CHAPTER 6

SPATIAL STABILITY IN A SHEARED PLASMA
WITH FINITE LARMOUR RADIUS

6.1. Introduction

In this chapter, the spatial stability of sheared plasma with finite larmour radius, will be discussed. Rosenbluth, Krall and Rostoker (1962) showed that the finite ion-gyration radius, which exhibit itself in the form of magnetic viscosity in the equations governing the fluid motion, has a stabilizing influence on the stability of a plasma in a gravitational field. But Lehnert (1961) pointed out that the conclusion of Rosenbluth et al was not correct and the stabilizing role of finite larmour radius is exactly cancelled by one more term occurring in the equations leaving only a null effect. This was previously reported by him in 1961. Roberts and Taylor (1962) have critically investigated this aspect and they have supported what Rosenbluth et al have claimed. They pointed that the scalar pressure as used by Lehnert should be modified to pressure tensor so as include certain transport terms. The collision effects were neglected in the analysis with two motivations; namely,

(i) The inclusion of collision effects and finite Larmour radius both together introduces such complications in the analysis
that it may be difficult to know about either of them.

(ii) Neglecting collisions, which is of course one real property of plasma, does provide a better understanding of the effects introduced by finite Larmour radius.

The modified pressure tensor includes the effects of finite Larmour radius through \( \nu \) which equals \( NT/4\omega_H P_0 \) and has the dimension of viscosity. Therefore \( \nu \) is called the magnetic viscosity because of dimension consideration only, through the physical mechanism of viscosity and this magnetic viscosity is physically entirely different.

**Surinder Singh and Hari Kishana Hans (1966)** also discussed the effects of magnetic viscosity on the Rayleigh–Taylor and Kelvin–Helmholtz insability problem where in the second situation, the magnetic field was taken in the direction transverse to the streaming direction. They showed a stabilizing character of finite Larmour radius and thus supported the results of Roberts and Taylor.

**Aggarwal and Rastogi (1977)** investigated the effects of finite Larmour radius on a sheared plasma in which the applied uniform magnetic field is in the direction transverse to the streaming direction. The problem discussed by them depends upon two non-dimensional parameters; namely, the magnetic viscosity parameter \( M_\nu \) and the Richardson number \( J \) given by

\[
M_\nu = \frac{\nu \beta}{U_d} \quad \text{and} \quad J = \frac{\beta g}{U^2}
\]
where $\nu = \frac{NT}{4\omega_H \rho_0}$

$\omega_H, N, T, g$ and $\beta = \left( -\frac{D\rho_0}{\rho_0} \right)$ respectively denote the ion-gyration frequency number density, ion-temperature, acceleration due to gravity and the heterogeneity factor. They discussed the temporal stability of the system.

In this chapter the same problem has been taken and the spatial stability has been discussed.

6.2. Formulation of the Problem.

Let the plasma be confined between two rigid horizontal plates situated at $z = 0$ and $z = d$. The x-axis is chosen along the lower plate, on z-axis transverse to the plates with y-axis perpendicular to the x-axis and lying in the horizontal plane. The applied uniform magnetic field is taken in the x-direction and the plasma has a general streaming in the y-direction. The fluid has been assumed to be non-viscous, incompressible and non-heat conducting. The plasma equations of motion governing the system can be written as

$$\begin{align*}
\frac{\partial U}{\partial t} + (U \cdot \nabla) U &= -\frac{1}{\rho} \nabla \cdot P + \frac{\mu_e}{4\pi\rho} (\nabla \times H) \times H + g + \rho \\
\frac{\partial \rho}{\partial t} + (U \cdot \nabla) \rho &= 0 \\
\nabla \cdot U &= 0 \\
\text{and} \quad \nabla \cdot H &= 0
\end{align*}$$

...(6.2.1)

Here the finite Larmour radius is conveniently taken in the pressure tensor $P$ through $\nu$ and $\mu_e$ is the magnetic permeability. Basic solution of the system under discussion is given by
\[
\begin{align*}
U &= (0, U(z), 0) \\
H &= (H, 0, 0) \\
\rho &= \rho_0(z) \\
p &= p_0(z) \\
g &= (0, 0, -g)
\end{align*}
\]...

**(Boundary Conditions.**

The fluid is confined between two rigid planes at \(z = 0\) and \(z = d\). The normal component of velocity must vanish on these planes, \(i.e.\)

\[w = 0\] at \(z = 0\) and \(z = d\)....

**(Perturbed and Linearised Equations.**

The stationary state characterized by (6.2.2) satisfies the equation given by (6.2.1) and the boundary conditions (6.2.3). For studying the instability of this basic flow, the perturbed state may be taken as

\[
\begin{align*}
U (x, y, z, t) &= (0, U(z), 0) + u' (x, y, z, t), \\
H (x, y, z, t) &= (H, 0, 0) + h' (x, y, z, t) \\
p (x, y, z, t) &= p_0(z) + p' (x, y, z, t) \\
\rho (x, y, z, t) &= \rho_0(z) + \rho' (x, y, z, t)
\end{align*}
\]...

Let the components of the perturbations in velocity and magnetic field be \((u', v', w')\) and \((h'_x, h'_y, h'_z)\) respectively. Then the stress tensor \(P\) will have the components.
\[ p_{xx} = p_0 \]
\[ p_{yy} = p_0 - \rho_0 \nu \left[ \frac{\partial w'}{\partial y} + \frac{\partial (v' + U)}{\partial z} \right] \] \hspace{1cm} ...(6.2.5)
\[ p_{zz} = p_0 + \rho_0 \nu \left[ \frac{\partial w'}{\partial y} + \frac{\partial (U' + U)}{\partial z} \right] \]

Following the linear theory of stability the linearised governing equations are given by

\[ \rho_0 \left[ \frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial y} \right] = -2 \nu D \rho_0 \frac{\partial u'}{\partial y} \] \hspace{1cm} ...(6.2.6)

\[ \rho_0 \left[ \frac{\partial u'}{\partial t} + U \frac{\partial v'}{\partial y} + w' DU \right] = - \left[ \frac{\partial p'}{\partial y} - \rho_0 U \left( \frac{\partial^2 w'}{\partial y^2} + \frac{\partial^2 w'}{\partial z^2} \right) \right. \]
\[ + D \rho_0 \nu \left( \frac{\partial v'}{\partial y} - \frac{\partial w'}{\partial z} \right) - \nu \frac{\partial p'}{\partial y} DU \]
\[ \left. + \frac{\mu e H}{4\pi} \left( \frac{\partial h'x}{\partial x} - \frac{\partial h'x}{\partial z} \right) \right] \] \hspace{1cm} ...(6.2.7)

\[ \rho_0 \left[ \frac{\partial w'}{\partial t} + U \frac{\partial w'}{\partial y} \right] = - \frac{\partial p'}{\partial z} + \rho_0 \nu \left( \frac{\partial^2 v'}{\partial y^2} + \frac{\partial^2 v'}{\partial z^2} \right) \]
\[ + \nu D \rho_0 \left( \frac{\partial w'}{\partial y} + \frac{\partial v'}{\partial z} \right) \]
\[ + \nu \frac{\partial p'}{\partial z} DU + \nu p'D^2 U + \frac{\mu e H}{4\pi} \left( \frac{\partial h'x}{\partial x} - \frac{\partial h'x}{\partial z} \right) - \rho' g, \] \hspace{1cm} ...(6.2.8)

\[ \frac{\partial \rho'}{\partial t} + w \frac{\partial \rho_0}{\partial z} + U \frac{\partial \rho'}{\partial y} = 0, \] \hspace{1cm} ...(6.2.9)
\[
\frac{\partial h'_x}{\partial t} + U \frac{\partial h'_x}{\partial y} = H \frac{\partial v'}{\partial x}, \quad \ldots(6.2.10)
\]
\[
\frac{\partial h'_y}{\partial t} + U \frac{\partial h'_y}{\partial y} = H \frac{\partial v'}{\partial x} + h'x \frac{\partial U}{\partial z}, \quad \ldots(6.2.11)
\]
\[
\frac{\partial h'_z}{\partial t} + U \frac{\partial h'_z}{\partial y} = H \frac{\partial w'}{\partial x}, \quad \ldots(6.2.12)
\]
\[
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad \ldots(6.2.13)
\]
\[
\frac{\partial h'_x}{\partial x} + \frac{\partial h'_y}{\partial y} + \frac{\partial h'_z}{\partial z} = 0. \quad \ldots(6.2.14)
\]

Taking the solutions of the form

\[f'(z) = f(z) \exp \imath k'(y - ct)\]

where \(|k|\) is the wave number \(c = \frac{w}{k'}\) is the wave velocity, the equations \((6.2.6)\) to \((6.2.14)\) reduces to

\[\rho_0 (U - c) v = -2 \nu D \rho_0 u, \quad \ldots(6.2.15)\]
\[\imath k' \rho_0 (U - c) v + \rho_0 w D U = -\imath k' p - \rho_0 \nu (D^2 - k'^2) w + D \rho_0 \nu (\imath k v - Dw) - \imath k v \rho D U - H D v, \quad \ldots(6.2.16)\]
\[\imath k' \rho_0 (U - c) w = -D p + \rho_0 \nu (D^2 - k'^2) v + D \rho_0 \nu (\imath k' w + D \nu + \nu D \rho D U + \nu \rho D^2 U + \frac{\mu \varepsilon H}{4\pi} (-D h_x) - \rho g, \quad \ldots(6.2.17)\]
\[ i \kappa' (U - c) \rho + w D \rho_0 = 0, \] ... (6.2.18)

\[ i \kappa' (U - c) h_x = 0, \] ... (6.2.19)

\[ i \kappa' (U - c) h_y = h_x D U, \] ... (6.2.20)

\[ i \kappa' (U - c) h_z = 0, \] ... (6.2.21)

\[ i \kappa' v + Dw = 0, \] ... (6.2.22)

\[ i \kappa' h_y + D h_z = 0. \] ... (6.2.23)

Eliminating various quantities from equation (6.2.15) to (6.2.23) we get the final equation

\[ D (\rho_0 W_1 dw) - k^2 \rho W_1 w - \frac{dgD\rho_0 w}{W} - D \left[ \rho_0 \frac{W_1 W'_1}{W} w \right] = 0. \] ... (6.2.24)

where

\[ W_1 = U - c - \frac{2\nu\beta}{d} \]

\[ W = U - c \]

\[ \beta = -\frac{D\rho_0}{\rho_0} \]

\[ D \equiv \frac{d}{dy} \]
and \( kd = k \).

The boundary conditions reduce to

\[ w = 0 \text{ at } z = 0 \text{ and } z = 1. \]

Using the transformation \( w = WF \) in equation (6.2.24) we get

\[ D [\rho W_1 W DF] - k^2 \rho W_1 W F - Dg\rho F = 0. \quad \ldots(6.2.25) \]

The boundary conditions become

\[ F = 0 \text{ at } z = 0 \text{ and } z = 1. \quad \ldots(6.2.26) \]

6.3. Analysis.

For spatial stability \( k (= k_r + i k_i) \) denotes the complex wave number and \( \omega (> 0) \) the real frequency.

The complex wave velocity \( c \) is given by

\[ c = \frac{\omega}{k}. \quad \ldots(6.3.1) \]

This gives

\[ c_r = \frac{\omega k_r}{|k|^2} \quad \text{and} \quad c_i = -\frac{\omega k_i}{|k|^2}. \quad \ldots(6.3.2) \]

The phase velocity of the perturbations is defined as

\[ c_p = \frac{\omega}{k_r} \quad \ldots(6.3.3) \]
The flow is unstable if the equation (6.2.25) along with the boundary conditions (6.2.26) have a non-trivial solution with \( k_i < 0 \). But if \((F, k)\) is a solution of (6.2.25) and (6.2.26), for a given \( \omega \), then \((\overline{F}, \overline{k})\) is also a solution of it. Therefore growing as well decaying modes occur simultaneously. It suggests that \( k_i \neq 0 \) will give unstable modes.

Let \( F \) be a non-trivial solution of (6.2.25) and (6.2.26) with \( k_i \neq 0 \).

Multiplying equation (6.2.25) by \( \overline{F} \), the complex conjugate of \( F \) and integrating over the flow domain \([0, 1]\), we get

\[
\int \rho_0 W_1 W \left[ |DF|^2 + k^2 |F|^2 \right] + \int dg \, D \rho_0 |F|^2 = 0. \quad \text{(6.3.4)}
\]

Separating the real and imaginary parts of (6.3.4), we get

\[
\int \rho_0 \left( U - c_r \right) \left( U - c_r - \frac{2\nu\beta}{d} \right) \left[ |DF|^2 + (k^2_r - k^2_i) \right] |F|^2
- 4k^2_c r \int \left( U - c_r - \frac{\nu\beta}{d} \right) |F|^2 + \int dg \, D \rho_0 |F|^2 = 0. \quad \text{(6.3.5)}
\]

\[
\int \rho_0 \left( U - c_r - \frac{\nu\beta}{d} \right) \left[ |DF|^2 + (k^2_r - k^2_i) \right] |F|^2
+ \frac{k^2_r}{c_r} \int \rho_0 \left[ (U - c_r) \left( U - c_r - \frac{2\nu\beta}{d} \right) - c^2_r \right] |F|^2. \quad \text{(6.3.6)}
\]

Equation (6.3.6) can be written as
\[ \int \rho_0 \left( U - c_r - \frac{v \beta}{d} \right) |DF|^2 + \int \rho_0 \left( U - c_p - \frac{2v \beta}{d} \right) U \]

\[ \times \frac{|k|^2}{c_p} |F|^2 - \int \rho_0 (U - c_p) \frac{v \beta}{c_p} \frac{|k|^2}{|F|^2} = 0 \] ... (6.3.7)

For \( k_r > 0, c_p > c_r \). Therefore we have

\[ U - c_p < U - c_r. \] \( ... (6.3.8) \)

We consider the following two cases:

**Case 1.** \( \beta > 0 \)

In this case we have

\[ U - c_r - \frac{v \beta}{d} > U - c_p - \frac{v \beta}{d}, \] \( ... (6.3.9) \)

For \( k_1 \neq 0 \), the equation (6.3.7) will hold only if

\[ U - c_r - \frac{v \beta}{d} > 0 \]

at least at one point in the flow domain.

It gives

\[ c_r < b - \frac{v \beta_{\text{min}}}{d} \] \( ... (6.3.10) \)

where \( b = U_{\text{max}} \)
In terms of \((k_r, k_i)\) the relation (6.3.10) can be expressed as

\[
\left[ k_r - \frac{\omega}{2 \left( \frac{b - \nu \beta_{\min}}{d} \right)} \right]^2 + k_i > \left[ \frac{\omega}{2 \left( \frac{b - \nu \beta_{\min}}{d} \right)} \right]^2. \quad \text{...(6.3.11)}
\]

We have thus established the following result:

**Theorem. (6.3.1) (Circle Theorem).**

For a spatially growing unstable mode with \(k_r > 0\), \((k_r, k_i)\) must lie outside the circle whose centre is \(\left( \frac{\omega}{2 \left( \frac{b - \nu \beta_{\min}}{d} \right)}, 0 \right)\) and radius \(\frac{\omega}{2 \left( \frac{b - \nu \beta_{\min}}{d} \right)}\).

Equation (6.3.7) can be written as

\[
\int \rho_0 \left( U - c_r - \frac{2\nu \beta}{d} \right) |DF|^2 + \int \rho_0 \left( U - c_p - \frac{2\nu \beta}{d} \right) \frac{U |k|^2}{c_p} |F|^2
\]

\[
+ \int \rho \frac{\nu \beta |k|^2}{d} |F|^2 = 0. \quad \text{...(6.3.12)}
\]

Obviously

\[
U - c_p - \frac{2\nu \beta}{d} < U - c_r - \frac{\nu \beta}{d}.
\]

Therefore for \(k_i \neq 0\) the equation (6.3.12) will hold only if
\[ U - c_p - \frac{2\nu\beta}{d} < 0, \]

at least at one point in the flow domain.

It gives

\[ c_p > a - \frac{2\nu\beta_{\max}}{d}. \]  \hspace{1cm} \text{...(6.3.13)}

where \( a = U_{\min} \).

Expression (6.3.13) can be expressed as

\[ k_r < \frac{\omega}{a - \frac{2\nu\beta_{\max}}{d}}. \]  \hspace{1cm} \text{...(6.3.14)}

The results given by Lemma (6.3.1) and the relation (6.3.14) do not depend on the nature of velocity profile.

**Case II.** \( \beta < 0. \)

In this case, we have

\[ U - c_r - \frac{\nu\beta}{d} > U - c_p - \frac{\nu\beta}{d} > U - c_p. \]  \hspace{1cm} \text{...(6.3.15)}

Therefore for \( k_i \neq 0 \) the equation (6.3.7) will hold only if

\[ U - c_r - \frac{\nu\beta}{d} > 0 \]

at least at one point in the flow domain.
It gives
\[ c_r < b + \frac{\nu |\beta|_{\text{max.}}}{d}. \] ...(6.3.16)

In terms of \( k_r, k_i \) the relation (6.3.16) can be expressed as
\[
\left[ k_r - \frac{\omega}{2 \left( b + \frac{\nu |\beta|_{\text{max.}}}{d} \right)} \right]^2 + k_i^2 > \left[ \frac{\omega}{2 \left( b + \frac{\nu |\beta|_{\text{max.}}}{d} \right)} \right]^2
\] ...(6.3.17)

We have thus established the following result:

**Theorem. (6.3.2) (Circle Theorem).**

For a spatially growing unstable mode with \( k_r > 0 \), \((k_r, k_i)\) must lie outside the circle whose centre is \( \left( \frac{\omega}{2 \left( b + \frac{\nu |\beta|_{\text{max.}}}{d} \right)}, 0 \) and radius \( \frac{\omega}{2 \left( b + \frac{\nu |\beta|_{\text{max.}}}{d} \right)} \).

Taking substitution
\[ F = W^{-1/2} G, \] ... (6.3.18)
in (6.2.25) and dividing the resulting equation by \( W^{1/2} \) we get
\[(\rho_0 W_1 G')' + \frac{1}{4} \frac{\rho_0 W_1 U^2 G}{W^2} - \frac{1}{2} \frac{(\rho_0 W_1 U')' G}{W} - k^2 \rho_0 W_1 G - \frac{d g D \rho_0 G}{W} = 0. \] ...

Further taking the substitution

\[G = W_1^{-1/2} H \] ...

in (6.3.19) and dividing the resulting equation by \(W_1^{1/2}\), we get

\[\left(\rho_0 H'\right)' + \frac{1}{4} \rho_0 U^2 \left(\frac{1}{W_1^2} + \frac{1}{W^2}\right) H \]

\[- \frac{1}{2} (\rho_0 U')' \left[\frac{1}{W_1} + \frac{1}{W}\right] H - \frac{1}{2} \frac{\rho_0 U^2 H}{W W_1} \]

\[\rho_0 k^2 H - \frac{d g D \rho_0 H}{W W_1} = 0. \] ...

The associated boundary conditions are

\[G = 0 \text{ at } z = 0 \text{ and } z = 1, \] ...

and \[H = 0 \text{ at } z = 0 \text{ and } z = 1. \] ...

Multiplying equation (6.3.19) by \(\overline{G}\), the complex conjugate of \(G\) and integrating over the flow domain \((0, 1)\), we get

\[\int \rho_0 W_1 \left(|G'|^2 + k^2 |G|^2\right) - \frac{1}{4} \int \frac{\rho_0 W_1 U^2}{W^2} |G|^2 \]
\[ + \frac{1}{2} \int \left( \frac{\rho_0 W_1 U_r}{W} \right) |G|^2 + \frac{dg D \rho_0}{W} |G|^2 = 0. \quad \text{...(6.3.24)} \]

Multiplying equation (6.3.21) by \( \overline{H} \), the complex conjugate of \( H \) and integrating over the flow domain \((0, 1)\) we get

\[ \int \rho_0 (|H'|^2 + k^2 |H|^2) - \frac{1}{4} \int \rho_0 U_r^2 \left( \frac{1}{W_1^2} + \frac{1}{W^2} \right) |H|^2 \]

\[ + \frac{1}{2} \int (\rho_0 U')' \left[ \frac{1}{W_1} + \frac{1}{W} \right] |H|^2 + \frac{1}{2} \int \rho_0 U_r^2 |H|^2 \]

\[ + \int \frac{dg D \rho_0}{WW_1} |H|^2 = 0. \quad \text{...(6.3.25)} \]

Separating the real and imaginary parts of (6.3.24) and (6.3.25), we have

\[ \int \rho_0 \left( U - c_r - \frac{2y}{d} \right) \left[ |G'|^2 + (k_r^2 - k_r^2) |G|^2 \right] \right] - 2k_1 c_r \int \rho_0 |H|^2 \]

\[ - \frac{1}{4} \int \rho_0 \left[ \left(U - c_r - \frac{2y}{d} \right) \left(U - c_r^2 - c_r^2 + c_r^2 \right) \right] \frac{U_r^2}{|W|^4} |G|^2 \]

\[ + \frac{1}{2} \int \frac{(\rho_0 U')' \left[ (U - c_r) \left( U - c_r - \frac{2y}{d} \right) + c_r^2 \right]}{|W|^2} |G|^2 \]

\[ + \int \left[ \frac{\rho_0 U_r^2}{2} + dgD \rho_0 \right] \frac{(U - c_r)}{|W|^2} |G|^2 = 0, \quad \text{...(6.3.26)} \]
\[-c_i \int \rho_0 \left[ |G|^2 + (k_r^2 - k_i^2) |G|^2 + 2k_r k_i \int \rho_0 \left( U - c_r - \frac{2\nu \beta}{d} \right) |G|^2 \right. \]

\[-\frac{1}{4} \int \frac{\rho U^2}{|W|^4} \left[ -c_i [(U - c_r)^2 - c_i^2] + 2c_i (U - c_r) \left( U - c_r - \frac{2\nu \beta}{d} \right) \right] |G|^2 \]

\[+ \frac{1}{2} \int \frac{(\rho U')'}{|W|^2} \left[ -c_i (U - c_r) + c_i \left( U - c_r - \frac{2\nu \beta}{d} \right) \right] |G|^2 \]

\[+ c_i \int \frac{1}{|W|^2} \left[ \frac{\rho U^2}{2} + \frac{dgD\rho_0}{|W|^2} \right] |G|^2 = 0, \quad \ldots (6.3.27) \]

\[\int \rho_0 \left[ |H'|^2 + (k_r^2 - k_i^2) |G|^2 \right] \]

\[-\frac{1}{4} \int \rho_0 U^2 \left[ \left( U - c_r - \frac{2\nu \beta}{d} \right)^2 - c_i^2 \right] \left( \frac{1}{|W|^4} \right) + \left( \frac{(U - c_r)^2 - c_i^2}{|W|^4} \right) \]

\[+ \frac{1}{2} \int (\rho U')' \left[ \frac{U - c_r - \frac{2\nu \beta}{d}}{|W|^2} + \frac{U - c_r}{|W|^2} \right] |H|^2 \]

\[+ \int \frac{\left( dgD\rho_0 + \frac{\rho_0 U^2}{2} \right)}{|W|^2 |W_1|^2} \left[ (U - c_r) \left( U - c_r - \frac{2\nu \beta}{d} \right) - c_i^2 \right] |H|^2 = 0 \]

\[\quad \ldots (6.3.28) \]

and
\[2k_n k_i \int \rho |H|^2 - \frac{1}{4} \int \rho_0 U'^2 \left[ \frac{2c_i \left( U - c_r \right)}{|W_1|^4} \right] + \left[ \frac{2c_i (U - c_r)}{|W|^4} \right] |H|^2 + \frac{1}{2} \int (\rho_0 U')' \left[ \frac{c_i}{|W|^2} + \frac{c_i}{|W|^2} \right] |H|^2 \]

\[+ \int \left( \frac{\partial g D \rho_0 + \rho_0 U'^2}{|W|^2 |W_1|^2} \right) \left[ 2c_i \left( U - c_r - \frac{v \beta}{d} \right) |H|^2 \right] = 0 \quad (6.3.29)\]

Equation (6.3.27) for \( c_i \neq 0 \) can be written as

\[\int \rho_0 \left[ |G'|^2 + (k_r^2 - k_f^2) |G|^2 \right] + \frac{2k_r^2}{c_r} \int \rho_0 \left( U - c_r - \frac{2v \beta}{d} \right) |G|^2 \]

\[+ \frac{1}{4} \int \frac{\rho_0 U'^2}{|W|^4} \left[ (U - c_r)^2 + c_i^2 - (U - c_r) - \frac{2v \beta}{d} \right] |G|^2 \]

\[+ \int \frac{(\rho_0 U')'}{|W|^2} \frac{v \beta}{d} |G|^2 - \int \left[ \frac{\rho_0 U'^2}{2} + \partial g D \rho_0 \right] \frac{|G|^2}{|W|^2} = 0 \]

or

\[\int \rho_0 |G'|^2 + \int \rho \left( \frac{2U - 4v \beta}{c_p} - 1 \right) |k|^2 |G|^2 \]

\[+ \frac{1}{2} \int \frac{\rho_0 U'^2 v \beta (U - c_r)}{d |W|^4} |G|^2 \]

\[+ \int \frac{(\rho_0 U')'}{|W|^2} \frac{v \beta}{d} |G|^2 - \int \left( \frac{\rho_0 U'^2}{4} + \partial g D \rho_0 \right) \frac{|G|^2}{|W|^2} = 0.\]
or \[ \int \rho_0 |G'|^2 + \int \rho_0 \left[ \frac{2U - \frac{4\nu\beta}{d}}{c_p} - 1 \right] |k|^2 |G|^2 \]

\[ - \frac{1}{2} \int \frac{\rho_0 \nu^2 \beta (U - c_t)}{d |W|^4} |G|^2 \]

\[ + \int \left[ \left( \frac{\rho_0 U'}{d} \right)' \nu \beta - \frac{\rho_0 U'^2}{4} - dgD\rho_0 \right] \frac{|G|^2}{|W|^2} = 0. \text{ ...(6.3.30)} \]

From (6.3.30) we see that if

\[ \begin{cases} 
2U - \frac{4\nu\beta}{d} - c_p > 1 \\
and \left( \frac{\rho_0 U'}{d} \right)' \nu \beta - \frac{\rho U'^2}{4} - dgD\rho_0 > 0 \end{cases} \text{ ...(6.3.31)} \]

throughout the flow domain then for the validity of this equation

\[ U - c_t < 0 \text{ ...(6.3.32)} \]

at least at one point within the flow domain.

Expression (6.3.31) and (6.3.32) implies that if

\[ L_{\text{min}} \frac{\nu}{d} + dJ_{\text{min}} - \frac{1}{4} > 0 \text{ and } \frac{\omega}{k_r} < 2d - \frac{4\nu\beta}{d} \]

then

\[ \left( k_r - \frac{\omega}{2a} \right)^2 + k_i^2 < \frac{\omega^2}{4a^2} \]
where \( L = \frac{(\rho_0 U')'}{\rho_0} \)

and \( J = -\frac{D\rho_0 g}{\rho_0 U^2} \).

Thus we have the following result:

**Theorem. (6.3.3).** For spatially growing modes with \( k_r > 0 \), \( (k_r, k_i) \) must satisfy the relations

\[
\frac{\omega}{k_r} < 2d - \frac{4v\beta}{d}
\]

and

\[
\left( k_r - \frac{\omega}{2a} \right)^2 + k_i^2 < \frac{\omega^2}{4a^2}
\]

provided

\[
L_{\min} \frac{v\beta}{d} + d J_{\min} - \frac{1}{4} > 0
\]

Further equation (6.3.29) can be written as

\[
\frac{k_r^2}{c_r} \int \rho_0 |\dot{H}|^2 + \frac{1}{4} \int \rho_0 U^2 \left[ \frac{U-cr-2v\beta}{|W_1|^4} + \frac{(U-cr)}{|W|^4} \right] |H|^2
\]

\[
- \frac{1}{4} \int (\rho_0 U')' \left[ \frac{1}{|W_1|^2} + \frac{1}{|W|^2} \right] |H|^2
\]
\[-\int \frac{(dD\rho_0 + \rho_0 U^2)}{|W|^2 |W_1|^2} \left[U - c_r - \frac{v\beta}{d}\right] |H|^2 = 0. \quad \text{(6.3.33)}\]

From equation (6.3.33) we see that if \((\rho U)' < 0\) and \(dD\rho + \rho_0 U' < 0\) then for unstable modes with \(k_r > 0\),

\[U - c_r - \frac{2v\beta}{d} < 0\]

at least at one point in the flow domain.

It implies

\[c_r > a - \frac{2v\beta}{d}\]

or

\[|k|^2 < \frac{\omega k_r}{\left(a - \frac{2v\beta}{d}\right)}\]

or

\[\left[k_r - \frac{\omega}{2 \left(a - \frac{2v\beta}{d}\right)}\right]^2 + k_i^2 < \left[\frac{\omega}{2 \left(a - \frac{2v\beta}{d}\right)}\right]^2.\]

Thus we have the following result:

**Theorem. (6.3.4)** If \((\rho U)' < 0\) and \(dD\rho_0 + \rho_0 U' < 0\) every where inside the flow region then for unstable modes with \(k_r > 0\),

\((k_r, k_i)\) lies with the circle with centre \(\left(\frac{\omega}{2 \left(a - \frac{2v\beta}{d}\right)}, 0\right)\) and radius

\[\left(\frac{\omega}{2 \left(a - \frac{2v\beta}{d}\right)}\right).\]

Here the spatial stability in a sheared plasma with finite larmour radius has been discussed. Eigen value spectrum has been obtain for unstable modes under certain conditions. It has been shown that finite larmour radius has a stabilizing effect.
The region of instability when $L_{\min} \frac{\nu \beta}{d} + d \frac{J_{\min}}{4} > 0$

I. \( \left( k_r - \frac{\omega}{2a} \right)^2 + k_i^2 = \left( \frac{\omega}{2a} \right)^2 \)

II. \( k_r = \frac{\omega}{2d - \frac{4\nu \beta}{d}} \)