CHAPTER. VIII.

FREE CONVECTION NON-NEWTONIAN FLOW WITH AND WITHOUT HEAT SOURCES IN A CIRCULAR PIPE

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8.1. INTRODUCTION.

The free convection heat transfer problem of a fully developed laminar flow between vertical heated plated has been investigated by Ostrach in a number of reports and papers in which he has included the effects due to frictional heating and distributed heat sources in the fluid. In the last chapter we have also studied the free convection flow of a non-Newtonian liquid between two parallel plates. Morton (1960) has studied the free convection flow of a viscous liquid in a vertical circular pipe and has given an exact solution for the problem when the pipe is heated or cooled uniformly. Tao(1960) has used an entirely different approach for the same problem. Both of them have neglected the frictional heating terms in the energy equation. This is not justified since the viscous dissipation and heat conduction terms are of the same order of magnitude. Nanda and Sharma (1962) have considered the fully developed free convection flow in a vertical circular pipe maintained at constant temperature, including the effects of frictional heating and distributed heat sources or sinks. In their problem they have taken the coefficient of viscosity to be a constant.

In this chapter our aim is to study the fully developed free convection flow and heat transfer of a particular type non-Newtonian liquid with and without heat sources or sinks in a circular cylinder, maintained at constant temperature. An iterative process has been used to obtain an approximate solution for the velocity and temperature distributions. The solution of the equations in the non-dimensional form depends on three parameters ζ (depending on the heat added by heat sources), K (characterising the buoyancy effects) and R (non-Newtonian parameter). The liquid model considered in this problem has been discussed in Section 1.4 [Eq. (1.4.10) - Eq.(1.4.20)] So the coefficients of viscosity μ and cross-viscosity μ_c are arbitrary scalar functions of the flow invariants I_1, I_2 and I_3. The equations of motion, continuity and heat energy are respectively

$$\rho\left[\frac{\partial v^i}{\partial t} + v^j v^i_{,j}\right] = -p_{,i} + p^{ij}_{,j} + \rho F^i . \qquad (8.1.1)$$

$$I_1 = v_{i,i} = 0 , \qquad (8.1.2)$$

and

$$\rho c \frac{DT}{Dt} = k \nabla^2 T + \Phi + Q. \qquad (8.1.3)$$

where ρ is the density of the medium , p is pressure, T is temperature, C and k are respectively the specific heat capacity and thermal conductivity with

$$\Phi = p_{ij} \cdot d_{ij} \ ,$$

and

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} + \frac{v}{r} \frac{\partial}{\partial \theta} + w \frac{\partial}{\partial z}$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

are respectively dissipation function, material derivative and Laplace operator in cylindrical polar coordinates , Q is a constant which denotes the heat added due to heat sources, F_i is the body force in the ith direction on unit mass of the liquid.

8.2. FORMULATION OF THE PROBLEM.

We consider fully developed laminar free convection flow of the non-Newtonian incompressible liquid in a long circular pipe. We use cylindrical polar coordinate system (r, ϕ z). The velocity field compatible with the continuity equation (8.1.2) can be taken as

$$u = 0, \qquad v = 0 ; \qquad w = w(r) \qquad (8.2.1)$$

The surviving stress components are

$$\left.\begin{aligned} p_{rz} &= \mu\left(\frac{dw}{dr}\right) \\ p_{zz} &= \mu_c\left(\frac{dw}{dr}\right)^2 \end{aligned}\right\} \qquad (8.2.2)$$

In this problem F_i has only one component

$$F_r = 0 \quad ; \quad F_0 = 0 \; ; \quad F_z = f_z \qquad (8.2.3)$$

The dissipation function Φ reduces to

$$\Phi = \mu\left(\frac{dw}{dr}\right)^2 \qquad (8.2.4)$$

The equations of motion (8.1.1) give

$$0 = -\frac{\partial p}{\partial r} + \frac{\partial p_{rr}}{\partial r} + \frac{1}{r} p_{rr} \qquad (8.2.5)$$

$$0 = -\frac{\partial p}{\partial z} + \rho f_z + \frac{d p_{rz}}{dr} + \frac{1}{r} p_{rz} \qquad (8.2.6)$$

The energy equation (8.1.3) gives

$$0 = k\left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right] + \mu\left(\frac{dw}{dr}\right)^2 + Q \, . \qquad (8.2.7)$$

In equations (8.2.2), (8.2.4) and (8.2.7) the coefficient of viscosity is a function of I_1, I_2, and I_3. In this problem

$$I_1 = v_{i,i} = 0 \quad \text{from (8.1.2)}$$

$$I_3 = \det(e_{ik})$$

$$= \begin{vmatrix} d_{rr} & d_{r\theta} & d_{rz} \\ d_{\theta r} & d_{\theta\theta} & d_{\theta z} \\ d_{zr} & d_{z\theta} & d_{zz} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & \frac{1}{2}\frac{dw}{dr} \\ 0 & 0 & 0 \\ \frac{1}{2}\frac{dw}{dr} & 0 & 0 \end{vmatrix} = 0.$$

Hence μ is a function of I_2 only where

$$I_2 = \frac{1}{2}\left[-e_{ik}e_{ik} + (e_{ii})^2\right]$$

$$= -\frac{1}{2}\left[2\, e_{rz}\, e_{rz}\right] = -\frac{1}{4}\left(\frac{dw}{dr}\right)^2.$$

We confine our discussions to the particular class of liquids characterized by

$$\mu = \mu_0\left[1 - \alpha\frac{dw}{dr}\right] \qquad (8.2.8)$$

Substituting (8.2.8) into (8.2.2) and (8.2.4) we get

$$p_{rz} = \mu_0 \left[1 - \alpha \frac{dw}{dr} \right] \frac{dw}{dr} \tag{8.2.9}$$

$$\Phi = \mu_0 \left[1 - \alpha \frac{dw}{dr} \right] \left(\frac{dw}{dr} \right)^2 \tag{8.2.10}$$

Substituting (8.2.9) into (8.2.6) we get

$$\frac{\partial p}{\partial z} = \mu_0 \frac{d}{dr} \left[\frac{dw}{dr} - \alpha \left(\frac{dw}{dr} \right)^2 \right] + \frac{\mu_0}{r} \left[\frac{dw}{dr} - \alpha \left(\frac{dw}{dr} \right)^2 \right] + \rho f_z \tag{8.2.11}$$

and (8.2.5) gives

$$\frac{\partial p}{\partial r} = 0 \tag{8.2.12}$$

Equation (8.2.7) gives

$$k \left[\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} \right] + \mu_0 \left[1 - \alpha \frac{dw}{dr} \right] \left(\frac{dw}{dr} \right)^2 + Q = 0 \tag{8.2.13}$$

The boundary conditions to be satisfied on the velocit and temperature fields.

$$\left. \begin{array}{l} w = 0 \quad \text{and} \quad T = T_w \quad \text{when} \quad r = a \\ w \quad \text{and} \quad T \quad \text{are finite when} \quad r = 0 \end{array} \right\} \tag{8.2.14}$$

We shall now express the body force term as a buoyancy term. This can be done by considering the hydrostatic state. If the subscript ' s ' refers to the hydrostatic condition when the wall and the liquid are at the same temperature and there is no flow, then (8.2.11) gives:

$$\rho_s f_z - \frac{\partial p_s}{\partial z} = 0. \qquad (8.2.15)$$

Now

$$\rho f_z - \frac{\partial p}{\partial z} = (\rho - \rho_s) f_z + \rho_s f_z - \frac{\partial p_s}{\partial z} - \frac{\partial p_D}{\partial z}$$

$$= (\rho - \rho_s) f_z - \frac{\partial p_D}{\partial z} \qquad (8.2.16)$$

where $p_D = p - p_s$. If we confine ourselves to the case of small temperature differences the coefficient of volumetric expansion β can be written as

$$\rho - \rho_s = \rho\beta (T_s - T) \qquad (8.2.17)$$

Thus, the buoyancy term in (8.2.16) can be written as

$$(\rho - \rho_s) f_z = -\rho\beta f_z (T - T_s) = -\beta\rho f_z \theta \qquad (8.2.18)$$

where $\theta = T - T_s$

Now from (8.2.11), (8.2.16) and (8.2.18) we get

$$\mu_0\left[\frac{d^2\varpi}{dr^2}+\frac{1}{r}\frac{d\varpi}{dr}\right]-\mu_0\alpha\left[\frac{d}{dr}\left(\frac{d\varpi}{dr}\right)^2+\frac{1}{r}\left(\frac{d\varpi}{dr}\right)^2\right]$$

$$-\rho\beta f_z\theta-\frac{\partial p_D}{\partial z}=0 \qquad (8.2.19)$$

Equation (8.2.12) shows that p is a function of z alone, but ϖ and θ are functions of r alone.

Hence, from (8.2.19), we have

$$\frac{\partial p_D}{\partial z}=f(r),\text{ a function of } r. \qquad (8.2.20)$$

As $p_D = p - p_S$, we have from (8.2.20),

$$\frac{\partial p}{\partial z}-\frac{\partial p_S}{\partial z}=f(r). \qquad (8.2.21).$$

Differentiating (8.2.21) with respect to r, we get

$$\frac{\partial^2 p}{\partial r\,\partial z}-\frac{\partial^2 p_S}{\partial r\,\partial z}=f'(r) \qquad (8.2.22).$$

But from (8.2.12) and (8.2.22), we have

$$\frac{\partial^2 p_S}{\partial r\,\partial z}=-f'(r)\ ,$$

which on integration with respect to 'r' gives

$$\frac{\partial p_S}{\partial z} = A_1 - f(r) \tag{8.2.23}$$

where A_1 is a constant. Adding (8.2.20) and (8.2.23) we have

$$\frac{\partial p_D}{\partial z} + \frac{\partial p_S}{\partial z} = A_1 \tag{8.2.24}$$

From (8.2.15) and (8.2.24)

$$\frac{\partial p_D}{\partial z} + \rho_s f_z = A_1 \tag{8.2.25}$$

Since $\rho_s f_z$ is a constant, from (8.2.20) and (8.2.25) we have

$$\frac{\partial p_D}{\partial z} \text{ is a constant.}$$

Hence from (8.2.21), it follows that the pressure gradient $\frac{\partial p_S}{\partial z}$ differs from $\frac{\partial p}{\partial z}$ by at most a constant. But, since the pressure difference required to accelerate the fluid from the equilibrium state to fully developed state and the pressure difference to decelerate it back to the hydrostatic state must be finite, and the pipe being long enough, it follows that the pressure gradient in the pipe must be equal to the hydrostatic pressure gradient. Utilising this fact (8.2.19) can be written as

$$\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - \alpha\left[\frac{d}{dr}\left(\frac{dw}{dr}\right)^2 + \frac{1}{r}\left(\frac{dw}{dr}\right)^2\right] + \rho\beta f\theta = 0, \quad (8.2.20)$$

where $f = -f_z$.

We now introduce the dimensionless variables

$$\left.\begin{aligned} w^* &= \frac{wK}{W} \; ; \quad \tau = \frac{\theta K}{\theta_w} \; ; \quad \eta = \frac{r}{a}, \\ K &= \frac{f^2\beta^2\rho^2 a^4\theta_w}{k\mu_0} \; ; \quad W = \frac{f\beta a^2\theta_w}{\gamma_0}, \\ \zeta &= \frac{Qa^2}{k\theta_w} \; ; \quad R = \frac{\alpha W}{Ka}. \end{aligned}\right\} \quad (8.2.21)$$

From Equations (8.2.20), (8.2.13) and the transformations (8.2.21) we get

$$\frac{d^2 w^*}{d\eta^2} + \frac{1}{\eta}\frac{dw^*}{d\eta} - R\left[2\frac{dw^*}{d\eta}\frac{d^2 w^*}{d\eta^2} + \frac{1}{\eta}\left(\frac{dw^*}{d\eta}\right)^2\right] + \tau = 0 \quad (8.2.22)$$

$$\frac{d^2\tau}{d\eta^2} + \frac{1}{\eta}\frac{d\tau}{d\eta} + \left(\frac{dw^*}{d\eta}\right)^2 - R\left(\frac{dw^*}{d\eta}\right)^3 + \zeta K = 0, \quad (8.2.23)$$

The boundary conditions from (8.2.14) are

$$\left.\begin{aligned} \eta = 0 &: \quad w^* \text{ and } \tau \text{ are finite} \\ \eta = 1 &: \quad w^* = 0 \text{ and } \tau = K \end{aligned}\right\} \quad (8.2.24)$$

We can eliminate τ between (8.2.22) and (8.2.23) and can get an equation in w^* only. But the resulting equation will be highly complicated and it is also not possible to obtain a close-form solution for w^*.

8.3. SOLUTION OF EQUATIONS

To solve the equations (8.2.22) and (8.2.23) we shall define an iterative procedure, following Collatz (1959), which determines a sequence of functions $w_0^*, w_1^*, \cdots$ and $\tau_0, \tau_1, \cdots$ in the following manner:

$$w_n^{*\prime\prime} + \frac{1}{\eta} w_n^{*\prime} - R\left[2 w_{n-1}^{*\prime} w_{n-1}^{*\prime\prime} + \frac{1}{\eta}\left(w_{n-1}^{*\prime}\right)^2\right] + \tau_n = 0, \qquad (8.3.1)$$

$$\tau_n'' + \frac{1}{\eta}\tau_n' + \left(w_{n-1}^{*\prime}\right)^2 - R\left(w_{n-1}^{*\prime}\right)^3 + \zeta K = 0, \qquad (8.3.2)$$

From the boundary conditions (8.2.23), we have

$$\left.\begin{aligned} &w_n^*(1) = 0 ; \quad w_n^*(0) = \text{Finite} \\ &\tau_n(1) = K ; \quad \tau_n(0) = \text{Finite} \\ &\text{with} \quad w_{-1}^{*\prime} = 0 = w_{-1}^{*\prime\prime} . \end{aligned}\right\} \qquad (8.3.3)$$

The zeroth order iterative equations are given by

$$\overset{*}{w}_0{}'' + \frac{1}{\eta}\overset{*}{w}_0{}' + \tau_0 = 0 \,. \tag{8.3.4}$$

$$\tau_0'' + \frac{1}{\eta}\tau_0' + \zeta K = 0 \,. \tag{8.3.5}$$

Solving (8.3.5) with the boundary conditions (8.3.3) we get

$$\tau_0 = \frac{1}{4}K\left[\zeta + 4\right] - \frac{1}{4}\zeta K \eta^2 \,. \tag{8.3.6}$$

Substituting the value of τ_0 in (8.3.4) and integrating, we get

$$\overset{*}{w}_0 = K\left[\frac{1}{4} + \frac{3\zeta}{64} - \frac{1}{16}(\zeta + 4)\eta^2 + \frac{1}{64}\zeta\eta^4\right] \,. \tag{8.3.7}$$

This is the first approximation when we neglect the frictional heating term in (8.2.23) and the terms due to non-Newtonian character of the fluid.

The first order iterative equations are given by

$$\overset{*}{w}_1{}'' + \frac{1}{\eta}\overset{*}{w}_1{}' - R\left[2\overset{*}{w}_0{}'\overset{*}{w}_0{}'' + \frac{1}{\eta}\left(\overset{*}{w}_0{}'\right)^2\right] + \tau_1 = 0 \,. \tag{8.3.8}$$

and

$$\tau_1'' + \frac{1}{\eta}\tau_1' + \left(\overset{*}{w}_0{}'\right)^2 - R\left(\overset{*}{w}_0{}'\right)^3 + \zeta K = 0 \,. \tag{8.3.9}$$

Substituting the value of $\overset{*}{w}_0$ from (8.3.7) into (8.3.9) and integrating with respect to η, subject to the boundary conditions (8.3.3) we get

$$\tau_1 = B_1 + RK^3 \left[\frac{\zeta^3 \eta^{11}}{495616} - \frac{(3\zeta^3 + 12\zeta^2)}{165888}\eta^9 + \frac{3\zeta(4+\zeta)^2\eta^7}{50176} - \frac{(4+\zeta)^3}{12800}\eta^5 \right] - K^2 \left[\frac{\zeta^2\eta^8}{16384} - \frac{(\zeta^2+4\zeta)}{2304}\eta^6 + \frac{(4+\zeta)^2}{1024}\eta^4 \right] - \frac{\zeta K}{4}\eta^2 \qquad (8.3.10)$$

where

$$B_1 = \frac{1}{4} K (4+\zeta) + K^2 \left[\frac{89\zeta^2}{147456} + \frac{7\zeta}{1152} + \frac{1}{64} \right] + RK^3 \left[\frac{1}{200} + \frac{7008\zeta}{2508800} + \frac{9001\zeta^2}{16934400} + \frac{563939\zeta^3}{16392499200} \right]. \qquad (8.3.11)$$

Substituting the values of τ_1 and $\overset{*}{w}_0$ from (8.3.10) and (8.3.7) and integrating with repsect to η and using the boundary conditions (8.3.3) we get

$$w_1^* = -\frac{1}{4} B_1 \eta^2 + \frac{\zeta K}{64} \eta^4 + K^2 \left\{ \frac{\zeta^2 \eta^{10}}{1638400} - \frac{(\zeta^2 + 4\zeta)\eta^8}{147456} + \frac{(4+\zeta)^2}{36864} \eta^6 \right\} + RK^2 \left\{ \frac{(4+\zeta)^2}{192} \eta^3 - \frac{\zeta(4+\zeta)}{320} \eta^5 + \frac{\zeta^2 \eta^7}{1792} \right\} - RK^3 \left\{ \frac{\zeta^3 \eta^{13}}{83759104} - \frac{(\zeta^3 + 4\zeta^2)\eta^{11}}{6690816} + \frac{\zeta(4+\zeta)^2}{1354752} \eta^9 - \frac{(4+\zeta)^3 \eta^7}{627200} \right\} + B_2 \qquad (8.3.12)$$

where

$$B_2 = \frac{1}{4} B_1 - \frac{\zeta K}{64} - K^2 \left\{ \frac{103\zeta^2}{4915200} + \frac{7\zeta}{36864} + \frac{1}{2304} \right\} - RK^2 \left\{ \frac{71}{26880} \zeta^2 + \frac{7\zeta}{240} + \frac{1}{12} \right\} - RK^3 \left\{ \frac{2753059\,\zeta^3}{2770332364800} + \frac{28329\,\zeta^2}{2049062400} + \frac{137\,\zeta}{2116800} + \frac{1}{9800} \right\} \qquad (8.3.13)$$

The solution can be carried to higher order of approximations, but the algebraic effort involved becomes very great. It is clear from (8.3.10) and (8.3.12) that the solution turns out to be a series in ascending powers of K and ζ, consequently it will give reliable results only for small values of K and ζ.

It is of some physical interest to calculate the rate of heat transfer through the pipe walls to the fluid per unit area of the pipe surface. It is given by

$$q = -k\left(\frac{\partial T}{\partial r}\right)_{r=a} = -\frac{k\,\theta_w}{aK}\left(\frac{\partial T}{\partial \eta}\right)_{\eta=1}$$

$$= \frac{k\,\theta_w}{aK}\Big[\frac{1}{2}\zeta K + K^2\left\{\frac{11}{6144}\zeta^2 + \frac{1}{48}\zeta + \frac{1}{16}\right\}$$

$$+ RK^3\left\{\frac{533}{4730880}\zeta^3 + \frac{107}{53760}\zeta^2 + \frac{27}{2240}\zeta + \frac{1}{40}\right\}\Big] \qquad (8.3.14)$$

It may also be expressed in terms of the Nusselt number as

$$N_u = \frac{q\,a}{k\theta_w} = \frac{1}{2}\zeta + K\left\{\frac{11}{6144}\zeta^2 + \frac{1}{48}\zeta + \frac{1}{16}\right\}$$

$$+ RK^2\left\{\frac{533}{4730880}\zeta^3 + \frac{107}{53760}\zeta^2 + \frac{27\zeta}{2240} + \frac{1}{40}\right\}, \qquad (8.3.15)$$

From (8.2.2), the stress component p_{rz} is

$$p_z^r = \mu\left(\frac{dw}{dr}\right)$$

$$= \mu_0\left[\frac{dw}{dr} - \alpha\left(\frac{dw}{dr}\right)^2\right]. \qquad (8.3.16)$$

With the help of transformations (8.2.21), the relation (8.3.16) reduces to

$$\frac{Ka}{\mu_0 W} p_z^r = \frac{dw^*}{d\eta} - R\left(\frac{dw^*}{d\eta}\right)^2$$

$$= \frac{dw_1^*}{d\eta} - R\left(\frac{dw_1^*}{d\eta}\right)^2 \qquad (8.3.17)$$

taking the first iterate of w^* in (8.3.17). Substituting the expression for w_1^* in (8.3.17) from (8.3.12), we have shear stress at the wall

$$\tau_w = -\frac{Ka}{\mu_0 W} p_z^r\Big]_{\eta=1} = -\left[\frac{dw_1^*}{d\eta}\right]_{\eta=1} + R\left[\frac{dw_1^*}{d\eta}\right]^2 .$$

where

$$\left[\frac{dw_1^*}{d\eta}\right]_{\eta=1} = -k\left(\frac{1}{2} + \frac{\zeta}{16}\right) + K^2\left\{\frac{26893\zeta^2}{31850496} - \frac{15925248}{8153726976}\zeta\right.$$

$$\left. - \frac{1}{192}\right\} + RK^2\left\{\frac{\zeta^2}{256} + \frac{\zeta}{16} + \frac{1}{4}\right\} - RK^3\left\{\frac{8834211727\zeta^3}{7848138486\,98880}\right.$$

$$+ \frac{506611\,\zeta^2}{2839710720} - \frac{637\,\zeta}{658560} + \frac{49}{20440} \Big\} .$$

8.4. CONCLUSIONS.

Fully developed free convection flow and heat transfer of a non-Newtonian fluid with and without heat sources or sinks in a circular pipe, maintained at constant temperature, has been studied in this chapter. The effects of frictional heating term in the energy equation has been taken into account. The flow phenomenon has been characterized by the non-dimensional parameters R (non-Newtonian parameter) K (characterising the buoyancy effects), and ζ (depending on the heat added by heat sources) and the effects of these numbers on the velocity and temperature fields, rate-of heat transfer from the surface of the cylinder and the shearing stress at the cylinder have been studied in detail.

A. VELOCITY FIELD:- Figure. 8.1 shows the effect of the non-Newtonian parameter R on the velocity field. An examination of this figure shows that for a fixed value of ζ and K , the velocity at any point decreases as R increases.

Figures. 8.2 and 8.3 exhibit the influence of the heat source parameter ζ . This figure shows that for a fixed value of R and K , the velocity at any point increases as ζ increases.

Figure 8.4 represents the effect of the parameter K (characterising the buoyancy parameter) on the velocity field. As obvious, the velocity at any point increases with the increase of K . Figure 8.5 also gives the same result.

B. TEMPERATURE FIELD:-

Figure 8.6 shows the effect of the non-Newtonian parameter R on the temperature distribution. This figure shows that the temperature at any point increases with the increase of R .

Figure. 8.7 exhibits the effect of the heat source parameter ζ on the temperature field. An examination of this figure gives that the temperature at any point increases with the increase in the value of ζ . The effect of the non-Newtonian parameter is to increase the temperature as is seen in Fig. 8.6.

Figures 8.8 and 8.9 show the effect of the parameter K ,characterising the buoyancy effects. Figure 8.8 shows that when there is no heat sources,th temperature profiles are straight lines for a viscous liquid (R = 0) and the temperature at any point increases with the increase in the value of K . An examination of Fig. 8.9 shows that the temperature distribution is no more a straight line as in the case when ζ = 0, that is , there are no heat sources in the pipe , but the temperature gradually falls as the wall is approached (when there are heat sources in the pipe). A comparison of the Figures 8.8 and 8.9 shows that the effect of the non-Newtonian parameter is to increase the temperature at any point in the pipe.

C. SHEARING STRESS AT THE WALL:-

Table. 8.1 presents the numerically computed values of the skin-friction at the wall. This table shows that the skin-friction decreases with the increase in the value of the non-Newtonian parameter. The shearing stress increases with the increase in the value of K , the parameter characterizing the buoyant effect. The table also shows that the skin-friction increases with the increase in the value of ζ , the heat source parameter

D. NUSSELT NUMBER.

Table. 8.2 presents the computed values of the Nusselt number which is the rate of heat transfer from the cylinder. This table shows that the non-Newtonian character of the liquid increases the Nusselt number. The Nusselt number increases with the increase in the value of the parameter K. This table also shows that the rate at which the heat is transferred from the cylinder increases with the increase in the value of ζ.

Table . 81

SKIN FRICTION

ζ \ K	0	0.1	0.2	R
-0.2	0.00000	+0.04879	+0.09769	0
	0.00000	+0.04874	+0.09726	0.5
	0.00000	+0.04857	+0.09597	1.0
0	0.00000	+0.05005	+0.10020	0
	0.00000	+0.04999	+0.09974	0.5
	0.00000	+0.04981	+0.09836	1.0
0.2	0.00000	+0.05130	+0.10272	0
	0.00000	+0.05124	+0.10223	0.5
	0.00000	+0.05105	+0.10074	1.0
0.4	0.00000	+0.05256	+0.10524	0
	0.00000	+0.05249	+0.10471	0.5
	0.00000	+0.05228	+0.10311	1.0

Table. 8.2

Nusselt Number.

ζ \ K	0.0	0.1	0.2	R
-0.2	-0.1000	-0.0941	-0.08831	0
	-0.1000	-0.0940	-0.0878	0.5
	-0.1000	-0.0939	-0.0874	1.0
0.0	0.0000	0.0062	0.0125	0
	0.0000	0.0063	0.0130	0.5
	0.0000	0.0065	0.0134	1.0
0.2	0.1000	0.1066	0.1133	0
	0.1000	0.1068	0.1138	0.5
	0.1000	0.1069	0.1144	1.0
0.4	0.2000	0.2071	0.2142	0
	0.2000	0.2072	0.2148	0.5
	0.2000	0.2074	0.2154	1.0

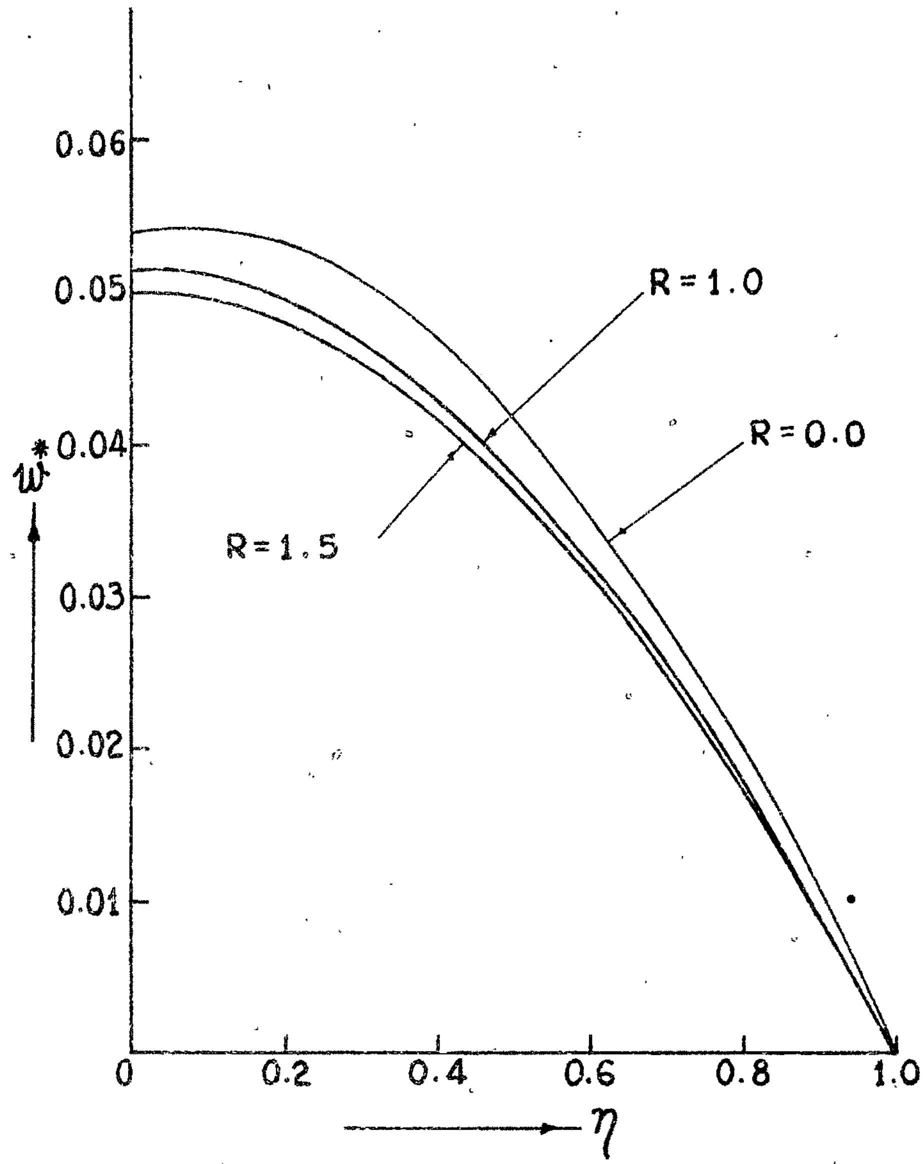

FIGURE: 8.1

VELOCITY DISTRIBUTION FOR DIFFERENT VALUES OF NON-NEWTONIAN PARAMETER R

ξ = 0.5 and K = 0.2

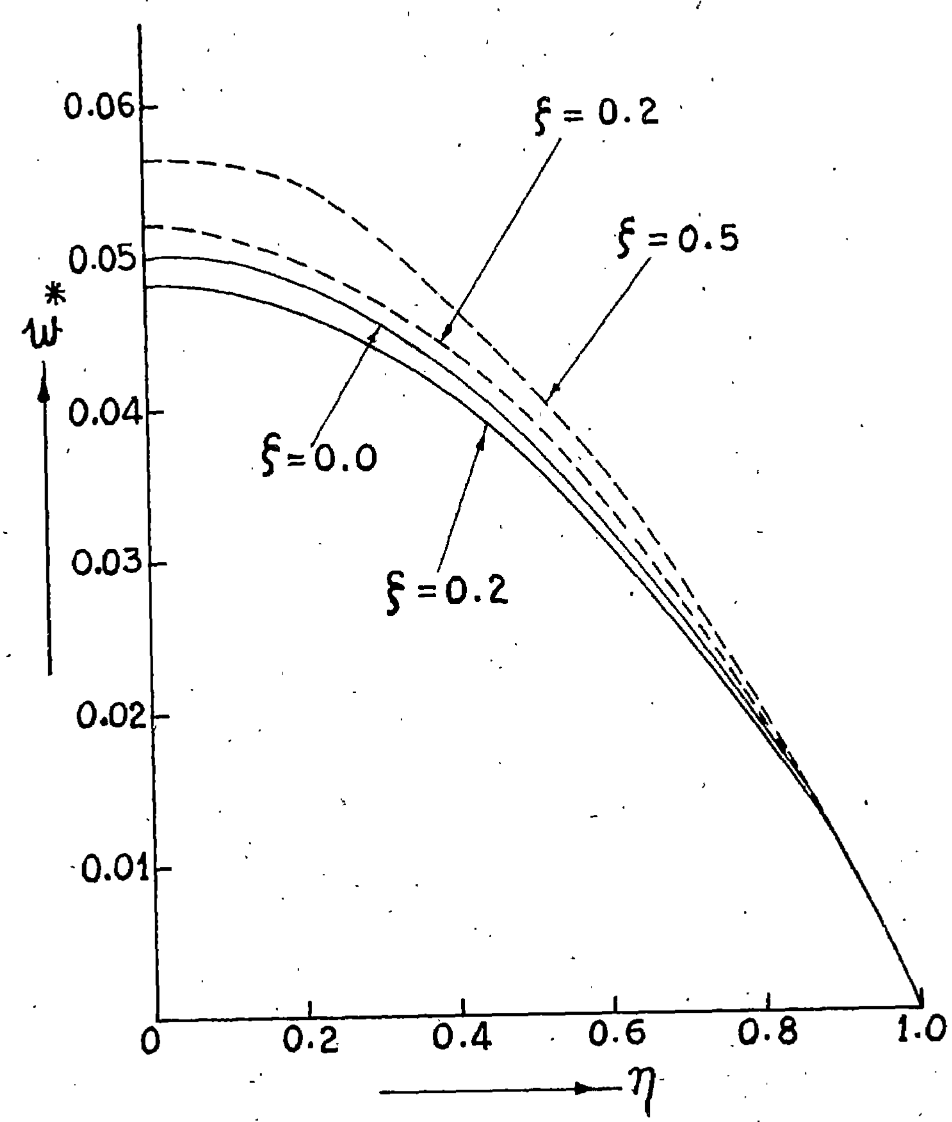

FIGURE: 8.2

VELOCITY DISTRIBUTION FOR DIFFERENT VALUES OF THE HEAT SOURCE PARAMETER

R = 0.0 and K = 0.2

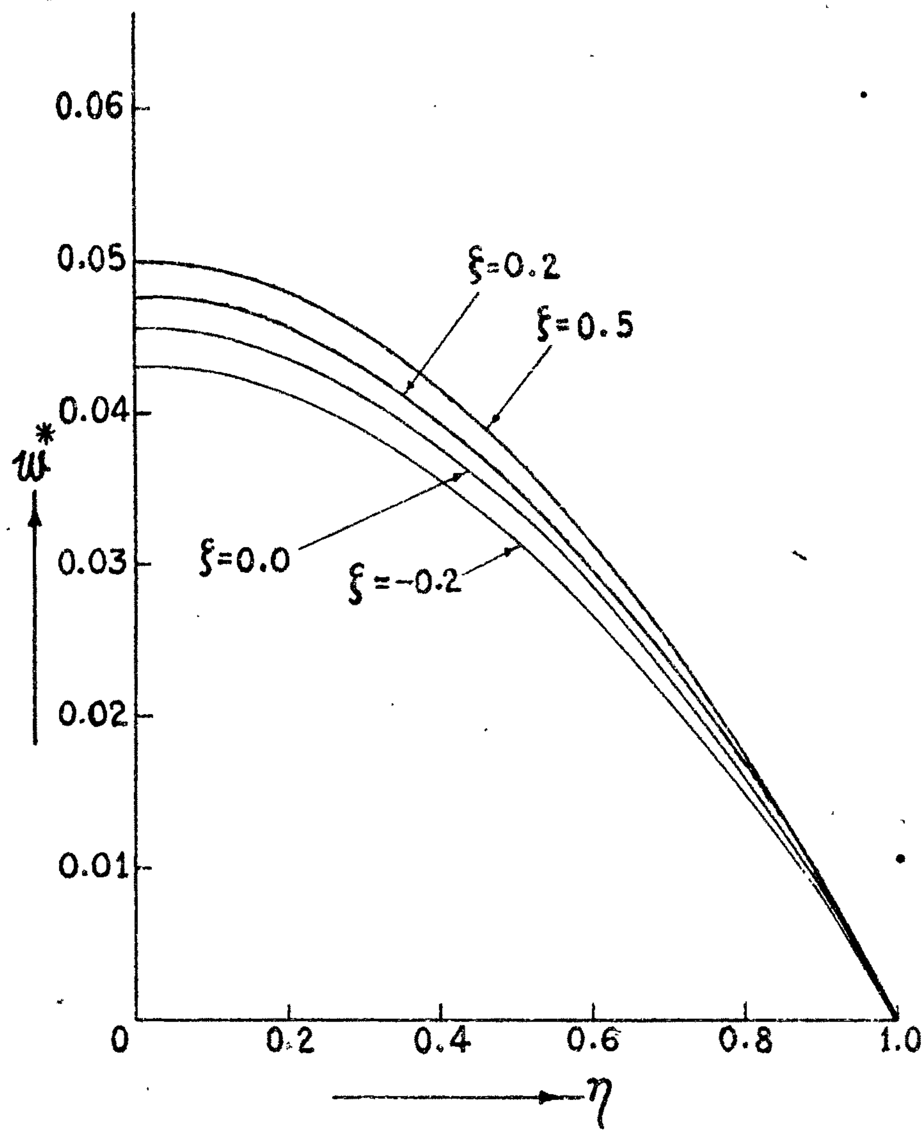

FIGURE: 8.3

VELOCITY DISTRIBUTION FOR DIFFERENT VALUES OF THE HEAT SOURCE PARAMETER

R = 0.5 and K = 0.2

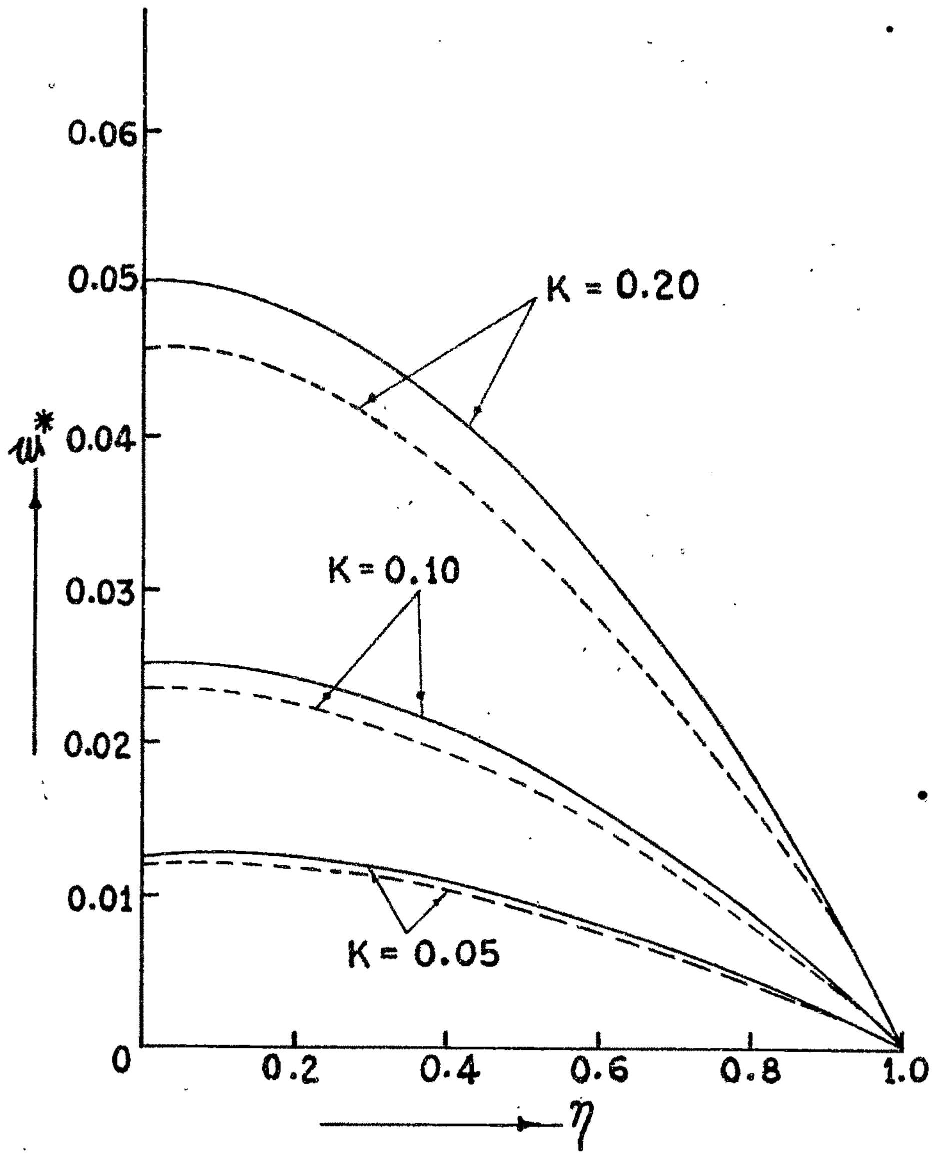

FIGURE: 8.4

VELOCITY DISTRIBUTION FOR DIFFERENT VALUES OF THE BUOYANCY PARAMETER K

$\xi = 0.0$

R = 0.0 ———— , R = 1.5 - - - - -

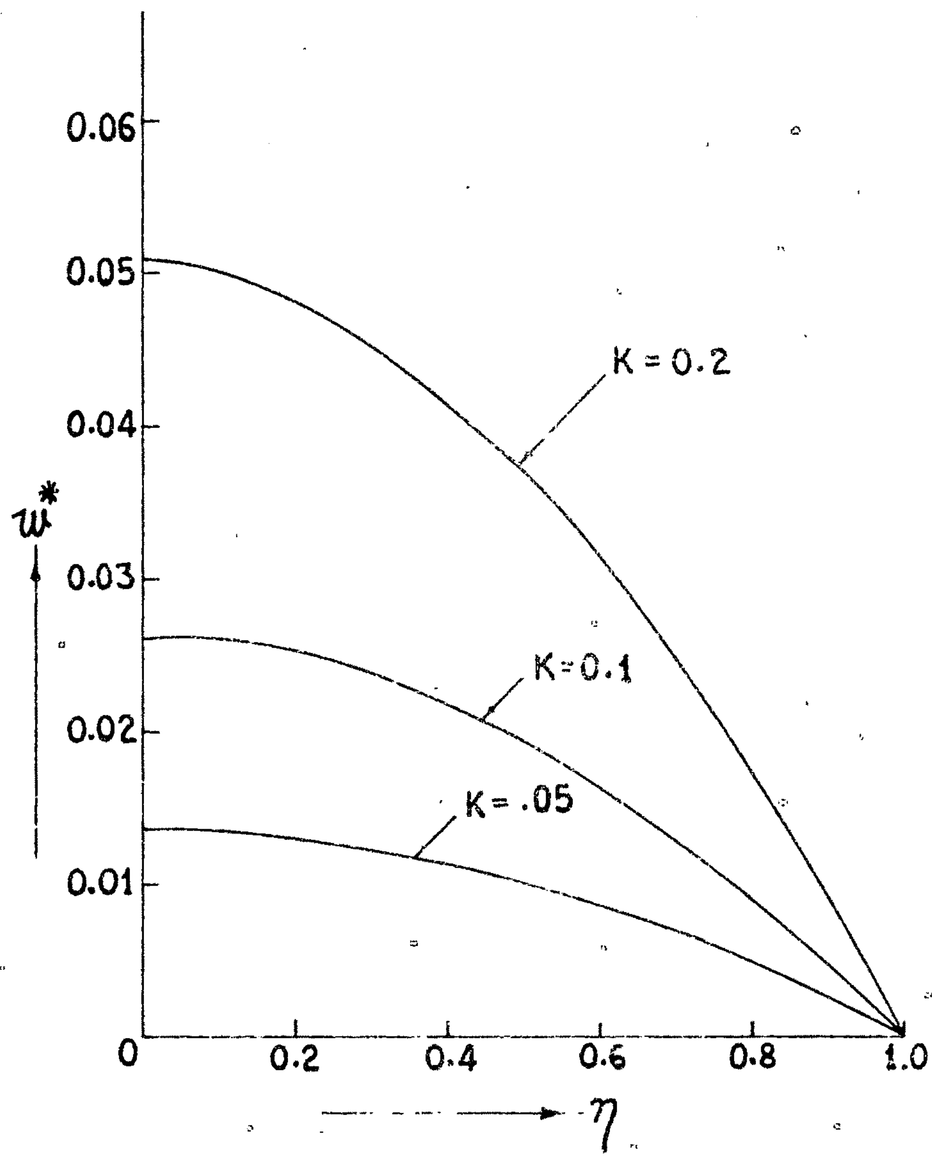

FIGURE: 8.5

VELOCITY DISTRIBUTION FOR DIFFERENT VALUES OF THE BUOYANCY PARAMETER K

ξ = 0.5 and R = 1.5

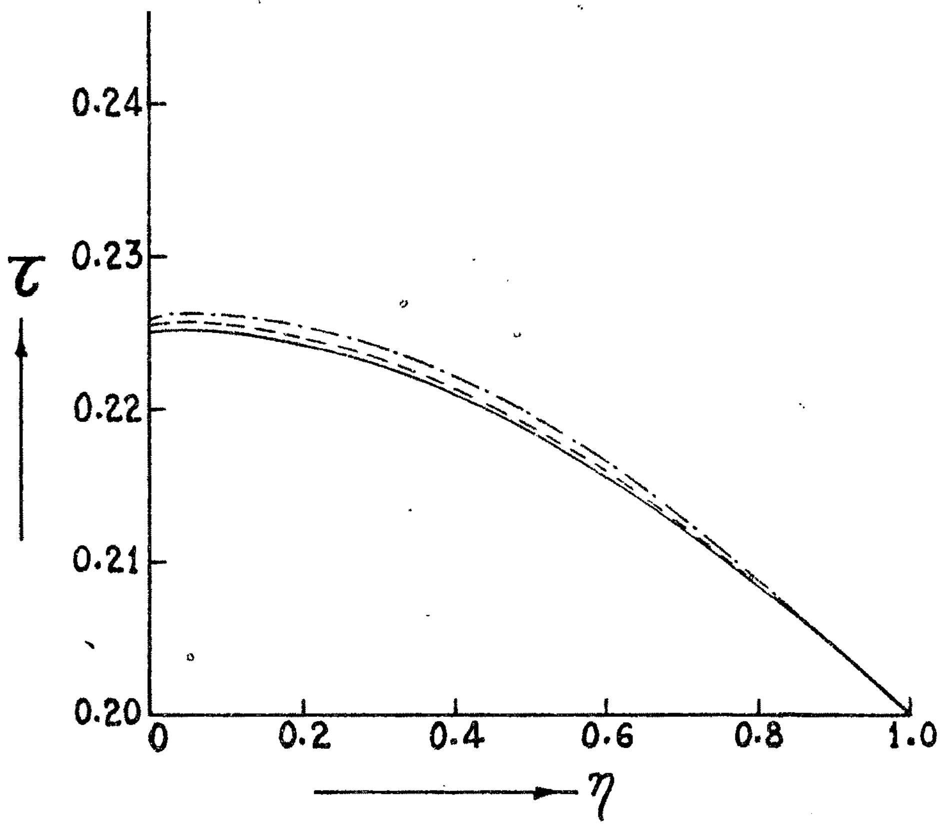

FIGURE: 8.6

TEMPERATURE DISTRIBUTION FOR DIFFERENCE VALUES OF THE NON-NEWTONIAN PARAMETER

$\mathcal{S}$ = 0.2 and K = 0.2

R = 0.0 ———— , R = 1.0 – – – – –

R = 1.5 –·–·–·–·–

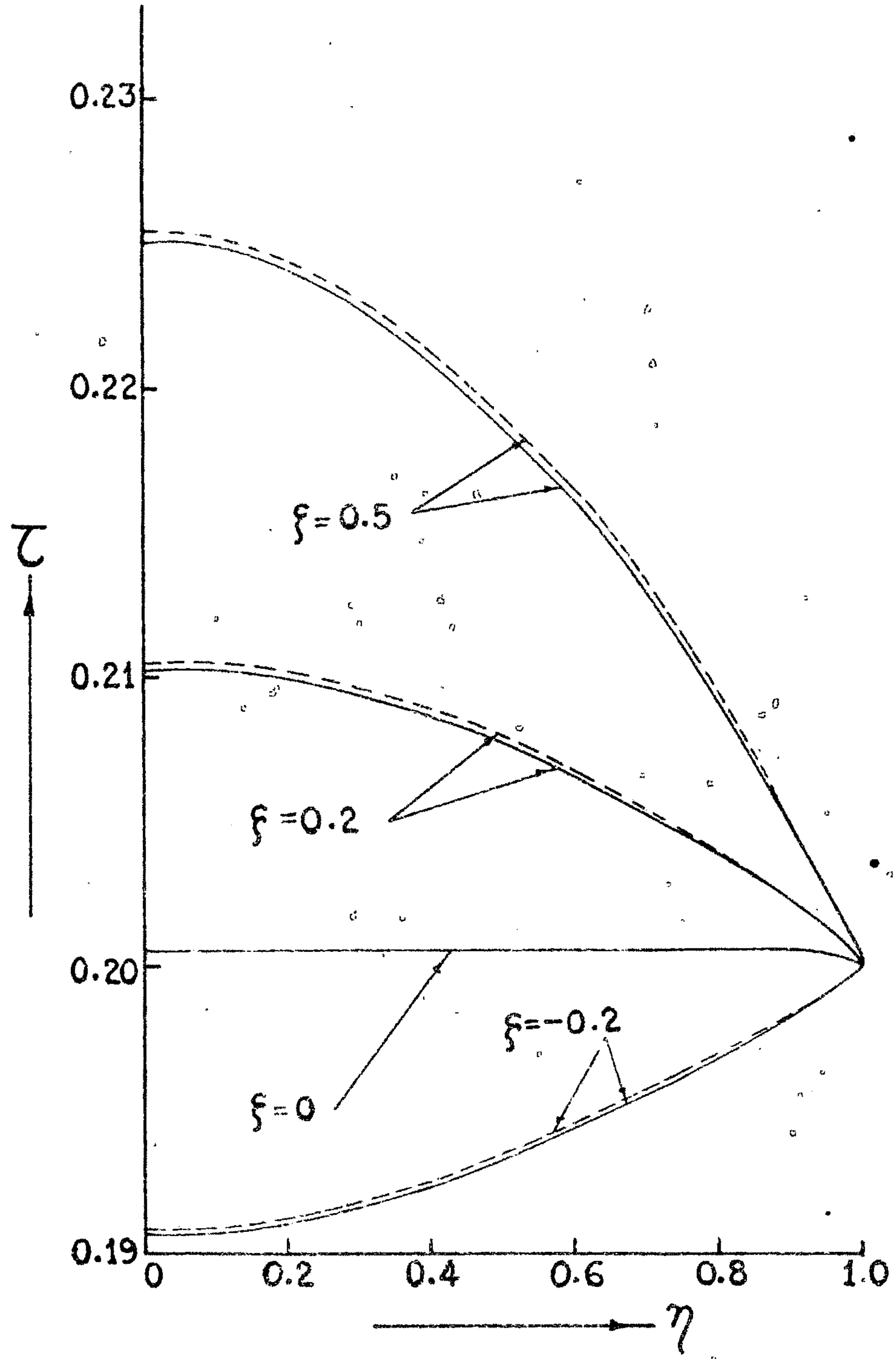

FIGURE: 8.7

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF HEAT SOURCE PARAMETER WHEN

K = 0.2 and

R = 0.0 ———— , R = 1.5 - - - - -

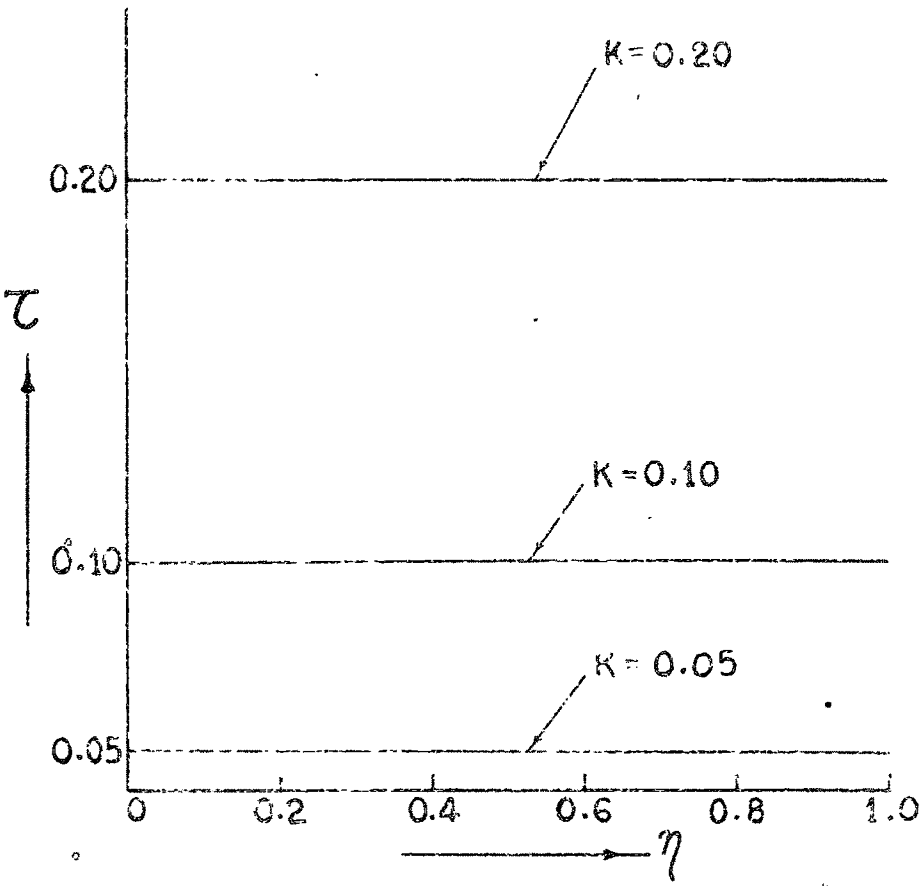

FIGURE: 8.8

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF K WHEN

ξ = 0.0 and R = 0.0

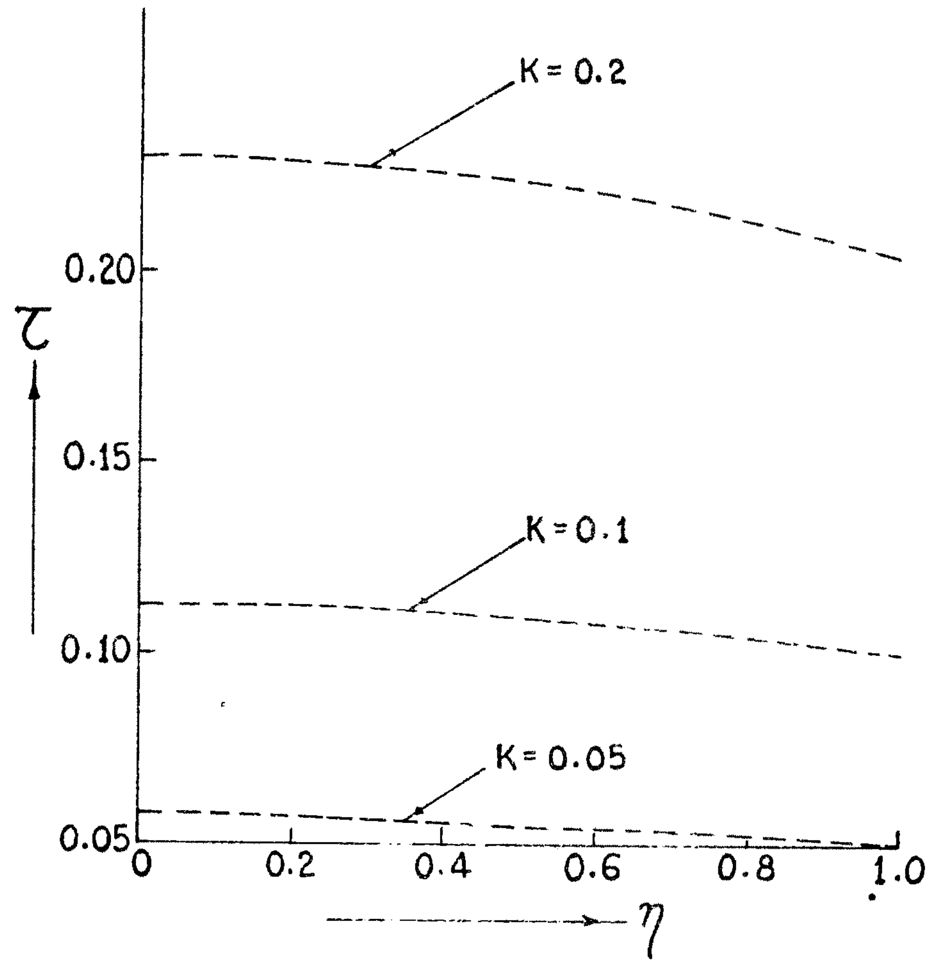

FIGURE: 8.9

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF K WHEN

ξ = 0. 5 and R = 1.5