1.1 Introduction.

The fluid flows occur everywhere in nature and therefore it is quite natural to discuss the behaviour of these fluid flows. One can develop the theory of fluid flows and can investigate various flows both theoretically and experimentally. Fluid mechanics is the study of fluid motion based on fundamental laws of motion. The extensive applications of fluid mechanics has made it one of the most vital and fundamental subjects in the field of almost all engineering and applied scientific studies. The basic principles of fluid mechanics are used in various fields of engineering such as astronautics, aeronautics, hydronautics, meteorology, fluid machines, gas dynamics etc. as well as in astrophysics, biology, biomedicine, physical chemistry, plasmas physics and geophysics etc. The progress of aeronautical, chemical and mechanical engineering during the past few decades and exploration of space in the past few years have given added stimuli to the study of fluid mechanics, so that it now ranks as one of the basic subject in engineering and applied sciences.

Now a days it has become necessary to combine the knowledge of thermodynamics, heat and mass transfer, electromagnetic theory in
fluid mechanics to understand the physical phenomenon involved. The flight of birds in air, action of fishes in water and design of aeroplanes and ships are based on the theory of fluid mechanics.

Flow of fluids of variable density in gravitational field is a great interest because without gravity, heterogeneity of the fluids has only minor effects on the behaviour of the fluids.

In most of the problems in fluid mechanics, we take some approximations and simplifying assumptions regarding the nature of fluids and flow boundaries as they are not well experienced. If we try to study a realistic physical situation, the problem become more and more complicated and the mathematical tools available at hand become insufficient to deal with the problems in their original shape. Therefore it becomes necessary to assume suitable assumptions and approximations which not only simplify the mathematical formulation of the problem but also agree considerably with the physical requirement of the problem. The actual flow conditions or the boundary conditions may be slightly different than those taken in the theoretical analysis. If these small changes lead to large deviation in the flow variables, the theoretically obtained flow can not be realised physically. Further, since some disturbances are always present in the flow, one is expected to know whether these disturbances decay or grow with time. To decide this question, theoretically, the investigation of the stability of the fluid flows becomes very important or rather essential.
During the past few decades, the study of electrically conducting, fluid flows in presence of magnetic or electric fields have also become very important because of their wide applications. The motion of an electrically conducting fluid, like mercury, under a magnetic field, in general, gives rise to induced electric currents on which mechanical forces are asserted by the magnetic field. On the other hand, induced electric currents also produced induced magnetic field and thus the original magnetic field is also changed. Thus there is a two-way interaction between the flow field and the magnetic field, the magnetic field exerts force on the fluid by producing induced currents and the induced currents changed the original magnetic field. The flows of electrically conducting fluids in presence of a magnetic field are called the hydromagnetic flows. Hydromagnetic flows are more complex than the hydrodynamic flows.

Mathematical complexity follows from the fact that the hydromagnetic equations have three non-linear terms while the hydrodynamic equations have only one. The number of governing equation are also increased in hydromagnetic flows.

The governing equations of incompressible hydromagnetic flows are:

The mass conservation equation

\[ \frac{\partial u_j}{\partial x_j} = 0. \]  ...(1.1.1)

The equation of motion
\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = - \nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{J} \times \mathbf{B}. \] ... (1.1.2a)

In tensor notation, it becomes

\[ \rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i + \epsilon_{ijk} J_j B_k \] ... (1.1.2b)

The energy equation

\[ \rho c_v \frac{\partial T}{\partial t} = K \nabla^2 T + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{J^2}{\sigma}. \] ... (1.1.3)

The Maxwell equations

\[ \frac{\partial \mathbf{H}}{\partial t} = - \nabla \times \mathbf{E}, \] ... (1.1.4)

\[ \nabla \times \mathbf{H} = \mathbf{J}, \] ... (1.1.5)

and \[ \nabla \cdot \mathbf{H} = 0. \] ... (1.1.6)

where \( \mathbf{J} \times \mathbf{B} \) is the lorentz force, \( \frac{J^2}{\sigma} \) is the dissipation due to electric currents and \( \rho, \mathbf{v}, p, \mathbf{J}, \mathbf{B}, \sigma, \mathbf{H}, \mathbf{E}, \mu, c_v, K \) are respectively the density, the velocity, the pressure, the current density, magnetic flux, electrical conductivity, magnetic field, electric field, coefficient of viscosity, the specific heat at constant volume and the thermal conductivity.

The Ohm's law is

\[ \mathbf{J} = \sigma (\mathbf{E} + \nabla \times \mathbf{B}). \] ... (1.1.7)

Here the charge density and the Hall currents etc. have been neglected.
Substituting the value of $J$ from equation (1.1.5) in the equation of motion (1.1.2), we get

$$\rho \frac{Dv}{Dt} = -\nabla \left( k + \frac{1}{2} \mu_e H^2 \right) + \mu \nabla^2 v + \mu_e H \cdot \nabla H \quad \cdots (1.1.8)$$

Taking the curl of the equation (1.1.7) and eliminating $E$ between (1.1.4), (1.1.5) and (1.1.7) we get the induction equation

$$\frac{\partial H}{\partial t} = \nabla \times (\nabla \times H) + \frac{1}{\eta} \nabla^2 H, \quad \cdots (1.1.9)$$

where $\eta = \sigma \mu_e$ is called the electrical diffusivity of the fluid.

Now we non-dimensionalize the basic equations (1.1.8) and (1.1.9). For this, we take the characteristic velocity as $U_0$, magnetic field as $H_0$, length as $L$ and time as $\frac{L}{U_0}$ and non-dimensionatize the variables by the transformation:

$$v = U_0 \overline{v}, \quad H = H_0 \overline{H}, \quad p = eU_0 \overline{p}, \quad t = \frac{L}{U_0} t,$$

$$\nabla = \frac{1}{L} \nabla$$

Substituting these transformations in (1.1.8) and (1.1.9) and removing the bars over the variables, we get the non-dimensional equations as
\[ \frac{\partial v}{\partial t} + v \cdot \nabla v = - \nabla \left( p + \frac{\mu_e H_0^2 H^2}{\rho U_0} \right) + \frac{1}{R} \nabla^2 v + S^2 H \cdot \nabla H, \]

\[ \frac{\partial H}{\partial t} + v \cdot \nabla H - H \cdot \nabla v = \frac{1}{R_m} \nabla^2 H, \]

...(1.1.11) ... (1.1.12)

where \( R = \frac{\rho U_0 L}{\mu_e} \) is the Reynolds number, \( R_m = \sigma \mu_e L U_0 \) is the magnetic Reynolds number and \( S = \frac{H_0}{L} \sqrt{\frac{\mu_0}{\rho}} \) is the Stommense number (or magnetic force number).

In hydromagnetic flows, the boundary conditions are of vital importance and are not as simple as in hydrodynamic flows. The boundary conditions on velocity vector are simple. For an inviscid fluid flow, the normal component of the velocity should be continuous while the tangential component may be discontinuous and for viscous fluid flow, both the tangential and normal components of the velocity must be continuous. Further, the normal field should component of the magnetic be continuous at the interface between the two mediums. The tangential components of the magnetic field is continuous at the interface if both mediums are of finite conductivity. If any of the two mediums is of infinite conductivity, then the tangential component may be discontinuous. The tangential components of the electric field should always be continuous across any interface while its normal component may be discontinuous depending upon the conductivity of the two mediums. The boundary conditions of the current density is also simple. The normal component should always be continuous
across the interface while the tangential component may be discontinuous depending upon the conductivity of the two mediums.

1.2 Basic Concepts of Stability Theory.

When we discuss any realistic flow, theoretically, we usually take some suitable approximations and assumptions which not only simplify the problem but also agree with the physical requirements, in providing the best representations of the physical phenomenon under investigation. In fact, there are many factors which we may not be able to account for whenever we consider any fluid flow experimentally, we find that there are always some inherent disturbances which are may not be able to avoid. For example when we consider the flow through a pipe or a channel, we usually take, for theoretical discussions, the pipe or the channel to be of infinite length which is not practically possible and so due to end effects and finite length of the pipe or the channel we may have some disturbances or deviations in the flow. There may also be some irregular behaviour of the applied pressure gradient, applied forces. Further, the surface of the pipe or the channel may also not be perfectly smooth. Therefore, there may be some disturbances due to the roughness of the surface of the pipe or the channel. Moreover, there may also be other type of disturbances which we may not be accounting for. Thus we see that the disturbances can not be avoided completely in any problem in fluid dynamics. In order to obtain a steady laminar flow through a pipe or a channel physically, we should know the reactions of the flow
to such disturbances. If these disturbances decay with time then it will be possible to obtain that flow pattern. On the other hand, if the disturbances grow with time then after some time these disturbances will dominate over the basic flow and therefore there will be an irregular motion of the fluid. Thus the flow may tend to a turbulent flow. This turbulent flow is the more natural state of fluid flow. The first category of flows are the stable flows, while the flows of second category are the unstable flows.

Physically speaking, in any flow the disturbances (if exist) contain some kinetic energy and if there is a transfer of energy from basic flow into the kinetic energy of the disturbances, then these disturbance grow in magnitude and the flow becomes unstable. On the other hand, if there is a transfer of kinetic energy from the disturbances into the basic flow, then the magnitude of the disturbances will decrease and thus the flow will be stable. In a dissipative system, a flow will be unstable, if the energy transferred from the main flow into the disturbances exceeds the dissipative energy of the flow and stable if the dissipative energy exceeds the energy transferred to the disturbance from the main flow.

In discussing the stability of a system, we study the reaction of the system to small disturbances. If the system is disturbed, then there are following two cases:

(i) The disturbances gradually die down.
(ii) The disturbances grow in amplitude in such a way that the system progressively departs from the initial state and never reverts to it.

In the first case, we say that the system is stable with respect to the particular disturbance and in the second case, we say that the system is unstable. Thus a system is said to be stable, if it is stable with respect to each mode of disturbance to which it can be subjected to. If there is even one mode of disturbances with respect to which it is unstable the system is considered to be unstable. In other words, stability of a system implies that there is no mode of disturbance for which the system is unstable.

There is a state which separates the unstable and the stable states of a system. This particular state is called the marginal state or a state of neutral stability.

Mathematically, a system whose stability is to be discussed, is given some arbitrary perturbation (If $U_1$ is a solution of a system of equations governing the flow and satisfying the boundary conditions and $U_1 + \delta U_1$ is also a solution of the disturbed system satisfying the same boundary conditions then $\delta U_1$ is called the perturbation). If these perturbations tend to zero as time tends to infinity then the flow is said to be stable with respect to these perturbations otherwise it is said to be unstable.
In linear stability theory, we take the perturbations $\delta U_1$ to be arbitrary small and so we can neglect the non-linear terms containing the product of the perturbation quantities and their derivatives in the governing equations in comparison to the linear one. Thus we get a system of homogeneous linear differential equations with homogeneous boundary conditions. Therefore, in linear stability theory, the perturbations either grow exponentially or decay exponentially or the magnitude of the perturbations remain constant; if the perturbations grow exponentially, then the system is said to be unstable and if the perturbations decay exponentially then the system is said to be stable and if their magnitude remains constant then the system is said to be in the marginal state.

The subject of hydrodynamic stabilities is mainly concerned with the instabilities that precede turbulence. A century of investigations by talented theoreticians and experimentalists have produced enormous literature on the stability of fluid flows. Four distinct trends have emerged in these studies: temporal linear, temporal non-linear, energy or global approach and spatial stability analysis.

Temporal linear analysis is the classical theory of stability. The non-linear hydrodynamic equations are linearized and time growth or decay of an inherent initial perturbation, assumed to be infinitesimal, is studied. It is essentially an initial value problem and determines the primary instabilities of fluid flow. Chandra Sekhar (1961) and Lin (1955) could be regarded as contributions of classical work depicting this trend.
Linear analysis can indicate only instantaneous tendency of a laminar flow for small perturbations. The moment flow is unstable and shows a tendency of growth, linear analysis will no more be valid. Sizable perturbations and instabilities subsequent to primary instability become important. The nature and the number of these successive instabilities that can occur before flow becomes chaotic cannot be predicted with confidence. Significant contribution and start was made by Stuart (1958, 1960, 1971), Eckhaus (1961, 1963, 1965), Loverz (1963) and Slwinney and Gollub (1978). It is referred to as temporal non-linear stability of flows.

Parallel to temporal non-linear stability analysis, there has been another trend in the study of non-linear stability. It is being referred to as energy/integral/global approach. As the name suggests, the growth or decay of energy is considered and the stability of the flow is determined. This method provides sufficient conditions for stabilities. However these stability limits are far from reality, Serrin (1958) gave a definite direction to this trend Joseph (1976) embodied the entire work along these lines in two volumes.

Perturbations to flow are not always inherent and unexcited. They could be imparted over one of the domain and over entire time. In such situations, the pertinent question is to know whether two disturbances grow or decay in space. Such a situation is more realistic from the experimental point of view. Disturbances grow or decay in space rather than in time, in an experiment, especially in boundary layer flows.
To compare experimental results, a different theoretical model is desirable wherein growth or decay of a disturbance imparted at a point over entire time must be evaluated in space. Gaster (1962) and Waston (1962) contributed original works in this direction. It is referred to as spatial stability of flows.

The equations of hydrodynamics (the equations of mass conservation, momentum, energy and state), in spite of their complexity, allow some simple patterns of flow (such as between two parallel plates or rotation cylinders) as stationary (basic) solutions. However, the problem of discussing the stability of a hydrodynamic system which is in a stationary state, through conceptually very simple, is not easy mathematically. Even in the case of simplest possible flows, the resulting differential equations are of higher order, often having variable coefficients and some times singular as well. Therefore, the discussion of stability of the flows has been confirmed mainly to simple problems only, for example, static fluid layer, flow between parallel plates, Coutte and spiral flow between coaxeial cylinders etc.

There are two main techniques of solving a stability problem, namely

(i) Energy method

(ii) Normal mode technique

We shall discuss briefly both these methods.
1.3a. Energy Method

For analyzing the stability of a flow by this method, the kinetic energy of the perturbations is calculated. If this kinetic energy of perturbations increases with time, then the flow is unstable and if it decays with time, then the flow is stable. This method is global in nature and thus restricted in applications since the kinetic energy of the whole system is calculated. Though this gives a surest limit for the stability of the flow but it is crude in giving the unstable limits and also it gives very little information about the local behaviour of the perturbation.

For investigating the stability of a flow by energy method, specially when the fluid is confined within rigid boundaries, sometimes the vorticity of the perturbations is considered rather than the kinetic energy. Therefore we calculate

$$W = \int (P^2 + Q^2 + R^2) \, dv$$

where \((P, Q, R)\) are the components of the vorticity of the perturbations and the integration is taken over whole of the flow domain. Then the basic flow is stable or unstable according as whether \(\frac{dW}{dt}\) is negative or positive. This can be agreed in following way.

If \(\frac{dW}{dt}\) is negative then \(W \to 0\) as \(t \to \infty\) and so the perturbation tend to irrotational. Now the perturbations must vanish to the boundaries as these are taken to be rigid. But there can not be a non-trivial irrotational
flow which vanishes at the boundary. Therefore as $t \to \infty$ the velocity components of the perturbations must also tend to zero. Hence the flow is stable.

This method has been used mostly in the non-dissipative systems. In recent years, attempts have been made to use this technique for dissipative system also. This method is more useful in analysing the non-linear stability.

1.3b. Normal Mode Technique.

The normal mode technique is the most important technique that is widely used so far in determining the linear stability of a system because it is applicable in a wider class of problems. Therefore it will be worth while to describe in brief the normal mode technique of analysing the stability problems. In this method, in linear theory, the perturbations are assumed to be arbitrarily small in magnitude so that the non-linear terms in the perturbation variables and (or) their derivatives can be neglected as compared to the linear terms in the governing equations of the system. However, the perturbations are assumed to be the regular functions of space variable and therefore the fourier analysis of the perturbations is possible. Thus in this method, we express the perturbations into fourier components, called normal modes and see whether these modes grow or decay with time. If all the modes decay with time, then the flow is stable and if even a single mode grows with time then the flow is unstable because after some time this mode will dominate over the whole flow.
To illustrate this, we consider a system confined between two parallel planes in which the physical variables in the stationary state are functions of the coordinate, say $y$, which is normal to the plane. In this case we may analyse an arbitrary perturbation in terms of two dimensional periodic waves. Thus if $f(x, y, z, t)$ represents a typical amplitude describing the perturbation and if flow extends to infinity, say in $x$ and $z$-directions, then we take the fourier analysis of $f(x, y, z, t)$ with respect to $x$ and $z$. This is possible because at any instant the perturbations are taken to be the regular functions of the space variables and bounded as $x$ and $z$ tend to infinity. In linear stability theory, we also take the exponential dependence of $f$ on $t$. Thus we write

$$f(x, y, z, t) = \sum \phi(y, k_x, k_y, n) \exp \left[i \{k_x x + k_z z\} + nt\right]$$

...(1.3b.1)

where $\bar{k}(k_x, 0, k_z)$ is called the wave number vector, $k = |\bar{k}|$, the wave number and the summation is taken over all $k_x$ and $k_z$, $n$ is the complex wave velocity. If the real part of $n$ is positive then the perturbations grow exponentially with time in this case. If the real part of $n$ is negative then the perturbations decay exponentially with time and the flow will be stable. Here $n$, besides depending on other parameters depends on $k$ (the wave number) and therefore all possible values of $k$. 
In any flow problem, we have some parameters, say, \( R_1, R_2, \ldots R_m \) on which the flow depends. Substituting (1.3 b.1) in the linearized equations governing the stability of the flow, we get a system of linear homogeneous differential equations. Solving these equations and using the boundary conditions, we get a dispersion relation of the form

\[
F(k, n, R_1, R_2, \ldots R_m) = 0 \tag{1.3b.2}
\]

Since \( n = n_r + i n_i \) is in general complex, this relation gives us two equations by separating into real and imaginary parts. One can solve, at least in principle, these relations for \( n_r \) and \( n_i \) or one can eliminate \( n_r \) between these two equations so obtained. In either case, by equating \( n_i = 0 \) we get a relation characterising the marginal state such as.

\[
\phi(k, R_i, R_2, \ldots R_m) = 0 \tag{1.3b.3}
\]

After obtaining such a relation we fix all the parameters \( R_1, R_2, \ldots R_m \) except one, say, \( R_1 \) and plot a neutral curve between \( R_1 \) and \( k \) and thus we get the critical value of \( R_1 \). Similarly the critical value of other parameters can also be obtained.

1.4 Temporal Stability of Flows—A Brief Review

(a) Stability of Stratified Flows

The stability of a shear flow of a continuously stratified fluid is a fascinating phenomenon of great importance to meteorology. The
stability of parallel flows of inviscid fluid was first discussed in the later half of the nineteenth century. First attempt was made by Helmholtz (1868) who studied the instability of an interface dividing two layers of inviscid and incompressible fluids of different densities and in relative horizontal motion. Kelvin (1871) did the detailed and exhaustive study of this problem in context with the study of the generation of water waves. Rayleigh (1900) studied the instability of inviscid, heterogeneous and incompressible fluid and showed that the necessary and sufficient condition for a system to be stable is that density should decrease upwards everywhere in the fluid region. Taylor (1931) considered the stability of two superposed fluids of different densities separated by a transition layer of intermediate density in which the velocity of streaming varies continuously from that of lower fluid to that of upper fluid. Goldstein extended this work. Taylor (1950) investigated the problem of instability of the plane interface between two fluids. This is known as Rayleigh Taylor instability problem. Taylor conjectured that for stability, local Richardson number must exceed 1/4 everywhere in the flow domain Taylor’s conjecture was proved by Miles (1961). Synge (1933) obtained bounds for growth rate and generalized form of Rayleigh’s criterion for instability of stratified parallel shear flows. Yih (1957) and Drazin (1958) obtained the same results independently.

Miles (1961, 1963) investigated the effect of small perturbations on the stability of stratified shear flow of an inviscid and incompressible fluid of density $\rho(y)$. Howard (1961) released the condition taken by Miles
and gave a similar but simple proof for Miles sufficient condition for stability. He also proceed that for any unstable mode, the complex wave velocity $c$ must lie inside a semicircle, in the upper half plane, which has the range of basic velocity as diameter. This is one of the most beautiful result in the theory of hydrodynamic stability. But Howard's semi-circle did not incorporate stratification. Therefore attempts wave continued to improve upon it by incorporating stratification. Banerjee and Jain (1972), Banerjee, Gupta and Gupta (1974) and Shandel (1978) and Agarwal and Rastogi did work in this direction. But these results were confined only for some particular velocity and density distributions Kochar and Jain (1979) took up this problem and dicussed the effect of statically stable stratification in reducing the unstable region in $(c_1, c_2)$—plane in a more general fashion.

(b) Stability of Hydromagnetic Shear Flows

The subject, hydromagnetics, is concerned with the ways in which fluid magnetic field can affect the fluid behaviour. In the present time, Hartman (1938) was first to discuss both theoretically and experimentally the hydromagnetic flow between two parallel plates. But the real boost was given by Alfven (1942) when he established transverse waves in electrically conducting fluids and explained many astrophysical phenomenon with the help of it.

The effect of a homogeneous magnetic field on the instability of a stratified fluid has been investigated by Kruskal and Schwarzschild (1954) and by Hide (1955), taking respectively the effect of horizontal and vertical components of magnetic field on a two layer system. In instability problems, in general, the effect of magnetic field is to stabilize the flow. Drazin, P.G. (1960) discussed the stability of parallel flow in a parallel magnetic field at small magnetic Reynolds number. He obtained the sufficient conditions for both stability and instability and proved the stabilizing effect of magnetic field. But, however, there are certain examples where it destabilize the flow. Kent (1966, 1968) studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and showed that if $U'' < 0$, then the system is unstable under certain conditions while in the absence of magnetic field the system is known to be stable.

Agarwal and Agarwal (1969) have investigated the stability of
a heterogeneous shear flow of an inviscid, incompressible, non-heat conducting fluid with zero electrical resistivity in the presence of a uniform magnetic field applied in the streaming direction when the system is statically stable. **Kochar and Jain (1979)** investigated the hydromagnetic stability of stratified shear flows. They obtain semi-ellipse type region for the unstable modes in \((c_r, c_i)\)-plane. **Gupta and Kaushal (1988)** discussed hydromagnetic Rayleigh–Taylor instability. **Sharma and Bakshish Singh (1990)** discussed the stability of stratified fluid in presence of suspended particles and variable magnetic field. They proved the stabilizing effect of magnetic field.

**(c) Stability of the Flow of a Dusty Gas.**

The investigations in the atmosphere are of great importance due to the direct relevance in our day to day life. The dust particles are suspended in the air and bulk concentration of these dust particles plays a significant role in any atmospheric phenomenon and can altogether change the character of the problem. Therefore, the flow of dusty gases and liquids and their stability need a careful investigation. Marble (1970) has presented an excellent review of the dynamics of dusty gases.

The investigation of the stability of dusty flows have been carried out by many researchers. The fundamental paper on the stability of dusty gases is due to **Saffman (1962)**. He has investigated the stability of the laminar flow of a dusty gas. He provided the mathematical formulation of the problem taking same simplifying assempition and approximation.
Michael (1964) investigated the stability of plane poiseuille flow of a dusty gas. He followed the formulation given by Saffman. Further Michael (1965) studied the Kelvin—Helmhaltz instability of a plane vortex sheet. Kochar (1979) has considered the stability of inviscid parallel shear flow of the gas and the dust. The stability characteristics are determined by the solutions of Rayleigh equation but with the basic velocity profile replaced by a modified profile which, in general, is complex. He has shown that the complex wave velocity of an unstable mode must lie inside a semi—ellipse type region in the upper half plane and further the dust has a stabilizing effect.

Gupta and Agarwal (1987) discussed the stability of an inviscid, incompressible dusty gas flow of Boussinesg’s fluid. They obtained a sufficient condition for the existence of non-neutral modes. For fine dust, they obtained stability equation same as Howard–Miles equation for the stability of heterogeneous shear flow with one difference that g is replaced by g/ť. Following Howard (1962) and Agarwal and Agarwal (1969) they obtained a sufficient condition of stability. Finally they obtained the growth rate of non-oscillatory unstable mode and a semi-circle-region for the complex wave velocity of any unstable mode. They confirmed the earlier results of Saffman and proved the destabilizing role of fine dust.

(d) Instability In A porous Medium.

Flow of density stratified through a porous medium and the stability problem thereof have been of significant importance in literature.
Contributions to the problem of stability in a porous medium are well summerized in the books by Scheidegger (1960) and Yih (1980). The stability of the fluid interface moving in a porous medium is of significant importance for the ground water hydrology, petroleum, production engineering, civil engineering etc. Exetensive studies have been conducted on the stability of the interface between two fluids of different densities and viscosities through porous media when there is movement or displacement perpendicular to the interface. Several authors have endeavoured to study the thermal instability of a fluid saturated porous layer initiated by Horton and Roger (1945) and Lapwood (1948). An excellent review of the literature is provided by Joseph (1976). Further contributions in this direction are made by Patil and Rudraiah (1973), Sharma and Sharma (1982) and Chang and Jang (1989).

In all these analysis the fluid flow has been assumed to be governed by Darcy’s law. A general argument has been advanced since the experimental findings of Darcey (1956) that the inclusion of inertia is not interesting for the physics of flow through porous medium. But there are situations in engineering and Geophysics in which a departure from Darey’s law and the inertia effects not included in Darey’s model may become significant.

Jaimala and Agarwal (1991) investigated the stability of a density stratified fluid with horizontal through a porous medium. They obtained semi-circle type bounds on the complex wave velocity of unstable modes
(if exist) under certain conditions. They discussed the temporal stability of the system.

(e) **Stability of Rotational Flows.**

The instability of rotating flows was first considered by Rayleigh (1980, 1966). He considered a basic rotating flow of an inviscid fluid moving with angular velocity \( \Omega (r) \), \( r \) is the distance from the axis of rotation. By a simple physical argument, he derived his celebrated criterion for stability that a necessary and sufficient condition for stability to axis symmetric disturbances is that \( \phi \) should be non-negative every where in the flow domain where \( \phi \) is the Rayleigh discriminant defined by

\[
\phi (r) = \frac{1}{r^3} \frac{d}{dr} (r^2 \Omega^2)
\]

Banerjee, Pradhan and Jain (1975) discussed the stability of heterogeneous incompressible viscous rotatory couette flow. Pradhan (1979) considered the effect on the stability of couette flow between rotating circulars of an axial gravitational force in the presence of an axial non-homogeneity and radial temperature gradient separately and showed that if the density variations because of basic non-homogeneity were such that it decreases in the axial direction, then the effect of this decrease in density is stabilizing.

Chandrasekhar (1960) discussed the stability of inviscid flow between two coasial cylinders. He showed that in the case of pure axial flow a necessary condition for occurrence of over stable oscillations is
that \( \psi (r) \) must change sign where \( \psi (r) = r \frac{d}{dr} \left( \frac{1}{r} \frac{dW}{dr} \right) \), \( W \) being the axial velocity. He also showed the validity of Rayleigh’s criterion for the stability of pure rotational flow in the presence of arbitrary axial flow.

Motivated by the analogy between the stratified fluid and rotating flows, Howard and Gupta (1962) studied the stability of inviscid flows between concentric cylinders which have an axial velocity component \( W(r) \) in addition to swirling component \( V(r) \) depending on the radius \( r \) for axis symmetric perturbations. They have shown that the spiral flow is stable if a suitable defined Richardson number \( \left( \frac{\phi}{W^2} \right) \) exceeds \( \frac{1}{4} \) every where in the flow domain and the complex wave velocity, for an unstable mode, must lie inside a semi-circle in the upper half, which has the range of basic axial velocity \( W(r) \) as diameter. These results are similar to the results obtained by Miles and Howard for stratified parallel shear flows. Thus, they have further enriched the analogy between the stratified parallel shear flows and the swirling flows. Kochar and Jain (1979) improved the result of Howard and Gupta and showed that the complex wave velocity, for any unstable mode, must lie inside the semi-ellipse type of a region in the upper half plane.

1.5 Spatial Stability of Flows.

The growth rate obtained from linearised temporal stability theory does not tally with those obtained in experimental investigation of shear
layer instability. Similarly the phenomenon of reversal of phase was observed in experiments while the linear temporal stability theory does not indicate a phase reversal at all. This shows that linear temporal stability theory is inadequate. The detailed experimental work on stability has been carried out on some form of forced oscillations which naturally generates a train of waves growing or decaying spatially as they propagate away from the source rather than a wave system which grew in amplitude with respect to time. Therefore it is expected that a better collaboration may be obtained if the experimental results are compared with those obtained by the stability theory of spatially growing disturbances.

**Schubauer and Sloramstad (1947), Sato (1956, 1959), Freymuth (1966)** have observed that the growth rate obtained from experiments though differs from that obtained by temporal stability theory but nearly tallys with phase velocity times the temporal growth rate. We know that for x-directional wave, two dimensional perturbations, may be assumed to be of the form \( f(y) \exp [i(kx - \omega t)] \). For temporal stability \( k \) is regarded real and \( \omega \) as complex. For spatial growth of wave \( k \) can be regarded as complex and \( \omega \) as real. This outlook leads to the concept of spatial stability theory. **Gastor (1962)** obtained, on the above considerations, a relation between spatial and temporal growth rate. He proved that for small perturbations, spatial growth rate is equal to group velocity times the temporal growth rate. For small dispersion group velocity and phase velocity are nearly equal. Therefore we get that for small dispersion
the spatial growth rate is nearly equal to phase velocity time the temporal growth rate. The growth rate obtained by Schubauer and Sloramstad, Sao and Freymuth tallys with the spatial growth rate. Similarly the reversal of phase phenomenon can be described very well by the spatial stability theory. Thus it has been established that experimental results collaborate better with the results obtained by the spatial stability theory rather than the results obtained by temporal stability theory. This shows the necessity for study and development of stability theory from spatial point of view.

In the governing stability equations we see that the temporal derivative is of first order and with unit coefficient where as the spatial derivatives occur in higher order and in more complicated form. This makes spatial stability problem more complex and difficult than its counterpart the temporal stability problem.

The paper of Gastor (1962) provided impetus to the development of spatial stability theory. Freymuth (1966) after careful and detailed experiment on the amplification of disturbances in free boundary layers observed the phenomenon of phase reversal for neutral disturbances at critical layer, that is at the location of inflexion point. He concluded that the amplification of disturbances in free boundary layers can only be described by a stability theory of spatially growing disturbances. Watson (1962), who developed non-linear theory of spatially growing finite disturbances had made the same suggestion for plane Poiseuille flow. Gastor (1975) showed that for weak amplification only, a transformation from temporal
growth rate to spatial growth rate is possible by means of group velocity. He further showed that for strongly spatially amplified disturbances, as present in shear layers, the eigen value equation has to be solved for complex wave numbers in order to evaluate the spatial growth rate and carried out the calculations for the linear broken line velocity profile. Gill (1965) investigated the stability of spatially damped disturbances of Poiseuile flow in a tube and found good agreement with the experimental results of Leite (1965).

Michalke (1964) carried out stability calculations according to inviscid linearized temporal stability using the smooth hyperbolic tangent velocity profile which is a good approximation to the measured jet boundary layer profile and found no agreement between computed and experimental results. In 1965, he carried out calculations according to linearised spatial stability and found better agreement with experimental results of Freymuth. He also concluded that at least for small frequencies the growth of disturbance in free boundary layer can more precisely be described by the stability theory of spatially growing disturbances.

Betchov and Criminale (Jr) (1966) analysed the spatial instability of inviscid laminar jet and wake with symmetrical velocity profile numerically. They found that k is analytic function of ω (the complex frequency) or vice-versa except a isolated singular points called saddle points. By considering the analyticity of the characteristic function F(k, ω), Gaster (1968) used series expansion to show that the influence they exerts on the motion,
results from a pulse input.

Michalke (1969) by numerical calculation for spatial case of the hyperbolic tangent velocity found that even for spatially growing disturbances the amplification of three dimensional disturbances is smaller than for two-dimensional ones. Gaster (1970) proved the same result analytically.

Mattingly and Criminale (1972) investigated the growth of small disturbances in a two dimensional incompressible wake experimentally. Their analysis was based upon inside stability theory from both temporal as well as spatial point of view. They obtained the following results:

(i) Most unstable disturbances in the wake produce transverse oscillations in the mean velocity profile and correspond to growing waves that have a minimum group velocity.

(ii) The classic von Harman Vortese Street formed behind streamlined bodies has its origin in the near wake region and the initial stages of development are predicted by the linear spatial stability theory. This mechanism is in contrast to as suggested by Sato and Kuriki (1961).

(iii) Between symmetrical and asymmetric disturbances, the symmetrical disturbances are the more unstable as determined by their spatial amplification faction.
Mattingly and Chung (1974) investigated the growth of infinite singal disturbances on an axis symmetric jet column theoretically and experimentally and concluded that stability analysis should be performed from the spatial view point. Freymuth (1973) approximated a free shear flow by a velocity discontinuity and justified it for waves which are considerably longer than the thickness of the shear layer and called then modified Kelvin–Halmhaltz waves. He obtained spatial growth rate and phase velocity of such waves in absence of surface tension. Menton (1974) investigated the linear spatial stability of parallel inviscid shear flows. Certain bounds for complex wave number and complex wave velocity were obtained. He concluded that unlike temporal stability, an inflexion point in the mean velocity profile is not necessary for the existence of spatially growing fluctuations. Itch (1974) modified Watson's approach to non—linear spatially growing finite disturbances in such a way that dependence of mean flow distortion on distance down stream side as well as the terms of eigen functions of the equations for mean flow distortion are taken into account. He obtained numerical results for plane Poisuille flow and flat plate boundary layer.

For obtaining the eigen values of Orr-Sommerfield equation Gastor and Jordison (1975) expanded the complex wave number k as a convergent power series in complex frequency \( \omega \) in various region of \( \omega \) plane. The loci of the real and imaginary points of k have been computed from these series and studied the behavior in the neighborhood of the
branch point. **Gastor (1975)** discussed analytically the development of a wave packet in the boundary layer on a flat plate. **Gaster and Grant J (1975)** discussed the same problem experimentally. Concidence between the analytic and experimental results is satisfactory in the small amplification region.

**Aggarwal and Lin (1975)** investigated the spatial stability of a thin liquid film and obtained that spatially growing disturbances are qualitatively same as temporally growing disturbances.

**Garg and Roukau (1972)** presented a theoretical analysis considering symmetric and axisymmetric disturbances of infinitesimal extent, impressed upon a viscous incompressible fluid in steady flow within a rigid pipe. They found that Poiseuille flow in spatially stable for Reynolds numbers upto atleast $10^4$. They (1976) investigated spatial stability of Poiseuille flow in an arbitrary thick elastic tube to axisymmetric infinitesimal disturbances, and elastic tube to axisymmetric infinitesimal disturbances, and found that critical Reynolds number varies almost as the square root of Young’s modulus of the tube material.

**Kochar and Jain (1978)** investigated the spatial stability of a shear layer, spatial stability of stratified shear flows and the spatial stability of spiral flows. They obtained bound of $k$ in different cases for unstable modes. **Yadav, K.L. (1982)** investigated the spatial stability of a dusty shear layer and the spatial stability of stratified dusty shear flows. They investigated the effect of dust on Manton’s problem to account for more
realistic situation such as in atmosphere. They obtained the spectrum of eigen value and established some stability theorems. **Pradhan and Tripathy (1987)** discussed the spatial stability criteria for rotating, curved and axial shear flows. They established that neither the Hagen—Poiseuille flow profile nor the rotating Couette flow profile can be unstable to disturbances propagating down stream in the inviscid limit.

**1.6 A Brief Survey of the Work Done in Thesis.**

The present work is devoted to the investigation of spatial stability of some hydrodynamic and hydromagnetic stratified parallel shear flows and rotational flows of inviscid fluid. Macroscopic approach is adopted throughout the work and the normal mode technique is used to determine the stability or instability of the fluid flows. This thesis contains seven chapters.

In the first chapter, I have given the basic concepts of stability theory, stability techniques and the relevant literature on the concerned topics. A brief summary of the results obtained in the thesis are also given at the end of this chapter.

In the second chapter the spatial stability of stratified shear flows has been discussed. **Kocher (1979)** discussed the same problem and obtained the spectrum of eigen values and certain stability theorems for \( k_r > 0 \). In this chapter, I have obtained the spectrum of eigen values and some stability theorem for \( k_r < 0 \).
In the third chapter, the spatial stability of spiral flows has been discussed, Kochar (1979) discussed the same problem and obtained the spectrum of eigen values and certain stability theorem for \( k_r > 0 \). In this chapter, I have obtained the spectrum of eigen values and some stability theorems for \( k_r < 0 \).

In the fourth chapter, I have discussed the spatial stability of stratified dusty shear flow. Yadav, K.L. (1982) discussed this problem and obtained the spectrum of eigen values and established some stability theorems for \( k_r > 0 \) in three cases:

(i) Case of fine dust,

(ii) Case of course dust,

(iii) Case when \( r \) is small

I have discussed the problem for \( k_r < 0 \) and obtained the spectrum of eigen values and some stability theorems have been established. The stabilizing effect of course dust and destabilizing effect of fine dust have been confirmed.

Fifth chapter is devoted to the spatial stability of hydromagnetic stratified flows. The density is taken to be decreasing upwards and the magnetic field is being considered parallel to the basic flow. The spectrum of eigen values has been obtained for \( k_r > 0 \). Some stability theorems have been established. It has been shown that as in temporal stability,
the stratification and the magnetic field have the stabilizing effect.

In the sixth chapter the spatial stability in a sheared plasma with finite Larmour radius has been discussed. The spectrum of eigen values has been obtained under different conditions. Some stability theorems have been established.

In the seventh chapter I have discussed the spatial instability of shear flow in a porous medium. The spectrum of eigen values of stable modes has been obtained under certain conditions.