td>
<td>168</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>10</td>
<td>350</td>
</tr>
</tbody>
</table>

### Table 7.iii Summary of the optimal solutions ($L'$ in week)

<table>
<thead>
<tr>
<th>$B_0$</th>
<th>$L'$</th>
<th>$Q^*$</th>
<th>$\pi_x^*$</th>
<th>$EAC^w(Q^<em>, \pi_x^</em>, L')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>3</td>
<td>255.49</td>
<td>100.92</td>
<td>5632.04</td>
</tr>
<tr>
<td>0.35</td>
<td>3</td>
<td>253.96</td>
<td>100.92</td>
<td>5597.04</td>
</tr>
<tr>
<td>0.5</td>
<td>3</td>
<td>271.78</td>
<td>100.95</td>
<td>5556.27</td>
</tr>
<tr>
<td>0.65</td>
<td>3</td>
<td>265.72</td>
<td>100.94</td>
<td>5416.66</td>
</tr>
<tr>
<td>0.8</td>
<td>3</td>
<td>259.49</td>
<td>100.936</td>
<td>5273.67</td>
</tr>
<tr>
<td>0.95</td>
<td>3</td>
<td>253.07</td>
<td>100.931</td>
<td>5126.99</td>
</tr>
</tbody>
</table>
Table 7.iv Optimal values of safety stock, reorder point and backorder rate

<table>
<thead>
<tr>
<th>$B_0$</th>
<th>$A^*$</th>
<th>$r^*$</th>
<th>$B^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2</td>
<td>1824.248</td>
<td>0.1345</td>
</tr>
<tr>
<td>0.35</td>
<td>2</td>
<td>1824.2487</td>
<td>0.2355</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>1800</td>
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Table 7.v Comparison of the minimum total expected cost per unit with setup crashing cost

<table>
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<tr>
<th>$B_0$</th>
<th>$Q^*$</th>
<th>$EAC^w(Q^<em>, \pi_x^</em>, L^*)$</th>
<th>Min. cost per unit</th>
<th>$Q^*$</th>
<th>$EAC^w(Q^<em>, \pi_x^</em>, L^*)$</th>
<th>Min. Cost per unit</th>
<th>Cost savings per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>159.64</td>
<td>$3816.27$</td>
<td>$23.91$</td>
<td>255.49</td>
<td>$5632.04$</td>
<td>$22.04$</td>
<td>$1.87$</td>
</tr>
<tr>
<td>0.35</td>
<td>159.0</td>
<td>$3789.35$</td>
<td>23.83</td>
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<tr>
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<td>157.67</td>
<td>$3733.52$</td>
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<td>265.72</td>
<td>$5416.66$</td>
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<td>3.3</td>
</tr>
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<td>$3704.50$</td>
<td>23.59</td>
<td>259.49</td>
<td>$5273.66$</td>
<td>20.32</td>
<td>3.27</td>
</tr>
<tr>
<td>0.95</td>
<td>156.63</td>
<td>$3674.67$</td>
<td>23.46</td>
<td>253.07</td>
<td>$5126.99$</td>
<td>20.26</td>
<td>3.2</td>
</tr>
<tr>
<td>i</td>
<td>j</td>
<td>( L_i )</td>
<td>( \sum_{j=1}^{i} C_j )</td>
<td>( \sum_{j=1}^{i} f_j )</td>
<td>( Q )</td>
<td>( EAC )</td>
<td>( \text{In our developed Model} )</td>
</tr>
<tr>
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<td>----</td>
<td>-----------</td>
<td>-----------------</td>
<td>-----------------</td>
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<td>3459.63</td>
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</tr>
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<td>3236.45^*</td>
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<td>574</td>
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<td>4430.16</td>
<td>57.4</td>
<td>6894.95</td>
<td>337 6894.95</td>
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Table 7.vii Comparison of the minimum total expected cost per unit with backorder price discounts

<table>
<thead>
<tr>
<th>Model</th>
<th>Minimum cost per unit in $</th>
<th>Cost savings per unit in $</th>
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<tbody>
<tr>
<td>Cheng et al.,’s Model[111]</td>
<td>22.32</td>
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<tr>
<td>Our model</td>
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<td></td>
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<tr>
<td>$B_0 = 0.95$</td>
<td>20.26</td>
<td>2.06</td>
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</tbody>
</table>

7.5 CONCLUSION

The main aim of this study is to present an inventory model with backorder price discounts and variable lead time for minimizing total expected annual cost. Here the order quantity, backorder price discount and the lead time are considered as decision variables. We assume that the backorder ratio is dependent on the amount of price discount from a supplier which is very common in our day-to-day life. Here we do not know the probability distribution of the lead time demand and apply the distribution free procedure to solve the problem. We also consider the setup crashing cost. We showed that the significant savings can be achieved.