CHAPTER. IX.

IDEALIZED PLANE-SLIDER BEARING WITH A NON-NEWTONIAN LUBRICANT

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9.1. INTRODUCTION.

The object of a lubricant is to separate rubbing surfaces by a layer which is more easily sheared than the surfaces themselves. This has the immediate effect of reducing wear by causing the lubricant to be sheared rather than the surfaces and reduces the frictional force needed to slide one surface over the other. It has been estimated that $\frac{1}{3}$ to $\frac{1}{2}$ of the total energy produced in the world is consumed in friction. So there is some thing to be gained in friction reduction even if wear is not particularly important. Studies have shown that two distinct types of lubricant action exist. In the first case only partial separation of the rubbing surfaces is achieved and some solid to solid contact occurs. Hence, wear occurs. This is termed boundary lubrication. In the second case a complete separation of the surfaces by a lubricant film is achieved, and no sensible wear occurs. This is hydrodynamic or film lubrication, and represents the ideal state aimed at by the designer. It may be pointed out that film lubrication is not always attainable in a given machine, particularly while starting or stopping the machines under heavy loads. It is important to note that boundary lubricants function by chemical action and hydrodynamic lubrication is a mechanical action. In this chapter we have solved a simple problem on hydrohynamic theory of lubrication with a particular reference to non-Newtonian lubricant. The discussion of this chapter is based upon a paper by Lord Rayleigh (1918). Before we do so, we shall first determine the flow field between two parallel planes.

9.2. MATHEMATICAL ANALYSIS.

We assume the steady parallel flow of the liquid between two infinite parallel planes. The x-axis is chosen parallel to the walls and y-axis normal to this direction. The velocity field compatible with the continuity condition is

$$u = u(y) \; ; \qquad v = 0 \; ; \qquad w = 0 \tag{9.2.1}$$

The boundary conditions on velocity are

$$\left.\begin{array}{lll} u = U & \text{when} & y = 0 \\ u = 0 & \text{when} & y = h \end{array}\right\} . \tag{9.2.2}$$

Hence, one gets the stress component p_{xy} as

$$p_{xy} = \mu \frac{du}{dy} \tag{9.2.3}$$

We take μ, the coefficient of viscosity to be a function of the flow invariants I_1, I_2 and I_3 defined earlier. In this problem

$$I_1 = 0 \; ; \quad I_2 = -\frac{1}{4}\left(\frac{du}{dy}\right)^2 \; ; \; I_3 = 0 \tag{9.2.4}$$

Hence, μ is a function of I_2 only. So

$$\mu = f \left\{ \left(\frac{du}{dy} \right)^2 \right\}$$

Following Rivlin (1955), we make

$$\mu = \sum_{n=0}^{\infty} \alpha_n \left(\frac{du}{dy} \right)^{2n}, \tag{9.2.5}$$

where α_0 , α_1 , α_2 , ... are material constants. We confine our attention to the particular class of liquid where

$$\left. \begin{array}{l} \alpha_0 \neq 0 , \quad \alpha_1 \neq 0 \\ \text{and} \\ \alpha_2 = \alpha_3 = . . = \alpha_n = 0 \end{array} \right\} \tag{9.2.6}$$

Hence, from (9.2.3), (9.2.5) and (9.2.6) we get

$$p_{xy} = \alpha_0 \frac{du}{dy} + \alpha_1 \left(\frac{du}{dy} \right)^3 . \tag{9.2.7}$$

The equation of motion gives

$$\frac{dp}{dx} = \frac{dp_{xy}}{dy} \tag{9.2.8}$$

From (9.2.7) and (9.2.8), we get

$$\alpha_0 \frac{d^2u}{dy^2} + 3\alpha_1 \left(\frac{du}{dy} \right)^2 \frac{d^2u}{dy^2} = \frac{dp}{dx} \tag{9.2.9}$$

Let us make the following transformations

$$\left.\begin{aligned} x &= h\xi \;;\quad y = h\eta \;;\quad u = UF(\eta) \\ p &= \rho U^2 p_1 \;;\quad P = -\frac{\rho U h}{\alpha_0}\frac{dp_1}{d\xi} \;;\quad R = \frac{\alpha_1 U^2}{\alpha_0 h^2} \end{aligned}\right\} \qquad (9.2.10)$$

Equation (9.2.9) now gives

$$\frac{d^2F}{d\eta^2} + 3R\left(\frac{dF}{d\eta}\right)^2 \frac{d^2F}{d\eta^2} + P = 0 \qquad (9.2.11)$$

The boundary conditions (9.2.2) now become

$$F(0) = 1 \quad \text{and} \quad F(1) = 0 \qquad (9.2.12)$$

Equation (9.2.11) is a non-linear differential equation and it is difficult to obtain an exact solution. So we obtain an approximate solution by a successive approximation method. Assuming the non-Newtonian number $R \ll 1$, we expand F in the form

$$F = F_0 + RF_1 + R^2F_2 + \ldots \qquad (9.2.13)$$

Inserting (9.2.13) into (9.2.11) and equating the coefficients of different powers of R to zero, we get the following set of differential equations:

$$F_0'' + P = 0, \qquad (9.2.14)$$

$$F_1'' + 3F_0'^2 F_0'' = 0, \qquad (9.2.15)$$

$$F_2'' + 3F_0'^2 \cdot F_1'' + 6F_0'' F_0' F_1' = 0 \qquad (9.2.16)$$

and so on. The boundary conditions from (9.2.12) are now

$$\left.\begin{array}{l} F_o(0) = 1 \quad \text{and} \quad F_r(0) = 0 \quad \text{for } r \geqslant 1 \\ F_r(1) = 0 \quad \text{for } r \geqslant 0. \end{array}\right\} \tag{9.2.17}$$

Solving Eqs. (9.2.14) and (9.2.15) subject to boundary conditions (9.2.17), we get

$$F_o = (1 - \eta) + \frac{1}{2} P\eta (1 - \eta). \tag{9.2.18}$$

$$F_1 = P\left[\frac{1}{4} P^2 \eta^4 - \frac{1}{2} P^2 \eta^3 + P\eta^3 + \frac{3}{8} P^2 \eta^2 - \frac{3}{2} P \eta^2 + \frac{3}{2}\eta^2 - \left(\frac{1}{8} P^2 - \frac{1}{2} P + \frac{3}{2}\right)\eta\right]. \tag{9.2.19}$$

In this manner we get the expressions for F_2, F_3, . . . etc., but the algebra will be very complicated. Since we restrict our discussions to very small values of R, we can take

$$F = F_o + R F_1 + O(R^2). \tag{9.2.20}$$

The total flux per unit breadth across a plane perpendicu: to x is :

$$\int_0^h u \, dy = U h \int_0^1 F(\eta) \, d\eta$$

$$= Uh \left[\frac{1}{12} P + \frac{1}{2} - \frac{1}{4} RP \left(1 + \frac{P^2}{20}\right)\right] \tag{9.2.21}$$

The tangential stress at any point from (9.2.7), (9.2.10) and (9.2.20) is given by

$$\frac{h}{\alpha_0 U}\, p_{xy} = \frac{1}{2}\, P\, (1 - 2\eta\,) - 1 - \frac{1}{2}\, RP\,(\,3 + \frac{1}{2}\, P\,) \qquad (9.2.22)$$

which gives

$$\frac{\alpha_0 U}{h}\left[1 \pm \frac{1}{2}P - \frac{1}{2}RP\left(3 + \frac{1}{2}P\right)\right] \qquad (9.2.23)$$

as the drag per unit area on the boundaries. Having the velocity field to be (9.2.20), we can now study the problem on idealized plane slider bearing.

The term slider bearing is applied to two plane members one moving horizontally with uniform linear speed U, the other called shoe or pad, being stationary as shown in Figure. 9.1. The direction of motion and the inclination of planes are such that a converging oil film is formed between the surfaces, and the positive pressure that the developed in the oil film is capable of supporting a transverse load. The force F is applied to move the slider with velocity U, causing shearing stresses within the oil. Since the inclination of the plane faces is small and assuming that there is no side flow, (no flow in the z -direction), the equations developed in (9.2.20), (9.2.21) and (9.2.22) may now be applied to this bearing.

The condition of continuity demands that the total flux given in (9.2.21) must be independent of x. If h_0

is the value of h at points of maximum pressure, then from (9.2.21), we get

$$\int_0^{h_0} u\, dy = \frac{1}{2} U h_0 \tag{9.2.24}$$

since $P = 0$, that is, $\frac{dP}{dx} = 0$ at $h = h_0$. Since (9.2.21) is independent x, from this and (9.2.24), we have

$$3R P^3 + 20 P (3R-1) + 120 \left(\frac{h_0}{h} - 1\right) = 0 \tag{9.2.25}$$

This shows that P is a function of R. The value of P can be obtained by applying Cardan's method. But the solution by this method will not be in a useful form for subsequent use. Hence, we obtain an approximate value of P in the following manner. Since $R \ll 1$, let

$$P = P_0 + R P_1 \tag{9.2.26}$$

Inserting (9.2.26) into (9.2.25), and equating the coefficients, we have

$$P_0 = \frac{6(h_0 - h)}{h}$$

$$= 18 \frac{h_0 - h}{h} \left\{ 1 + \frac{9}{5} \left(\frac{h_0 - h}{h}\right)^2 \right\}$$

Hence, P can be written approximately as

$$P = \frac{6(h_0 - h)}{h} \left[1 + 3R \left\{ 1 + \frac{9}{5} \left(\frac{h_0 - h_1}{h}\right)^2 \right\} \right] \tag{9.2.27}$$

or

$$\frac{dp}{dx} = 6\alpha_0 U \frac{h-h_0}{h^3}\left[1+3R\left\{1+\frac{9}{5}\left(\frac{h_0-h}{h}\right)^2\right\}\right] \qquad (9.2.28)$$

But

$$h(b-a) = h_1\ (b-x) + h_2\ (x-a). \qquad (9.2.29)$$

Differentiation of Eq. (9.2.29) with respect to x gives

$$L\frac{dh}{dx} = h_2 - h_1\ , \qquad (9.2.30)$$

where $L = b-a$, the length of the bearing as shown in Fig. 9.1. Hence from equations (9.2.28) and (9.2.30), we have

$$\frac{dp}{dh} = \frac{6\alpha_0 UL}{h_2-h_1}\cdot\frac{h-h_0}{h^3}\left[1+3R\left\{1+\frac{9}{5}\left(\frac{h-h_0}{h}\right)^2\right\}\right],$$

which on integration gives

$$p = \frac{3\alpha_0 UL}{h_2-h_1}\left[\frac{h_0-2h}{h^2} + 3R\left\{\frac{h_0-2h}{h^2} + \frac{9}{20h^4}\left(h_0^3 - 4h_0^2 h + 6h_0h^2 - 4h^3\right)\right\} + C_1\right] \qquad (9.2.31)$$

The values of h_0 and C_1 can be determined from the conditions that

$$p = 0 \quad \text{when} \quad h = h_1 \quad \text{and} \quad h = h_2.$$

This gives the approximate values

$$h_0 = \frac{2h_1 h_2}{h_1 + h_2} \tag{9.2.32}$$

$$C_1 = \frac{2}{h_1 + h_2} + 3R\left[\frac{2}{h_1 + h_2} + \frac{9(h_1^2 + h_2^2)}{5(h_1 + h_2)^2}\right] \tag{9.2.33}$$

It is interesting to note that the point of maximum pressure remains unaffected due to the non-Newtonian nature of the fluid.

Load bearing capacity:-

Here we shall derive the load bearing capacity of the slider bearing, which is obtained by integrating the pressure over the area of the bearing. If we denote this by W, then

$$W = \int_a^b p\,dx \cdot B = \frac{BL}{h_2 - h_1}\int_{h_1}^{h_2} p\,dh$$

$$= \frac{6\alpha_0 U B L^2}{h_2^2} K_w \tag{9.2.34}$$

where B is the breadth of the bearing,

$$K_w = \frac{1}{(K-1)^2}\left[\log K - \frac{2(K-1)}{K+1} + R\left\{5.7 \log K\right.\right.$$

$$\left.\left. - 8.7\frac{(K-1)}{(K+1)} - 0.9 \cdot \frac{(K-1)(5K^2 + 2K + 5)}{(K+1)^3}\right\}\right]. \tag{9.2.35}$$

and

$$K = \frac{h_1}{h_2} \quad .$$

This K_w is a dimensionless quantity and may be called as 'load function'.

Frictional Resistance:-

The tangential stress on the moving plane from the equation (9.2.22) is

$$\left[p_{xy}\right]_{y=0} = \frac{\alpha_0 U}{h}\left[\tfrac{1}{2}P - 1 - \tfrac{1}{2}RP\left(3 + \tfrac{1}{2}P\right)\right] \qquad (9.2.36)$$

The total frictional resistance F_r can be derived by integrating the tangential stress over the area of the bearing and is given by

$$F_r = -\int_a^b \left(p_{xy}\right)_{y=0} \cdot B\,dx = \frac{\alpha_0 ULB}{h_2} K_F \qquad (9.2.37)$$

where

$$K_F = \frac{1}{K-1}\left[4 \log K - \frac{6(K-1)}{K+1} + 9R\left\{2.8 \log K - \frac{2(K-1)}{K+1} - 4.8 \frac{K^3-1}{(K+1)^3}\right\}\right]. \qquad (9.2.38)$$

Coefficient of Friction:-

The coefficient of friction f_c is the ratio between the frictional resistance F_r and the load W and is given by

$$f_c = \frac{F_r}{W} \,. \tag{9.2.39}$$

From Eqs. (9.2.34), (9.2.37) and (9.2.39), we have

$$f_c = \frac{h_2}{B} \times K_f \tag{9.2.40}$$

where

$$K_f = \frac{1}{6} \frac{K_F}{K_w} \tag{9.2.41}$$

9.3. CONCLUSIONS.

In this chapter, we have made a theoretical investigation of the performance of an idealized slider bearing with a non-Newtonian lubricant. The non-Newtonian nature of the lubricant is characterized by a non-dimensional number R. Lord Rayleigh's (1918) method has been used to study different aspects of the problem. The effects of the non-Newtonian nature of the lubricant on the pressure, load carrying capacity, frictional resistance and the coefficient of friction have been studied in detail.

Figure.9.2 shows that the pressure at each point of the bearing increases as the non-Newtonian parameter R increases.

Figure. 9.3 shows the effect of the non-Newtonian parameter on the load-carrying capacity of the bearing. An examination of this figure shows that the load carrying capacity of the bearing increases due to the presence of the non-Newtonian elements in the lubricant.

Figure. 9.4 represents the frictional resistance for different values of the non-Newtonian parameter R . This figure shows that the frictional resistance on the moving plane increases with the increase in the value of R .

Figure. 9.5 shows that frictional resistance per unit load decreases as the non-Newtonian nature of the liquid increases.

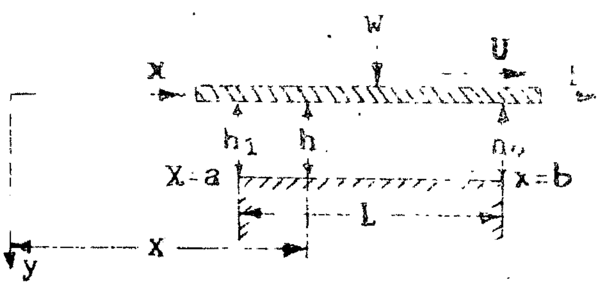

FIGURE: 9.1

SLIDER BEARING

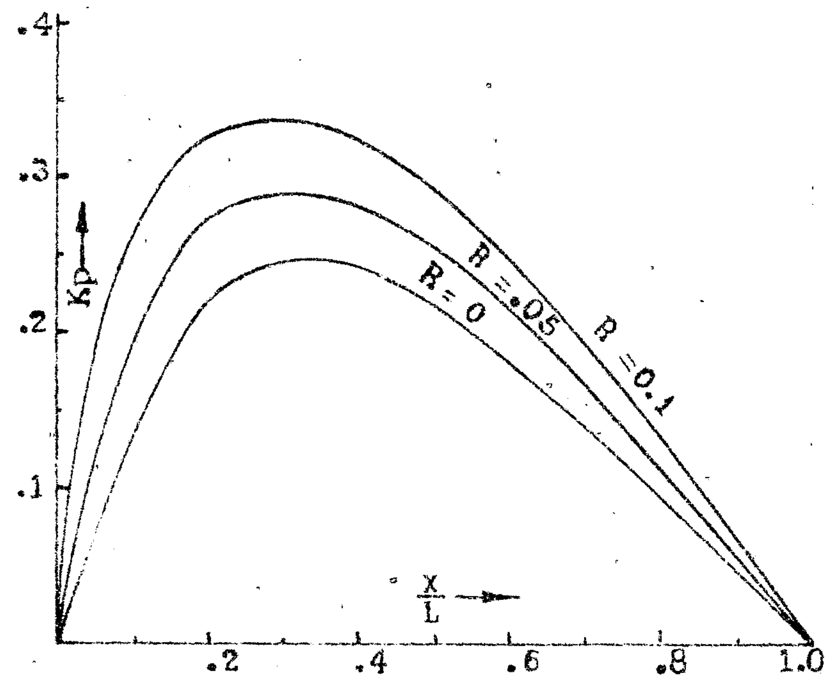

FIGURE: 9.2

PRESSURE DISTRIBUTION FOR DIFFERENT NON-NEWTONIAN PARAMETERS

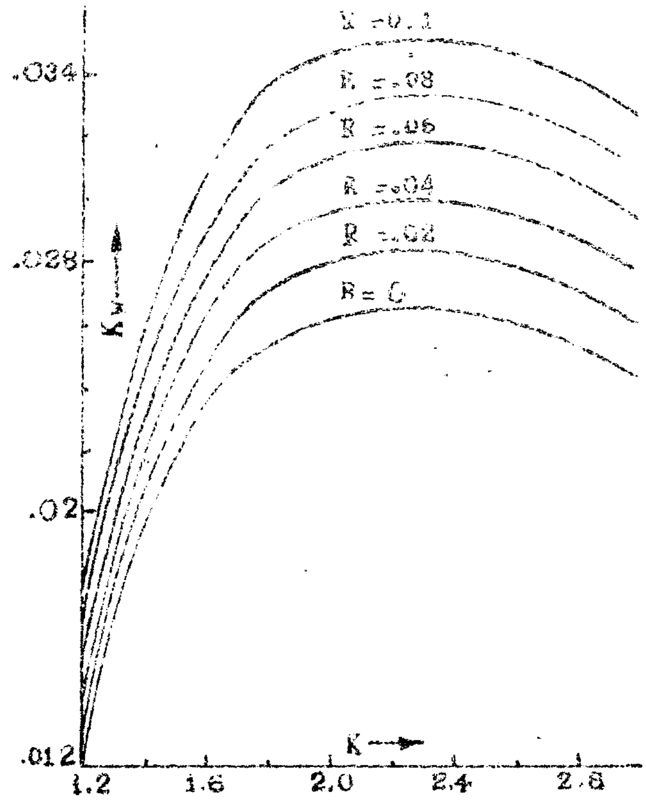

FIGURE: 9.3

LOAD CARRYING CAPACITY FOR DIFFERENT NON-NEWTONIAN PARAMETERS

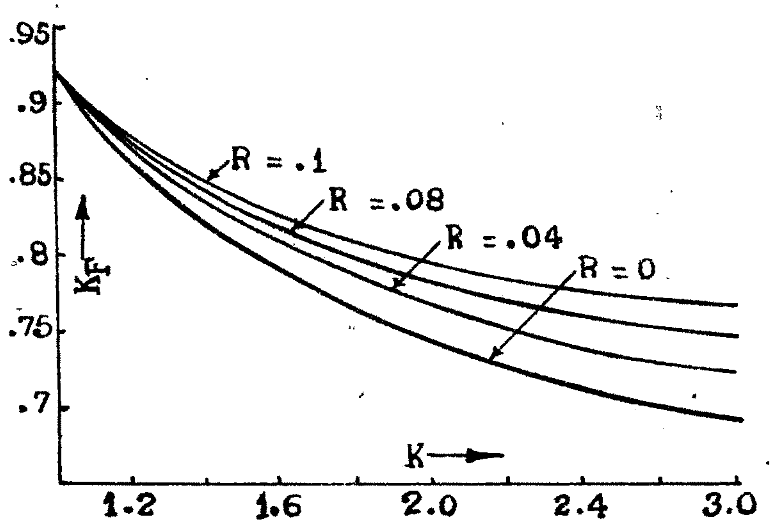

FIGURE: 9.4

EFFECT OF NON-NEWTONIAN PARAMETER ON FRICTIONAL RESISTANCE

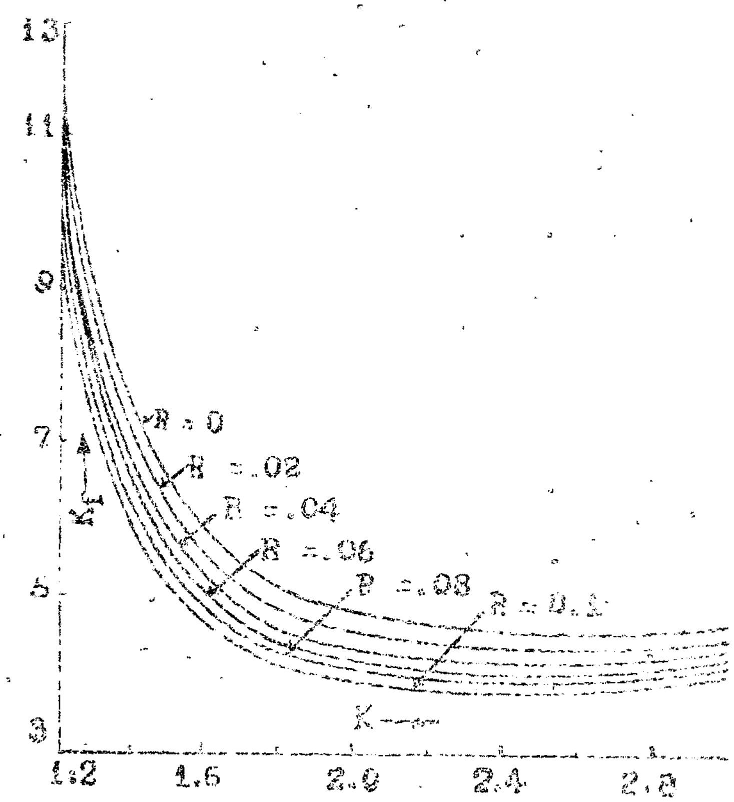

FIGURE: 9.5

VARIATION OF FRICTIONAL COEFFICIENT

FOR DIFFERENT NON-NEWTONIAN

PARAMETERS