3.1 POWER SYSTEM ISSUES RELATED TO CRITICAL FAILURES

When a power disturbance occurs at some bus in the network, an unbalance between the power input and power output occurs resulting in transient. When the transient subsides, a steady-state condition is reached. The power impact is shared by the various synchronous machines according to the steady-state characteristics which are determined by the steady-state droop characteristics of the various generators. During the transient period, the power impact is shared by the machines according to different criteria. If these criteria differ among the group of machines, each impact is followed by oscillatory power swings among the groups of machines to reflect the transition from initial sharing of the impact, to reflect the final adjustment, reached at steady-state. During the system operation, the impact of sudden addition or removal of loads will be followed by the power swings among group of machines.

3.11 Synchronous Generator Concept

Power plants convert potential, thermal and other forms of energy into electrical energy. There are three main components in a conventional power plant: a turbine, a generator, and a transformer. The turbine provides the mechanical energy to the generator. The generator converts this mechanical energy into electrical energy and a transformer raises the voltage levels to allow efficient transportation of the energy. The basic concept of synchronous generators helps to understand the islanding operation.

The basic principle of power generation is that a magnetic field changing across a stationary conductor induces a voltage in the conductor. In simple terms, generators are formed by an electrical circuit and a magnetic circuit interacting with each other. In today's generators, the electrical circuit is stationary and it is made up of armature windings embedded in stationary stator core. When the generator is running, voltage is induced in these windings by the rotating magnetic fields in the rotor. When a load is connected, current flows through the stator windings. The frequency of the electricity produced by a synchronous generator is related to the speed of the rotor and the number of poles in the stator as shown in equation (3.1).

$$f_e = \frac{n_m P}{120} \qquad \text{…. (3.1)}$$

Where,

f_e : electric frequency (Hz),

n_m : rotor speed(rev/Min),

P : number of poles.

The output capacity of the generator is expressed in MVA (Mega-Volt-Amperes). Typical values for the rating of a generator range from about 2 MVA, for a small turbine-generator intended for Navy applications, to 1,000 MVA for large power stations. The ranges of most of generators connected to the grid are the same, but some of generators at the distribution level like powering a home may be as low as 3 KVA.

3.12 Power System Instability

The different types of power system instabilities are described [7] by Kundur, while attention here is given to the instability aspects relevant to power system protection. In particular, the focus is on relation to the grid configuration and system instability, due to its importance in protection applications. For simplicity, the different types of power system instability are discussed separately. During a severe disturbance some of these phenomena may occur simultaneously.

3.13 Angle Instability – Power Oscillations

The fundamental phenomena appearing in a power system in case of angle instability is power oscillations. Depending on their severity and origin they are categorized as transient angle instability and small-signal angle instability.

An important security requirement is the relay coordination for ascertaining the dependability of the relaying scheme. The distance relays should not trip on power swings in case of swings due to system instability. The performance of the distance relaying under stable power swings to be evaluated with the help of ranking tools. In this context three tier hierarchy norms was proposed by S.A.Soman, et.al [34]. As branch norm, fault norm and system norm to solve the relay ranking, fault ranking and out of step detection problem. Ranking of faults identifies the critical faults, while ranking of relays identifies critical relays. Together these will pinpoint the faults and the relays that have to be investigated on power swings.

Generally, power oscillations can be divided into three different categories; 1) local plant mode oscillations or inter-machine oscillations with a frequency range of 0.7 - 2 Hz, 2) inter-area oscillations, where groups of generators are swinging against each other in the frequency range of 0.4 - 0.7 Hz and 3) large sub-systems oscillating against each other where the swinging frequency usually is in the order of 0.1 - 0.3 Hz. 4) If the swing frequency range is between 0.3 - 0.4 Hz, large sub systems oscillating against each other without involvement of some sub group generators.

3.14 Small-Signal Angle Instability

Small-signal stability is the ability of the power system to maintain synchronism when subjected to small disturbances. In this context, a disturbance is considered to be small if the equations that describe the resulting response of the system are to be linearized for the purpose of analysis. Such disturbances happen all the time due to small variations in loads and generation. The physical response of the system may be a steady increase in rotor angle due to lack of synchronizing torque or rotor oscillations of increasing amplitude due to lack of sufficient damping torque. It is important to observe that, although the initial phase can be described by **linear** behavior, the consequences of these oscillations could be **non-linear**.

Measures to counteract small-signal instability are usually based on closed-loop controls [35]. These devices provide dynamic control of electric quantities of the power system. Typical examples of closed loop control devices include generator excitation control, power system stabilizers (PSS), Static Var Compensators (SVCs) and series capacitors with a closed-loop controlled varying capacitance.

3.15 Voltage Instability

Voltage stability is concerned with the ability of a power system to maintain acceptable voltages at all buses in the system under normal conditions and after being subjected to a disturbance. A system enters a state of voltage instability when a disturbance such as an increase in load demand or a change in system conditions causes a progressive and uncontrollable decline in voltage. The main factor causing instability is the inability of the power system to meet the demand for reactive power. According to the time duration of load response, voltage instability can roughly be divided into two different categories: short term and long-term voltage instability. Induction motors restore their active power consumption within one second (short-term) while Load Tap Changers (LTCs) will restore voltage dependent loads within one to several minutes (long-term). Within this 1sec to 1minute time, the primary controllers will take action by way of AVR actuation of the generator, for maintaining the voltages at the respected buses up to their capability limits. Also, thermostatically controlled loads have a recovery time in the range of minutes. In case of long-term voltage instability, generator current limiters may be activated to protect the generators from thermal stresses. When current limiters are activated, the operating condition of the power system is often seriously aggravated. Particularly, armature current limiter activation often leads to blackouts.

Voltage instability may be caused by a variety of single and/or multiple contingencies. Typical initiating events are heavy load pick-up and grid weakening. Obviously, generator tripping also contributes to voltage instability; especially tripping of generators located close to the loads supporting the voltage control in that area. Generator tripping can be the event which initiates voltage instability, but it may also be an accelerating element when it occurs some time as a voltage instability event.

In case of short-term voltage instability, the system may collapse within a few seconds after the disturbance if no curative measures are taken [36].

For short-term voltage instability, the fault clearing time may be essential as an induction motor dominated load, e.g. air conditioning, may become unstable if the fault is present for a longer period [10].

During long-term voltage instability, the power system normally reaches a quasi-stable low voltage operating point after the initial disturbance. Voltage dynamics and angle dynamics can be decomposed into separate sub-systems. A saddle-node bifurcation occurs in voltage sub-system before angle instability. When the system reaches this saddle-node bifurcation point the system will eventually loose synchronism as the generator field flux slowly decays, which implies that the slow voltage decay phenomenon can be regulated with the help of voltage regulating devices like LTCs [37]. Because of the voltage drop LTCs will start to restore the voltage on regional transmission and distribution levels. In turn, the loads recover which leads to increased system loading. Consequently, the voltage will further decline and the LTCs will operate again to restore the loads. This interaction between LTCs and increased system loading continues while the voltage on the transmission level declines continuously. The reactive power demand of the system is continuously increasing as the voltage is decreasing. Eventually, the reactive power demand cannot be supplied by the generators and their current limiters operate. At this moment, the voltage decay accelerates significantly, which in turn leads to generator tripping due to low voltage and cascading line outages due to overload. The voltage decay due to LTCs is obviously restricted to the operating range of the tap changers.

In case of a heavy load pick-up in a highly meshed network, the crucial aspect with respect to voltage stability will initially be related to whether local generators manage to meet the increased load demand without hitting their limits. When the local units hit their limits, stability will depend on the capability of the surrounding system to provide active and reactive power. If the surrounding system is strong, it will support the highly loaded area successfully. However, if the surrounding system is weak, a sufficient support will most likely not be given and the situation may aggravate to a voltage collapse. It is observed that, the more the circuits and with shorter lengths in the grid, the stronger will be the network.

When a long distance separates a load centre from a significant amount of supply, a grid weakening between the two areas may lead to voltage instability. The remaining grid may not have the capability to transport the required power at a sufficient voltage level, which in turn increases the reactive power demand from the system and the tap-changer interaction. Moreover, this may lead to generators hitting their limits, cascading line outages because of (distance) relays operating on overload and eventually to a collapse.

The case where generators located close to loads are tripped, is similar to the case of a heavy load increase. Also, in this case, the stability will be decided by the capability of the surrounding system to provide active and reactive power to the area lacking generation. When the power system experiences such disturbances, as a remedial action plan to control the voltage instability that occurs due to over drawing of power from the grid, emergency load shedding scheme will be implemented [38-40]

To summarize, the voltage instability is likely to happen for the system configurations, where large amounts of power have to be transported long distances in a lightly meshed network due to weak interconnections between remote generation areas and load centers [41].

In addition to the network configuration, the following aspects are important as far as voltage stability is concerned [9]:

- Active and reactive power reserves. For example, generators, synchronous condensers and Static Var Compensators (SVCs). Both the amount of available reactive power and the location are important.
- Passive reactive power reserves such as capacitors.
- System loading levels. High loading levels are critical.
- Low power factor.
- Load characteristics. Especially, the load recovery due to LTC operation (or thermostatic heating).

3.16 Frequency Instability

Frequency stability describes the ability of a power system to maintain the system frequency within an acceptable range during normal operating conditions or after a severe disturbance. Thus, frequency instability occurs when there is a mismatch between load and supply, and the system cannot compensate for this mismatch before the frequency reaches an unacceptable value. Typical events which may lead to frequency instability are major outages of generating units and splitting of the system into isolated areas.

In case, normal frequency control measures fail to maintain the frequency within an acceptable range, it is still important to limit frequency excursions. Especially, generators are sensitive to fairly small frequency deviations. Normally generators can operate within a band of ±0.5 Hz related to nominal frequency (50 and 60 Hz systems) without any restrictions. Additionally, generators can operate outside these values for a limited time period given by manufacturing constraints.

Protection applications responding to frequency instability are normally rather straightforward. For example, unit devices protecting equipment from damage in the case of frequency excursions should respond to a certain frequency deviation present for a specified duration. Accordingly, simple frequency relays with settings relevant to the current application can be used. Similarly, the process for Special Protection Schemes responding to frequency instability is also straightforward, although extensive setting procedures in order to obtain the optimal curative measures may be required [42].

3.17 Transient Angle Instability

According to Kundur [7], transient stability is the ability of the power system to maintain synchronism when subjected to a severe disturbance such as a fault on transmission facilities, loss of generation, or loss of a large load. Usually, these types of disturbances lead to large excursions of generator angles and significant changes in active and reactive power flows, bus voltages, system frequency and other system variables. Accordingly, both the customers and the power system are confronted to these features where they have a more or less developed transient characteristic. In case, appropriate counteractions are not taken transient instability may result in extensive power system blackouts.

Loss of synchronism may include one single generating unit, a power plant represented by multiple generators, a region of the network or several interconnected regions. The loss of synchronism may occur during the first swing. In this case the mismatch between the electrical and mechanical torque is considered to be the main issue. In the second case the loss of synchronism is occurred after a number of divergent oscillations following the disturbance. In this case the loss of synchronism is associated with insufficient damping after a few swings.

Lightly meshed networks, large power flows and long distance power transport are features which contribute to transient angle instability. Accordingly tie-lines, bottlenecks and weak interconnections (between different areas) are typical sources of transient instability. As transient instability includes large voltage and power variations, fast tripping of power system devices may be initiated due to undesirable backup protection operations [43].

3.2 CRITICAL FAILURES - BLACKOUTS

The power systems operated by the utilities in developing countries suffer from large gap between the demand and generation, inadequate transmission capacity and non uniform location of load centers and generating stations, power sector reforms in deregulation scenario made the power grid vulnerable to blackouts due to occurrences of critical failures in power systems. Major power system breakdowns have been occurring historically in the inter-connected electrical grids. Certain technical factors played an important role in the recently occurred critical failures.

1. Incorrect operation of protective system.
2. Voltage instability.
3. Frequency instability.
4. Critical overloads.
5. Lack of control schemes to control rapid frequency decline following the disturbance.
6. Lack of supplementation of reactive resources to address the voltage collapse.
7. Lack of demand side and management techniques with automation to prevent cascade failures.

Thus, the critical failures in the power system have presented challenges to the power system planners and operators. This has made the study of critical failures as an immediate need drawing the attention from the power system engineers and academicians.

3.21 Blackout Occurrences

Some of the critical failure events occurred around the world are detailed below.

- **WSCC 14th December, 1994**

On 14th December, 1994 at 1:25AM the abnormal overload occurred from southern California to Northern California and form Northern California to North-West. A single phase to ground fault on 345kV line caused the inadvertent tripping of the additional 345kV lines feeding the same substation. The over load in the remaining lines caused the under voltage condition due to the weakened network, the lines tripped one after the other in domino effect style.

- **WSCC 2nd July, 1996**

At 1:25PM on 2nd July, 1996 a large disturbance occurred in WSCC system. A short circuit occurred on 345kV line and was tripped successfully. This disturbance caused the parallel lines to be tripped. An SPS scheme was initiated after tripping of the two lines which caused the shutdown of generating units at the Jim Bridger Plant. The under voltage and inter oscillations developed quickly throughout the system and resulted in loss of 2500MW of power.

- **WSCC 3rd July, 1996**

At 2:03PM on 3rd July, 1996 a sequence of initiating events that was similar to the previous day events occurred in WSCC system. After the shutdown of two generating units at Jim Bridger Plant, the under voltage problem began to develop. The system operator resorted to shut the load demand of 1200MW due to his previous day experience. As a result of this decisive action the system was saved and only limited number of customers were affected.

- **WSCC 10th August, 1996**

At 15:48hrs on 10th August, 1996, due to the prevailing of high temperatures throughout the west coast, which lead to high electricity demand. A number of transmission lines outage in Washington over a period of 1hr weakened the transmission system which lead to the growing oscillations in the system. Three 500kV pacific AC tie lines and ± 500kV pacific DC tie lines were lost due to the oscillations which caused the cascade outages without control [44].

- **Brazil 11th March, 1999**

At 22:16hrs on 11th March, 1999 a phase to ground fault occurred on 440kV bus bar and caused the opening of all the lines connected to the substation, where in there was no bus bar protection. The Brazil power system could withstand the multiple imitating contingency for only 10secs. The power system began to collapse at the instant of disconnection of the tie lines between two areas, the outage of major power plants, loss of HVDC links and 765kV AC links, finally lead to system separation resulting into the loss of 24,731MW load, affecting 75 million people.

- **US Midwest and North-East Canada 14th August, 2003**

Sequence of line trippings in the North-East Ohio, caused heavy loadings on number of transmission lines, the cascade blackout occurred as more than 508 generating units at 265 power plants were lost. The northern part of the whole eastern interconnection was broken apart into five islands. The blackout affected about 50 million people and caused the loss of 61,800 megawatts of electric load in the states of Ohio, Michigan, Pennsylvania, New York, Vermont, Massachusetts, Connecticut, New Jersey and the Canadian province of Ontario

- **Italy, 28th September, 2003**

In the night of 28th September, 2003 at 03:01:42, the 380kV transmission line connecting Italy and Switzerland had a flashover due to falling of trees and the line got tripped. This tripping caused some other transmission lines to become overloaded. About 25 minutes later, two transmission lines, the 380kv connecting Italy and Switzerland and also the 220kV lines of Italy and Switzerland, were tripped due to overload. At 03:25:26, a special protection scheme automatically disconnected the 380kV line of Austria and Italy to protect the Austria system. Immediately after the last contingency, the Italian grid lost its synchronism and got separated. A total of 27.7GW load was lost and 57 million people were affected.

- **India 30th and 31st July 2012**

There was a major grid disturbance in Northern Region at 02.33 hrs on 30-07-2012. Northern Regional Grid load was about 36,000 MW at the time of disturbance. Subsequently, there was another grid disturbance at 13.00 hrs on 31-07-2012 resulting in collapse of Northern, Eastern and North-Eastern regional grids. The total load of about 48,000 MW was affected in this blackout. On both the days, few pockets survived from blackout.

- **Brief Sequence of Events which led to the Grid Collapse on 30th and 31st July 2012**

(i) On 30th July, 2012, after NR (Northern Region) got separated from WR (Western Region) due to tripping of 400 kV Bina-Gwalior line, the NR loads were met through WR-ER (Eastern Region)-NR route, which caused power swing in the system. Since the center of swing was in the NR-ER interface, the corresponding tie lines tripped, isolating the NR system from the rest of the NEW grid system. The NR grid system collapsed due to under frequency and further power swing within the region.

(ii) On 31st July, 2012, after NR got separated from the WR due to tripping of 400 kV Bina-Gwalior line, the NR loads were met through WR-ER-NR route, which caused power swing in the system. On this day the center of swing was in the ER, near ER-WR interface, and, hence, after tripping of lines in the ER itself, a small part of ER (Ranchi and Rourkela), along with WR, got isolated from the rest of the NEW grid. This caused power swing in the NR-ER interface and resulted in further separation of the NR from the ER-NER system. Subsequently, all the three grids collapsed due to multiple tripping attributed to the internal power swings, under frequency and overvoltage at different places.

(iii) The WR system, however, survived due to tripping of few generators in this region on high frequency on both the days.

(iv) The Southern Region (SR), which was getting power from ER and WR, also survived on 31st July, 2012 with part loads remained fed from the WR and the operation of few defense mechanism, such as AUFLS and HVDC power ramping.

3.3 MODELING BLACKOUTS

Consider, checking combinations of failures in a power system model with n components. For practical models, for large blackouts, n is in the thousands or tens of thousands. Checking for single failures requires only n cases to be checked, but checking for combinations of k successive failures requires n^k cases to be checked, which rapidly becomes infeasible even with the fastest computers for modest values of k. But large blackouts can involve cascades of tens to hundreds of events. It is clear that exhaustively checking all possible combinations of cascading failures that could lead to blackouts in practical power system models is computationally infeasible.

Cascading phenomena are complicated because of the diversity of failures and the many different mechanisms by which failures can interact. There are varying modeling requirements and timescales (milliseconds for electromechanical effects and tens of minutes for voltage support and thermal heating). Combinations of several of types of failures and interactions can typically occur in large blackouts, including cascading overloads, failures of protection equipment, transient instability, forced or unforced initiating outages, reactive power problems and voltage collapse, software, communication, and operational errors, mismatch between planning studies and operational environment, rare and unusual failures or combinations of failures, operating mistakes and lack of situational awareness.

For investigation of some of the general features of interactions between the infrastructure system by using very simple models are found required, D. E. Newman, et al [45] proposed two such models. One is probabilistic model, cascade and the other model is dynamic complex system model (DCSM), which can work in self-organized critical state. In both the models the threshold above which the occurrences of cascading failures is characterized by the branching parameter λ. The percolation point is at λ=1, where the probability density of failures for cascade is a power law with exponent -1.5 while for DCSM it is somewhat closer to -1.0 . For the DCSM model it is found that large failures are more likely to be synchronized across the two dynamical systems, which is likely to be the power law found in the probability of failure with size is less steep with the coupling. This means that in coupled systems there is a greater probability of large failures. With DCSM model the other important aspect of infrastructure that can be explored is non-uniform and non-symmetric couplings.

Since an exhaustive computation of all possible combinations of failures is infeasible, and making a very detailed model of all possible failures and their interactions is beyond the state-of-the-art, compromises are needed in modeling and analyzing cascading failure, such as,

- Analyze the detailed failures and interactions in a single blackout after it occurs.
- Analyze a selection of most probable or high risk failures.
- Statistically, model the overall progression of cascading failures, while neglecting details of the interactions.
- Analyze a simplified power system model to explain the bulk properties of cascading failures, rather than modeling all of the equipment in detail.
- Analyze one or only a few of the cascading mechanisms.
- Analyze only an initial part of the sequence of failures; for example, up to a point of "no return."
- Use real-time information about the current power system configuration and the progress of the cascade (if slow enough) to help limit the possibilities to be considered.

3.31 Mechanism of Blackouts

Definition of Cascading

Cascading failure is defined as a sequence of dependent failures of individual components that successively weakens the power system. Cascading failure is the main mechanism of large blackouts. Failures successively weaken the system and make further failures more likely so that a blackout can propagate to disable large portions of the electric power transmission infrastructure upon which modern society depends. There are many different types of interactions by which failures can propagate during the course of a blackout [46, 47]. For example, a transmission line tripping can cause a transient, the overloading of other lines, the operation or misoperation of relays, reactive power problems, or can contribute to system instabilities or operator stress. However, for all these types of interaction, the risk of cascading failure generally becomes more severe as overall system loading increases. But exactly how does cascading failure become more likely as loading increases? There is a critical loading at which the probability of large blackouts and the mean blackout size start to rise quickly and at which the probability distribution of blackout sizes has power law dependence [17-20]. The significance of the power law in the probability distribution of blackout size is that it implies a substantial risk of large blackouts near the critical loading. It is important to verify and compare criticality in a variety of power system blackout models to find out the extent to which it is a phenomenon universally associated with cascading failure of power systems. Models of cascading failure in power systems are described in the following sections.

When new solutions are to be developed that aim at avoiding the large scale blackouts to occur, it is important to understand causes and involved mechanisms in the process leading to a blackout. Fig. 3.1 illustrates a general sequence of events that can lead to a blackout.

The main cause of blackout is a recurrent succession of failures, that disconnects the transmission lines or generators and failure of the survival of the formed island if any. A practical problem may start as a major fault that causes large variations in power flow and voltages which in turn can cause the damage of generation units or transmission lines which further leads to generation load imbalance.

3.32 Industry Practice

Transmission Operations is: “To ensure that the transmission system is operated so that instability, uncontrolled separation, or cascading outages will not occur as a result of the most severe single Contingency and specified multiple Contingencies.” and requires that: “Each Transmission Operator shall operate so that instability, uncontrolled separation, or cascading outages will not occur as a result of the most severe single contingency.” and that: “Each Transmission Operator shall, when practical, operate to protect against instability, uncontrolled separation, or cascading outages resulting from multiple outages, as specified by Regional Reliability Organization policy.”

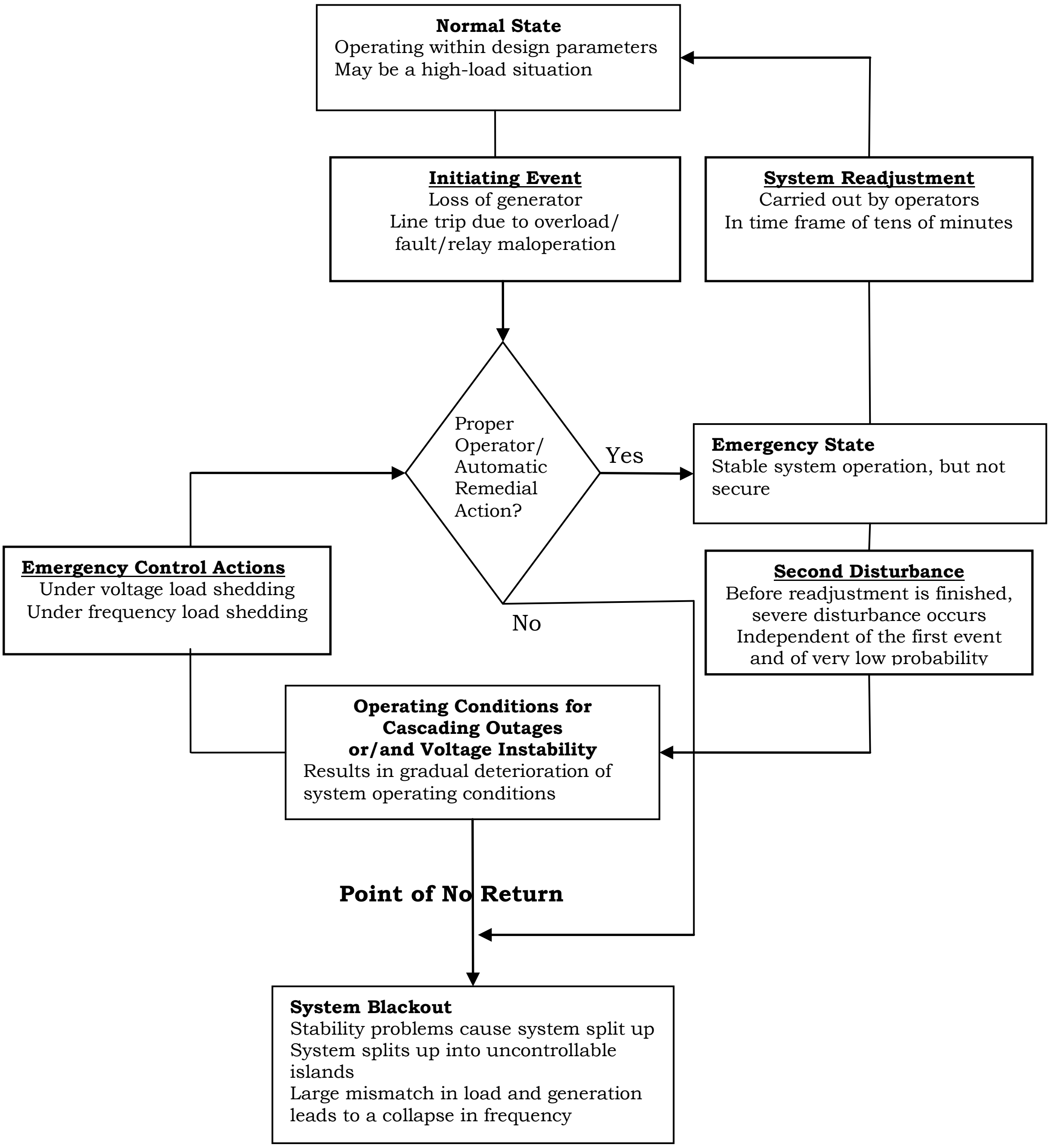

Fig. 3.1: Mechanism in the evolution of a Power System blackout

Emergency Preparedness and Operations requires that: "After taking all other remedial steps, a Transmission Operator operating with insufficient generation or transmission capacity shall shed customer load rather than risk an uncontrolled failure of components or cascading outages of the Interconnection." The purpose for is to ensure that "a Transmission Operator operating with insufficient generation or transmission capacity" has "the capability and authority to shed load rather than risk an uncontrolled failure of the Interconnection."

Different categories are defined in transmission planning to address the system operating condition under different categories.

Category A (normal system operations with no contingencies), Category B (event resulting in the loss of a single element or N-1), Category C (event(s) resulting in the loss of two or more (multiple) elements or N-2), and Category D extreme event resulting in two or more (multiple) elements removed.

Category A analysis is straightforward: if any element exceeds its applicable rating, then transmission planners will add new transmission facilities, or write and implement procedures to avoid the condition. In the operating time frame, the transmission system will be reconfigured or re-dispatched to eliminate any violations. Category B analysis follows a path similar to Category A, except that loss of each facility must be considered. Credible single contingencies are usually selected using engineering judgment, but modern computing capability allows analysis of all failures of single elements or at least a wide selection of them.

Category C analysis is complicated by the number of events that must be considered. Typically, this list is reduced to a manageable number of events using engineering judgment. Contingencies that do not cause applicable rating violations are dismissed. The operating standards focuses only on most severe single contingency [48].

The Category D standard used to include a prohibition against cascading.

3.33 Properties of Cascading

How much rarer are large blackouts than small blackouts? One might expect a probability distribution of blackout size to fall off exponentially as the size of the blackout increases. That is, doubling the blackout size squares its probability and so, after many squarings, the largest blackouts have vanishingly small probability. However, analyses of North American blackout statistics show that the probability distribution of blackout size does not decrease exponentially, but rather has an approximate power law region with an exponent [43], between – 1 and – 2.

The power law data from different countries suggests that large blackouts are much more likely than might be expected from the common probability distributions that have exponential tails. The power law region is always limited in extent by a finite cut off corresponding to the largest possible blackout.

3.34 Some Models of Blackouts

1. **Cascade Model**

The Cascade model is an analytically tractable probabilistic model of cascading failure that captures the weakening of the system as the cascade proceeds [46]. There are a large but finite number n of identical components and each component has a level of loading or stress. The initial load on each component is an s-independent uniform random variable over a fixed range of loading. There is an initial disturbance to the system that adds additional loading to each component. Each component has a maximum loading threshold and fails if this threshold is exceeded. When any component fails, all the other components are additionally loaded so that initial failures can lead to a cascading sequence of failures as components successively overload and additionally load the other components. The cascade continues until there are no further failures or all the components are failed. The total number of failed components in the Cascade model follows a saturating variant of the quasibinomial distribution. The main parameters are the size d of the initial disturbance and the amount p by which load of other components is incremented when a component fails, which controls the extent to which the cascade propagates [49].

Normalized cascade Algorithm

1. All n components are initially unfailed and have initial loads that are n s-independent random variables uniformly distributed in {0; 1}
2. Add the initial disturbance d to the load of each component. Initialize the generation number i to zero.
3. Test each unfailed component for failure: For j = 1,….,n, if component j is unfailed and its load > 1 then component j fails. Suppose if m_i components fail in this step.

4. If $m_i = 0$, stop, the cascading process ends.
5. If $m_i > 0$, then increment the component loads according to the number of failures m_i; Add m_{ip} to the load of each component for j=1… N.
6. Now, increment generation number i and go to step 2.

2. Branching Process Model

The branching process model of cascading failure is a standard Galton-Watson branching process [50] with Poisson offspring distributions, except that there are a finite number n of components. The failures are produced in generations. In generation zero, there is an initial Poisson distribution of failures with mean that represents the initial disturbance to the system. Each failure $\hat{\theta}$ in each generation produces further failures according to a Poisson offspring distribution with mean λ until no more failures are produced or all the components fail. The total number of failures follows a saturating variant of the generalized Poisson distribution. The main parameters are the mean size $\hat{\theta}$ of the initial disturbance and the mean number of offspring failures λ which controls the extent to which the cascade propagates.

3.35 Computational Study of Blackout Model

There are a finite number of components that can fail in a blackout, so it must be recognized that the cascading process will saturate when most of the components have failed. Moreover, many observed cascading blackouts do not proceed to the entire interconnection blacking out. The reasons for this may well include inhibition effects such as load shedding relieving system stress, or successful islanding, that apply in addition to the stochastic variation that will limit some cascading sequences. Understanding and modeling these inhibition or saturation effects is important. However, in estimating λ, we avoid this issue by analyzing the cascading process before saturation occurs. Analytic formulas for the total number of components failed can be obtained in some cases. For example, assume that there are M_0 initial failures, the offspring distribution is Poisson with mean λ, and the process saturates when n components fail. Then the total number of failures S is distributed according to a saturating Borel-Tanner distribution [19, 49]:

$$P[S=r]=\begin{cases} M_0\lambda(r\lambda)^{r-M_0-1}\dfrac{e^{-r\lambda}}{(r-M_0)!}; & M_0\le r<n \\ 1-\displaystyle\sum_{s=M_0}^{n-1} M_0\lambda(s\lambda)^{s-M_0-1}\dfrac{e^{-s\lambda}}{(s-M_0)!}; & r=n \end{cases} \quad \ldots (3.2)$$

Approximation of (3.1) for large $r < n$ using Stirling's formula and a limiting expression for an exponential yields

$$P[S=r] \approx \frac{M_0}{\sqrt{2\pi}}\lambda^{-M_0} r^{-1.5} e^{-r/r_0}; \quad 1 \ll r < n$$
$$\text{where } r_0 = (\lambda - 1 - \ln\lambda)^{-1} \quad \ldots (3.3)$$

In approximation (3.2), the term $r^{-1.5}$ dominates for $r \le r_0$ and the exponential term $e^{-r/r0}$ dominates for $r_0 \le r < n$. Thus (3.2), reveals that the distribution of the number of failures has an approximate power law region of exponent −1.5 for $1 < r \le r_0$ and an exponential tail for $r_0 \le r < n$.

3.36 Computation and Results

(1) By varying the number of failures (r), the corresponding values of p.d.f. are obtained, and plotted as log-log plot, (Fig. 3.2). The qualitative behavior of the distribution of blackout size as λ is increased can now be described. This behavior is illustrated in Fig. 3.2. For subcritical λ well below 1, r_0 is well below n and the exponential tail for $r_0 \le r < n$ implies that the probability of large blackouts of size near n is exponentially small. The probability of large blackouts of size exactly n is also very small. As λ increases in the subcritical range $\lambda<1$, the mechanism by which it develops a significant probability of large blackouts of size near n is that, r_0 increases with λ so that the power law region extends to the large blackouts. For near critical $\lambda\approx 1$, r_0 becomes large and exceeds n so that power law region extends up to $r = n$. For supercritical λ well above 1, r_0 is again well below n and there is an exponential tail for $r_0 \le r < n$. This again implies that the probability of large blackouts of size near n is exponentially small. However there is a significant probability of large blackouts of size exactly n and this probability of total blackout increases with λ.

(2) By varying the number of failures (r), the corresponding values of the blackout risk is computed and plotted as log-log plot (Fig. 3.3). Fig. 3.3 shows the distribution of risk with respect to the number of failures for the same values of λ considered in Fig. 3.2, the essential point is that, given an assumption about the blackout cost, as a function of blackout size, the branching process model gives a way to compute blackout risk in terms of λ, i.e., the nature of the blackout risk which is proportional to the probability distribution function, will exhibit the same variation with respect to the number of failures.

The MATLAB program for computation of the probability distribution function and risk assessment, for different values of number of failures (r), is given below.

- Matlab Program

```
close all; clear; clc;
M0=1/sqrt(2);
lmda=[0.7 1 1.3];
r=1:1001;
for i=1:length(lmda)
r0=1/(lmda(i)-1-log(lmda(i)));
P(i,:)=(M0/sqrt(2*pi))*(lmda(i)^M0).*(r.^(-1.5)).*exp(-r/r0);
end
loglog(r,P(1,:),'r^','MarkerFaceColor','r'); hold on
loglog(r,P(2,:),'gs','MarkerFaceColor','g'); hold on
loglog(r,P(3,:),'bo','MarkerFaceColor','b');
xlabel('r number of failures');
ylabel('p(s=r) PDF');
axis([1 1000 10^-8 1])   % %  Fig 1
figure,
loglog(r,P(1,:),'r^','MarkerFaceColor','r'); hold on
loglog(r,P(2,:),'gs','MarkerFaceColor','g'); hold on
loglog(r,P(3,:),'bo','MarkerFaceColor','b');
xlabel('r number of failures');
ylabel('p(s=r) blackouts');
axis([1 1000 10^-8 10^3]) %% Fig 2
```

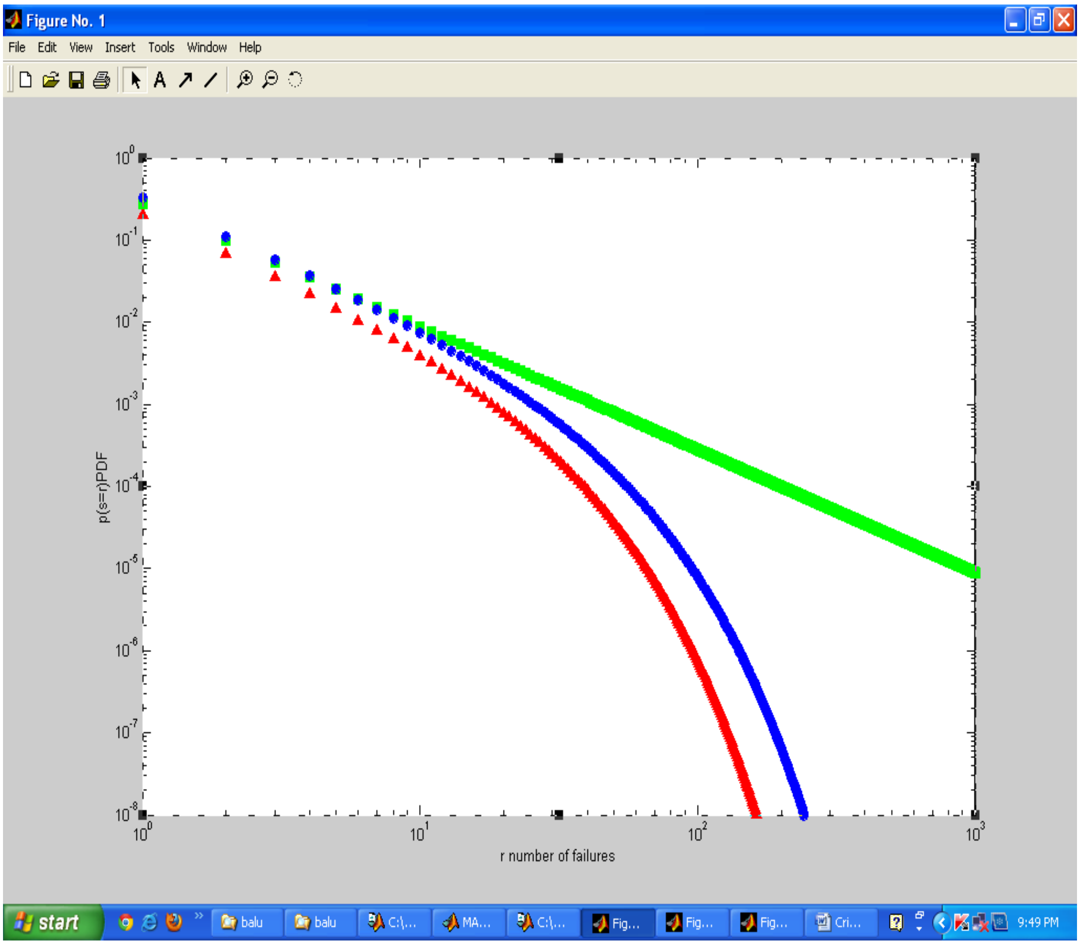

Fig. 3.2: Log log plot of PDF of total number of failures in branching process model for three values of λ. λ = 0.7 indicated by circle, λ = 1.0 indicated by square (critically) and λ = 1.3 indicated by triangle

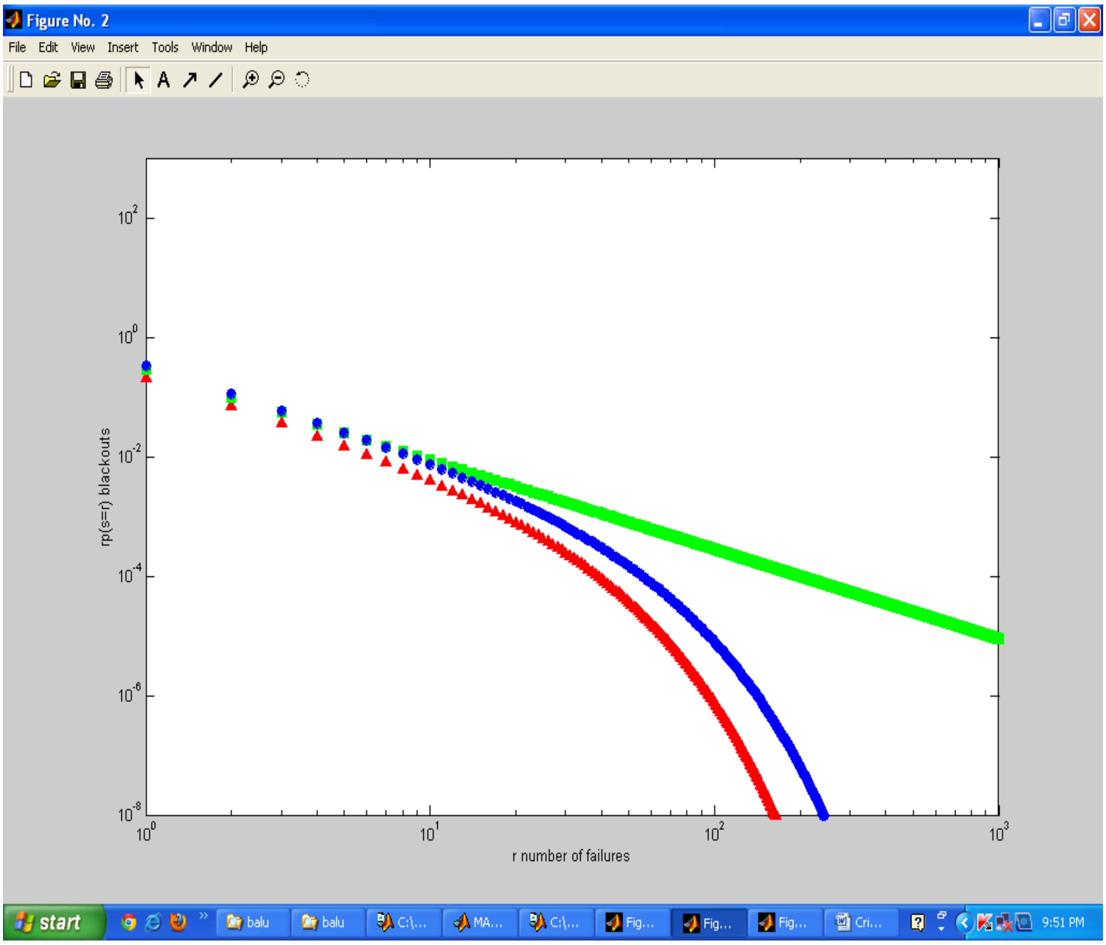

Fig. 3.3: blackout risk rp(s=r) and number of failures in branching process model for three values of λ. λ = 0.7 indicated by circle, λ = 1.0 indicated by square (critically) and λ = 1.3 indicated by triangle