CHAPTER. VII

FREE CONVECTION FLOW OF NON-NEWTONIAN FLUIDS BETWEEN PARALLEL WALLS

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7.1. INTRODUCTION.

Studies on laminar free convection flow and heat transfer of Newtonian viscous liquids have been reported in literature. Ostrach (1952,1954,1955,1955a, 1957) has studied the free convection flow of a viscous liquid in a vertical channel formed by two parallel planes and determined the velocity and temperature profiles. Jain (1962) has studied a similar problem for a visco-elastic liquid. In this chapter our aim is to study the laminar free convection flow and heat transfer of a non-Newtonian liquid with and without heat sources in channels with constant wall temperature. The flow phenomena have been characterized by the dimensionless parameters like R (non-Newtonian parameter), ζ (heat source parameter), m (ratio of the wall temperatures) and M (buoyancy parameter) and the effects of these parameters on the velocity and temperature distributions and Nusselt number have been studied. An iterative method has been employed to solve the equations of motion and energy.

The liquid model considered in this chapter has been discussed in detail in Section 1.4 [Eq.(1.4.10)- Eq. (1.4.20)]. The momentum equation for a laminar steady flow of a heat conducting non-Newtonian fluid subject to a body force f_i per unit mass is

$$\rho v_j v_{i,j} = \rho f_i + p_{ij,j} + \frac{\partial}{\partial x_i}(\lambda I_1) \quad (7.1.1)$$

where ρ is the density of the medium, p is the pressure, and λ is the bulk viscosity. The heat energy equation is

$$\rho c v_i \frac{\partial T}{\partial x_i} = Q - p v_{i,i} + \frac{\partial}{\partial x_j}\left(K \frac{\partial T}{\partial x_j}\right) + p_{ij} d_{ij} \quad (7.1.2)$$

where 'C' is the specific heat at constant volume, T is the temperature, Q is the heat added by heat source and K is the thermal conductivity of the fluid,

7.2. FORMULATION OF THE PROBLEM.

A fully developed free convection flow between two parallel infinitely stretched planes is assumed. X - axis is taken parallel to the plates and Y - axis perpendicular to the planes. So the velocity field is taken in the form

$$u = u(y) \;; \quad v = 0 \;; \quad w = 0 \quad (7.2.1)$$

It is easy to see that the velocity field is compatible with the continuity condition (1.4.13). The momentum equation (7.1.1) now reduces to

$$\rho f_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = 0 \tag{7.2.2}$$

$$\rho f_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left(\mu_c \frac{\partial u}{\partial y}\right) = 0 \tag{7.2.3}$$

The flow invariants in two-dimensional flow are

$$I_1 = 0 \ , \quad I_2 = -\frac{1}{4} \left(\frac{du}{dy} \right)^2 \ , \quad I_3 = 0 \tag{7.2.4}$$

Hence, μ and μ_c are functions of $\frac{du}{dy}$.
We shall confine our attention to the particular type of fluid characterized by

$$\mu = \mu_0\left[1 - \alpha \frac{du}{dy}\right], \text{ and } \mu_c = f\left(\frac{du}{dy}\right) \tag{7.2.5}$$

From (7.2.2), (7.2.3) and (7.2.5), we get

$$\rho f_x - \frac{\partial p}{\partial x} + \mu_0 \frac{d^2 u}{dy^2} - \alpha \mu_0 \frac{d}{dy}\left(\frac{du}{dy}\right)^2 = 0 , \tag{7.2.6}$$

$$\rho f_y - \frac{\partial p}{\partial y} + \frac{d}{dy}\left(\mu_c \frac{du}{dy}\right) = 0 \tag{7.2.7}$$

The energy equation (7.1.2) gives

$$K \frac{d^2 T}{dy^2} + \mu_0\left(\frac{du}{dy}\right)^2 - \mu_0 \alpha \left(\frac{du}{dy}\right)^3 + Q = 0 \tag{7.2.8}$$

In accordance with the usual practice in free convection flow the density ρ has been considered a variable only in forming the buoyant force. So Eq. (7.2.6) can be written as

$$\mu_0 \frac{d^2u}{dy^2} - \alpha \mu_0 \frac{d}{dy}\left(\frac{du}{dy}\right)^2 + \beta \rho f_x \theta - \frac{\partial p_D}{\partial x} = 0 \qquad (7.2.9)$$

and

$$\frac{\partial p_D}{\partial y} = 0 \text{ and } p_D = p - p_s - \mu_c \left(\frac{du}{dy}\right)^2 \qquad (7.2.10)$$

where β is the coefficient of thermal expansion, and

$$\theta = T - T_s \quad , \qquad (7.2.11)$$

T_s being the temperature at hydrostatic state and is the hydrostatic pressure. Equation (7.2.8) is now written as

$$K \frac{d^2\theta}{dy^2} + \mu_0 \left(\frac{du}{dy}\right)^2 - \mu_0 \alpha \left(\frac{du}{dy}\right)^3 + Q = 0 \qquad (7.2.12)$$

As u and θ are functions of y alone, it is evident that

$$\frac{dp_D}{dx} = \text{constant.}$$

Hence, the pressure gradient $\frac{dp}{dx}$ inside the channel differs from the hydrostatic pressure gradient by at most a constant, since

$$\frac{dp}{dx} = \frac{dp_s}{dx} + \frac{dp_D}{dx} = \frac{dp_s}{dx} + \text{Const.} \tag{7.2.13}$$

Equation (7.2.9) can be written as, following Ostrach (1952)

$$\mu_0 \frac{d^2u}{dy^2} - \alpha\mu_0 \frac{d}{dy}\left(\frac{du}{dy}\right)^2 + \rho\beta f_x\theta = 0 \tag{7.2.14}$$

The boundary conditions on the velocity and temperature fields are

$$\left.\begin{aligned} u(0) &= u(h) = 0 \\ \theta(0) &= T_{w(0)} - T_s = \theta_{w(0)} \\ \theta(h) &= T_{w(h)} - T_s = \theta_{w(h)} \end{aligned}\right\} \tag{7.2.15}$$

where h is the distance between the parallel plates. We adopt the following transformations:

$$\left.\begin{aligned} &\eta = \frac{y}{h} \;;\quad u = \frac{KU}{\rho\beta f_x h^2} \;;\quad \theta = \frac{\theta_{w(0)}}{M}\tau, \\ &\zeta = \frac{Qh^2}{M\,\theta_{w(0)}} ,\quad R = \frac{\alpha k}{\rho\beta f_x h^3} , \\ &M = P_r\,G_r\,\frac{\beta f_x h}{c} ,\quad m = \frac{\theta_{w(h)}}{\theta_{w(0)}} \end{aligned}\right\} \tag{7.2.16}$$

In view of the above transformations equations (7.2.12) and (7.2.14) give

$$U'' - 2RU'U'' + \tau = 0 , \qquad (7.2.17)$$

$$\tau'' - (U')^2 - R(U')^3 + \zeta M = 0 , \qquad (7.2.18)$$

where dashes denote differentiation with respect to η. The boundary conditions are from (7.2.15) and the transformations (7.2.16)

$$\left.\begin{aligned} U(0) &= U(1) = 0 \\ \tau(0) &= M \;; \quad \tau(1) = mM \end{aligned}\right\} \qquad (7.2.19)$$

7.3. SOLUTIONS OF EQUATIONS.

An exact solutions of the equations (7.2.17) and (7.2.18) cannot be obtained because of the inter-relations of the functions U and τ . To solve these two equations, we shall define an iterative procedure as suggested by Collatz (1959). We define a sequence of functions $U_0, U_1, U_2 \ldots\ldots\ldots$ and $\tau_0, \tau_1, \tau_2, \ldots\ldots$ The functions τ_0 is chosen arbitrarily satisfying the boundary conditions (7.2.19). Then $U_0, \tau_0, U_1, \tau_1, U_2, \tau_2, \ldots$ are calculated as the solutions of equations

$$U_n'' - 2R(U_{n-1}')(U_{n-1}'') + \tau_n = 0 \qquad (7.3.1)$$

$$\tau''_{n+1} + (U'_n)^2 - R(U'_n)^3 + M\varsigma = 0 \tag{7.3.2}$$

with

$$\left.\begin{aligned} U_n(0) &= U_n(1) = 0 , \\ \tau_{n+1}(0) &= M ; \quad \tau_{n+1}(1) = mM \\ U'_{-1} &= 0 , \quad U''_{-1} = 0 \end{aligned}\right\} \tag{7.3.3}$$

We start with the function

$$\tau_0 = M + M(m-1)\,\eta + A\eta\,(1-\eta) \tag{7.3.4}$$

which satisfies the boundary conditions

$$\tau_0(0) = M \quad \text{and} \quad \tau_0(1) = Mm$$

and A is an arbitrary constant to be determined. Substituting (7.3.4) in (7.3.1) with n = 0, we get

$$U''_0 = - M - M\,(m-1)\,\eta - A\eta\,(1-\eta) , \tag{7.3.5}$$

which on integration with the boundary conditions (7.3.3.) gives

$$U_0 = \left[\frac{1}{2} M + \frac{1}{6} M(m-1) + \frac{1}{12} A\right] \eta - \frac{1}{2} M\eta^2$$

$$- \frac{1}{6} M\,(m-1)\,\eta^3 + \frac{1}{6} A\,(\frac{1}{2}\eta^4 - \eta^3). \tag{7.3.6}$$

For determinating A , we assume that the first iterate U_0 is the same as if the fluid is only viscous with a constant coefficient of viscosity. Hence, from (7.3.2) with $n = -1$, we get

$$A = \frac{1}{2} M \xi \, .$$

Hence, from (7.3.6), we have

$$U_0 = M \sum_{r=1}^{4} \frac{a_r}{r} \eta^r \qquad (7.3.7)$$

and from (7.3.4), we get

$$\tau_0 = -M \sum_{r=1}^{3} r \, a_{r+1} \eta^{r-1} \qquad (7.3.8)$$

The next higher approximation can be obtained in the following way:

From (7.3.2), with $n = 0$, we have

$$\tau_1'' + (U_0')^2 - R(U_0')^3 + M\xi = 0 \qquad (7.3.9)$$

Substituting the value of U_0' from (7.3.7) into (7.3.9), one gets

$$\tau_1'' = \sum_{r=0}^{9} (r+1)\,(r+2)\, A_r \, \eta^r$$

which on integration gives

$$\tau_1 = M + M\,(m-1)\,\eta - \eta \sum_{r=0}^{9} A_r + \sum_{r=0}^{9} A_r \, \eta^{r+2} \qquad (7.3.10)$$

From (7.3.1), with $n = 1$, we get

$$U_1'' - 2R(U_0')(U_0'') + \tau_1 = 0 \cdot \qquad (7.3.11)$$

Substituting the values of U_0', U_0'' from (7.3.7) and the value of τ_1 from (7.3.10) into (7.3.11) and integrating with the boundary conditions (7.3.3), we get

$$U_1 = \sum_{r=0}^{5} \left[\frac{B_r}{(r+1)(r+2)} \left\{ \eta^{r+2} - \eta \right\} - \frac{A_{r+4}}{(r+7)(r+8)} \left\{ \eta^{r+8} - \eta \right\} \right], \qquad (7.3.12)$$

where

$$a_1 = \frac{1}{3}\left(1 + \frac{1}{2} M + \frac{1}{8} \zeta\right),$$

$$a_2 = -1$$

$$a_3 = \frac{1}{2}\left[1 - m - \frac{1}{2}\zeta\right],$$

$$a_4 = \frac{1}{6}\zeta$$

$$A_0 = -\frac{1}{2} M \left[\zeta + M a_1^2 - R M^2 a_1^3\right]$$

$$A_1 = \frac{1}{6} M^2 \left[3RM\, a_1^2 a_2 - 2 a_1 a_2\right]$$

$$A_2 = \frac{1}{12} M^2 \left[3RM(a_1 a_2^2 + a_1^2 a_3) - (a_2^2 + 2a_1 a_3)\right]$$

$$A_3 = \frac{1}{20} M^2 \left[RM(a_2^3 + 6a_1 a_2 a_3 + 3a_1^2 a_4) - 2(a_1 a_4 + a_2 a_3)\right],$$

$$A_4 = \frac{1}{30} M^2 \left[3RM(a_3 a_2^2 + 2a_1 a_2 a_4 + a_1 a_3^2) - (a_3^2 + 2a_2 a_4)\right],$$

$$A_5 = \frac{1}{42} M^2 \left[3RM \; (a_2^2 \; a_4 + 2a_1 a_3 a_4 + a_2 a_3^2 \;) - 2a_3 \; a_4 \right],$$

$$A_6 = \frac{1}{56} M^2 \left[RM \; (a_3^3 + 3a_1 a_4^2 + 6a_2 a_3 a_4 \;) - a_4^2 \right],$$

$$A_7 = \frac{1}{24} RM^3 \; (a_3^2 \; a_4 + a_2 \; a_4^2 \;),$$

$$A_8 = \frac{1}{30} RM^3 \; a_4^2 \; a_3 \;,$$

$$A_9 = \frac{1}{110} RM^3 \; a_4^3 \;,$$

$$B_o = 2RM^2 \; a_1 a_2 - M \;,$$

$$B_1 = 2RM^2 \; (2a_1 a_3 + a_2^2 \;) - M \; (\; m - 1 \;) + \sum_{r=0}^{9} A_r \;,$$

$$B_2 = 6RM^2 \; (a_1 a_4 + a_2 a_3 \;) - A_o \;,$$

$$B_3 = 4RM^2 \; (2a_2 a_4 + a_3^2 \;) - A_1 \;,$$

$$B_4 = 10RM^2 \; a_3 a_4 - A_2 \;,$$

$$B_5 = 6RM^2 \; a_4^2 - A_3 \;.$$

In a similar way the higher approximate iterative solutions can be obtained. But the algebra will be naturally very complicated. Equations (7.3.10) and (7.3.12) yield approximate solutions for the temperature and velocity distributions. The rate of heat transfer for natural convection process can be expressed in terms of Nusselt numbers. By using Eq.(7.3.10), the Nusselt number in the non-dimensional form for the wall at $\eta = 0$ when $m \neq 1$ is

$$N_{u(0)} = \frac{1}{M(m-1)}\left[M(m-1) - \sum_{r=0}^{q} A_r\right], \tag{7.3.13}$$

and for the wall at $\eta = 1$ is

$$N_{u(1)} = \frac{1}{M(m-1)}\left[M(m-1) + \sum_{r=0}^{q} (r+1) A_r\right]. \tag{7.3.14}$$

When the walls are at the same temperature, $m = 1$ and the Nusselt number in the non-dimensional form can be written as for the wall $\eta = 0$

$$N_{u(0)} = -\sum_{r=0}^{q} A_r . \tag{7.3.15}$$

7.4. CONCLUSIONS.

Laminar free convection flow of a non-Newtonian liquid between parallel walls with and without heat sources in channels with constant temperature has been studied. The flow phenomena have been characterized by the dimensionless parameters like R (non-Newtonian parameter), ζ (heat source parameter), m (ratio of the wall temperatures) and M (buoyancy parameter) and the effects of these parameters on the velocity and temperature distributions and Nusselt number have been studied. An iterative method has been employed to solve the equations of motion and energy.

Velocity distribution:-

Figure. 7.1 represents the velocity distribution for different values of the non-Newtonian number R when both the walls are at equal temperature. When the liquid is only viscous, we have $R = 0$, and the velocity field is symmetrical about the line $\eta = 0.5$, that is, the mid-plane of the channel. But the non-Newtonian character of the liquid makes it asymmetric. In the half of the channel near the wall $\eta = 0$, the velocity increases with the non-Newtonian parameter R . But in the part of the channel nearer to $\eta = 1$, an opposite effect is observed.

Figure. 7.2 represents the velocity distribution when the walls are at different temperatures. In this case both for Newtonian and non-Newtonian liquids the profile is asymmetrical. In the part of the channel nearer to the plane $\eta = 0$, the non-Newtonian character of the fluid increases the velocity at a point. But in the other part an opposite effect is observed.

Figure. 7.3 shows the velocity distribution for different values of the heat source parameter ζ , when the walls are at equal temperature. From this figure it is seen that the velocity profile is asymmetrical about the mid-plane and the velocity at any point increases with the increase in the value of the heat source parameter. The point of maximum velocity occurs near the wall $\eta = 0$.

Figure. 7.4 shows that velocity at any point increases with the increase in the value of m, the ratio of the wall-temperature.

Temperature Distribution:-

Figure. 7.5 represents the temperature distribution for different values of R when both the walls are at equal temperature. When R = 0, it is seen that the profile is symmetrical about the mid-plane. But the non-Newtonian nature of the liquid decreases the temperature in the part of the channel near the plane $\eta = 0$ and increases in the other part.

Figure. 7.6 represents the effect of the non-Newtonian parameter on the temperature field when the walls are at different temperatures and without any heat source. This figure shows that the temperature at any point decreases as the value of the non-Newtonia parameter increases.

Figure. 7.7 represents the temperature distribution for different values of the heat source parameter ζ when the walls are at equal temperature. As obvious, the temperature increases with the value of the heat source parameter.

Figure. 7.8 represents the temperature distribution for different values of m . The temperature decreases at a point with the increase in the value of m .

Rate of Heat Transfer from the Walls:-

The rate of heat transfer for natural convection process has been expressed in terms of Nusselt numbers and the numerical values are presented in Table. 7.1. This table gives us the following conclusions:

Nusselt number N_u for $m = 0$ decreases with the increase in the value of ζ at $\eta = 0$. But the non-Newtonian parameter increases the N_u . Nusselt number at $\eta = 1$ increases with the increase in the value of ζ , but the non-Newtonian parameter decreases it.

When the walls are at unequal ($m = 2$) temperature Nusselt number N_u at $\eta = 0$ increases with the increase in the value of heat source parameter and the non-Newtonian parameter decreases it. At the other plate, N_u decreases as ζ increases and the non-Newtonian parameter increases the Nusselt number.

When the walls are at equal temperature ($m = 1$). in this case the Nusselt number at $\eta = 0$ decreases with the increase in the value of ζ and the non-Newtonian parameter increases it. At $\eta = 1$, the Nusselt number increases with the increase in the value of ζ and the non-Newtonian parameter increases it further.

TABLE 7.1

NUSSELT NUMBER AT THE WALLS.

m \ ζ	0	10	15	η	R
0	0.9705	−4.2050	−6.8730	0	0.0
	1.0165	6.1735	8.8340	1	
	0.9885	−4.0861	−5.9185	0	0.5
	0.8541	5.2220	5.2830	1	
1	−0.0830	−5.1351	−7.7470	0	0.0
	0.0825	4.4360	8.0155	1	
	−0.0591	−4.9421	−6.3160	0	0.5
	0.3585	4.7195	8.2041	1	
2	1.1780	6.4980	9.2195	0	0.0
	0.7890	−4.5490	−7.2845	1	
	1.0700	5.4170	6.8970	0	0.5
	0.9111	−1.2235	−2.5370	1	

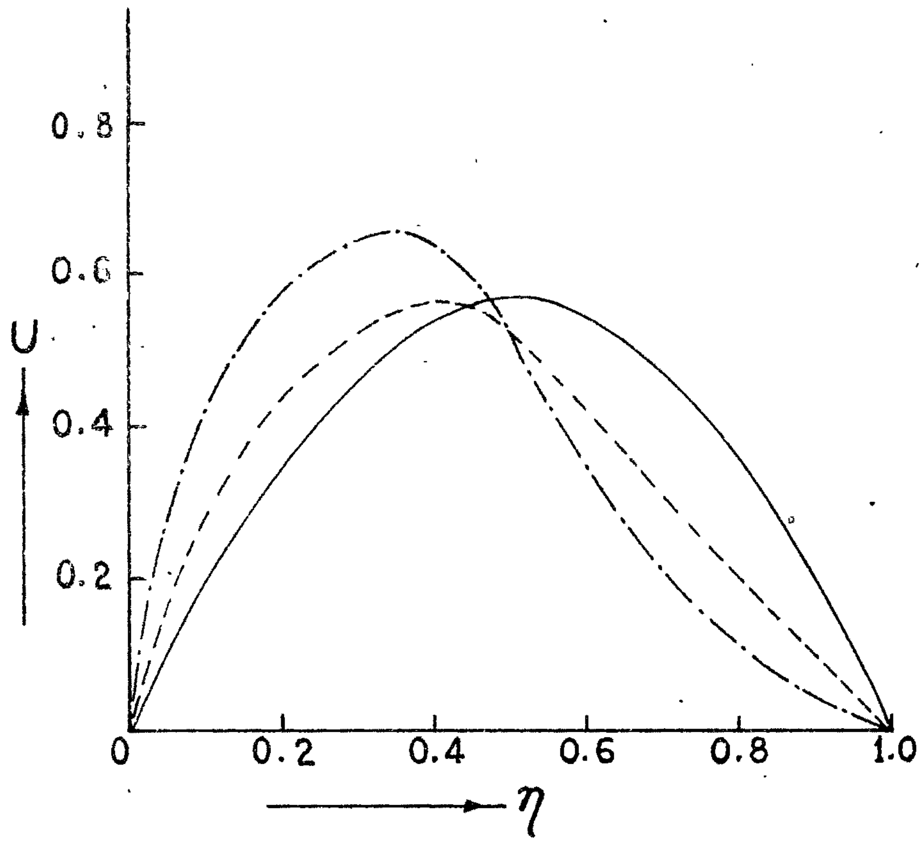

FIGURE: 7.1

VELOCITY DISTRIBUTION FOR DIFFERENT

VALUES OF R

M = 2 , m = 1 , ξ = 10

R = 0.0 ————————

R = 0.5 - - - - - - - - -

R = 1.0 -·-·-·-·-·-

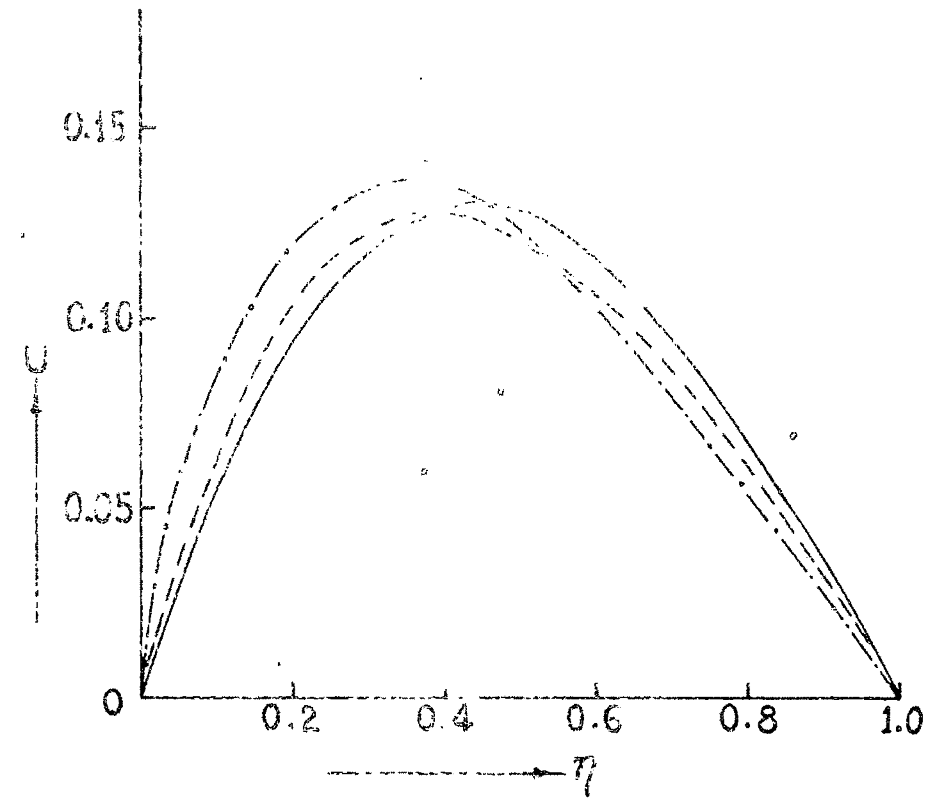

FIGURE: 7.2

VELOCITY DISTRIBUTION FOR DIFFERENT

VALUES OF R

M = 2 , m = 0 , ξ = 0

R = 0.0 ————————

R = 0.5 — — — — — —

R = 1.0 —·—·—·—·—·—

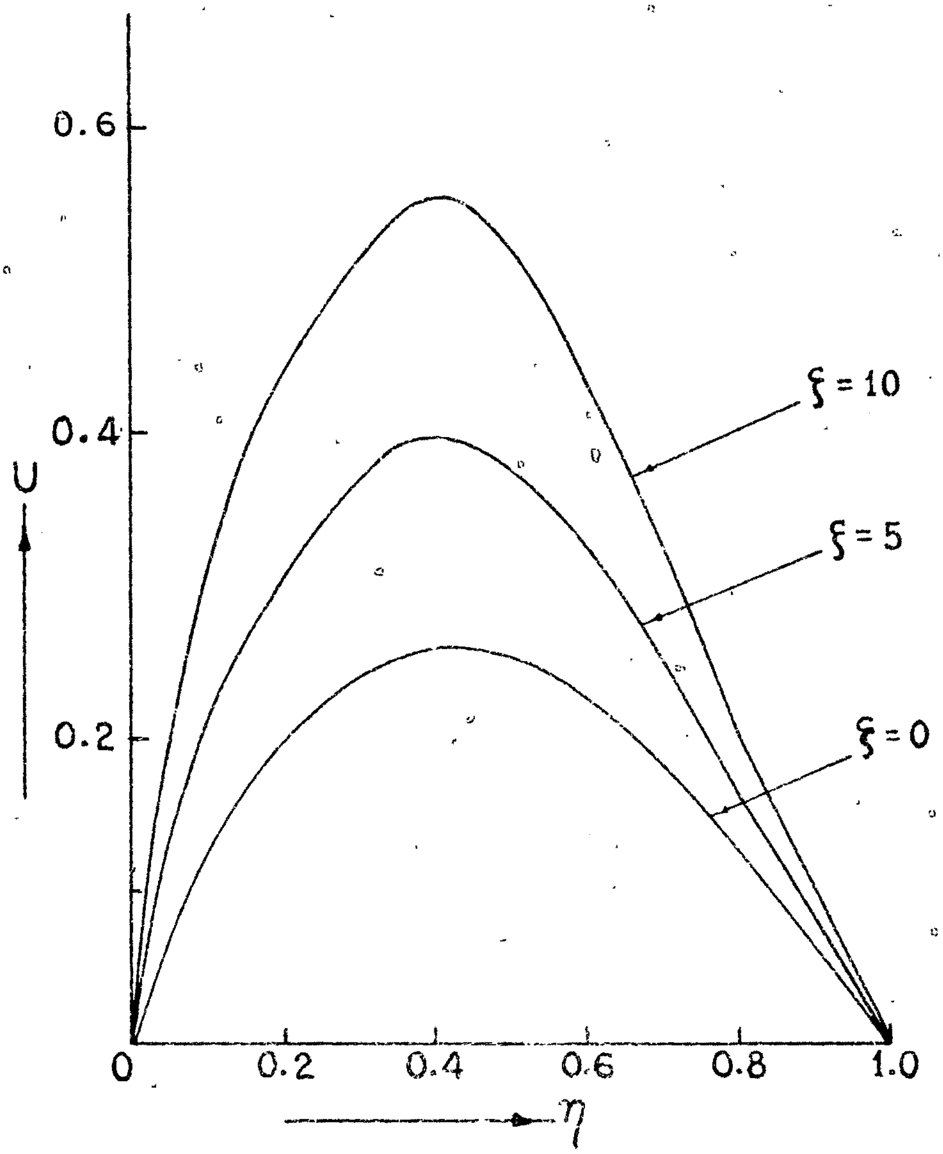

FIGURE: 7.3

VELOCITY DISTRIBUTION FOR DIFFERENT VALUES OF THE HEAT SOURCE PARAMETER

M = 2 , m = 1 , R = 0.5

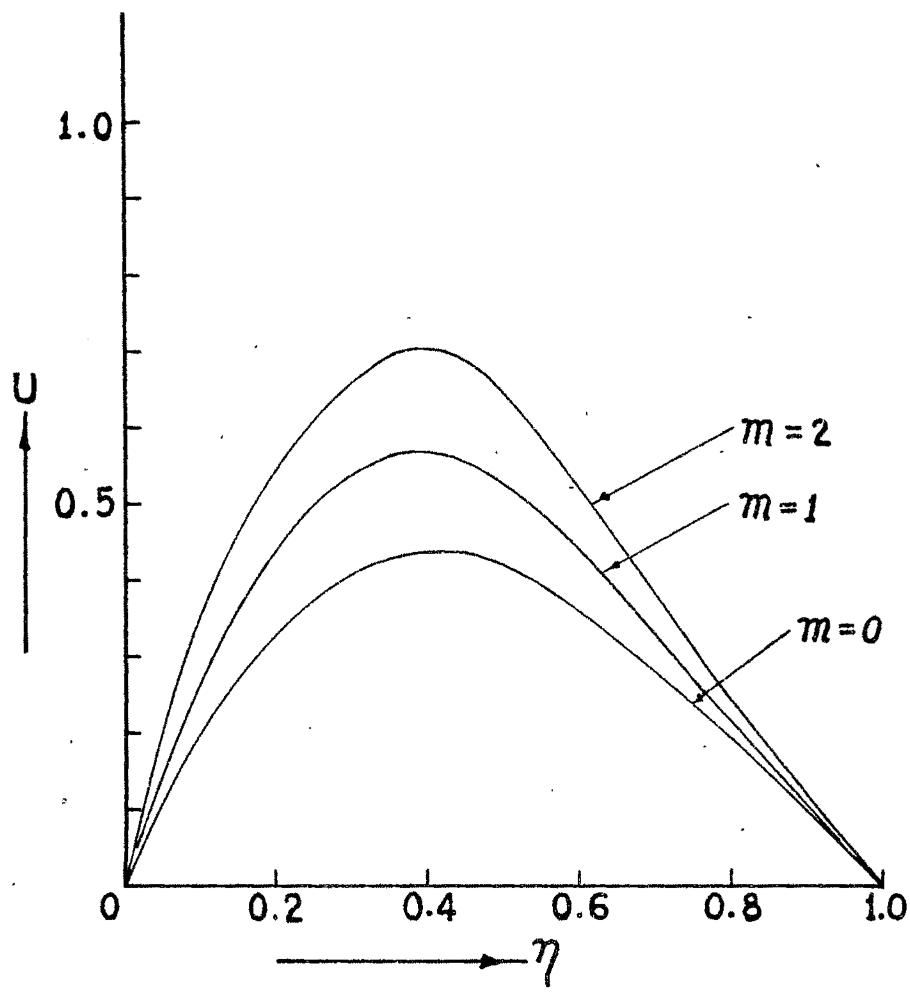

FIGURE: 7.4

VELOCITY DISTRIBUTION FOR DIFFERENT

VALUES OF m

$M = 2$, $\xi = 10$, $R = 0.5$

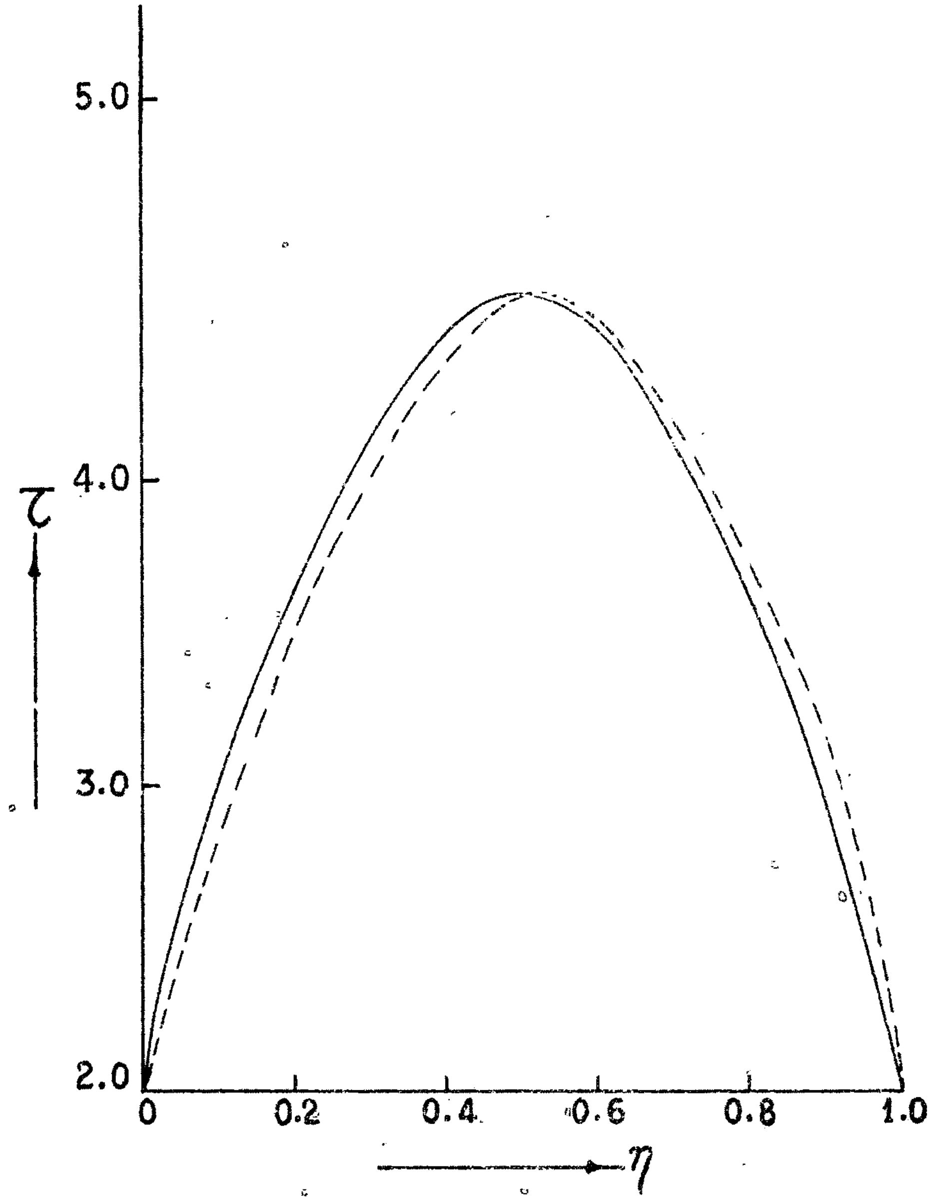

FIGURE: 7.5

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF R

M = 2 , m = 1 , ξ = 10

R = 0.0 ————————

R = 0.5 – – – – – – – –

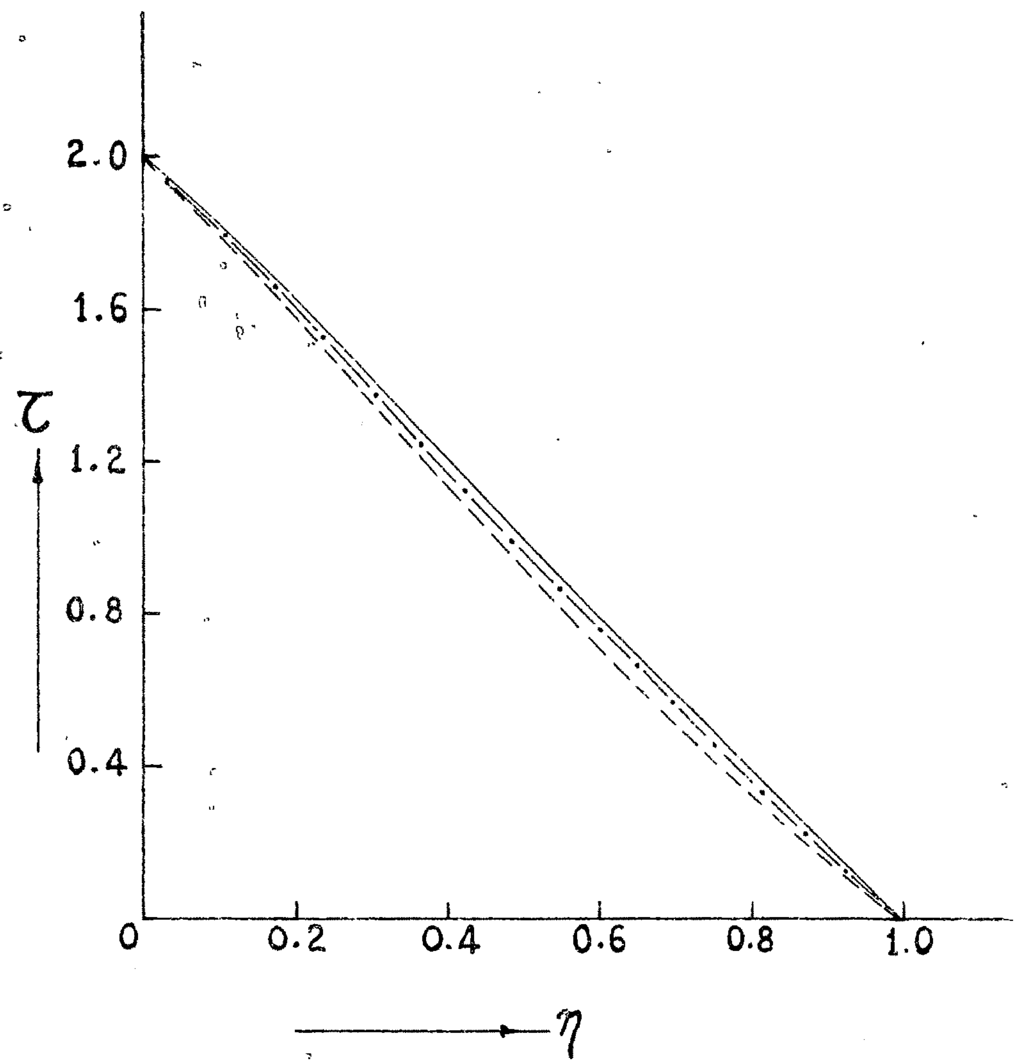

FIGURE: 7.6

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF R

M = 2 , m = 0 , ξ = 0

R = 0.0 ————————

R = 0.5 — · — · — · — · —

R = 1.0 — — — — — —

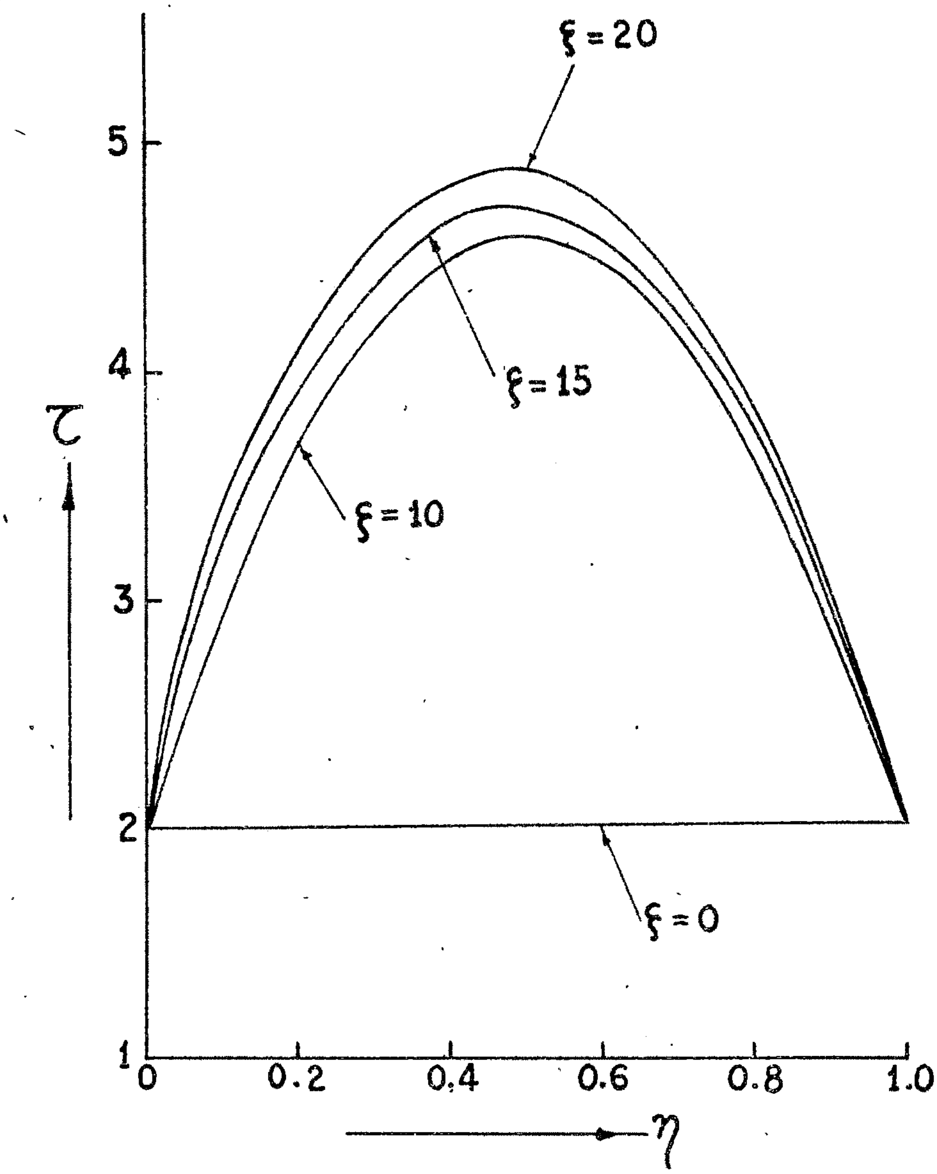

FIGURE: 7.7

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF HEAT SOURCE PARAMETER

M = 2 , m = 1 , R = 0.5

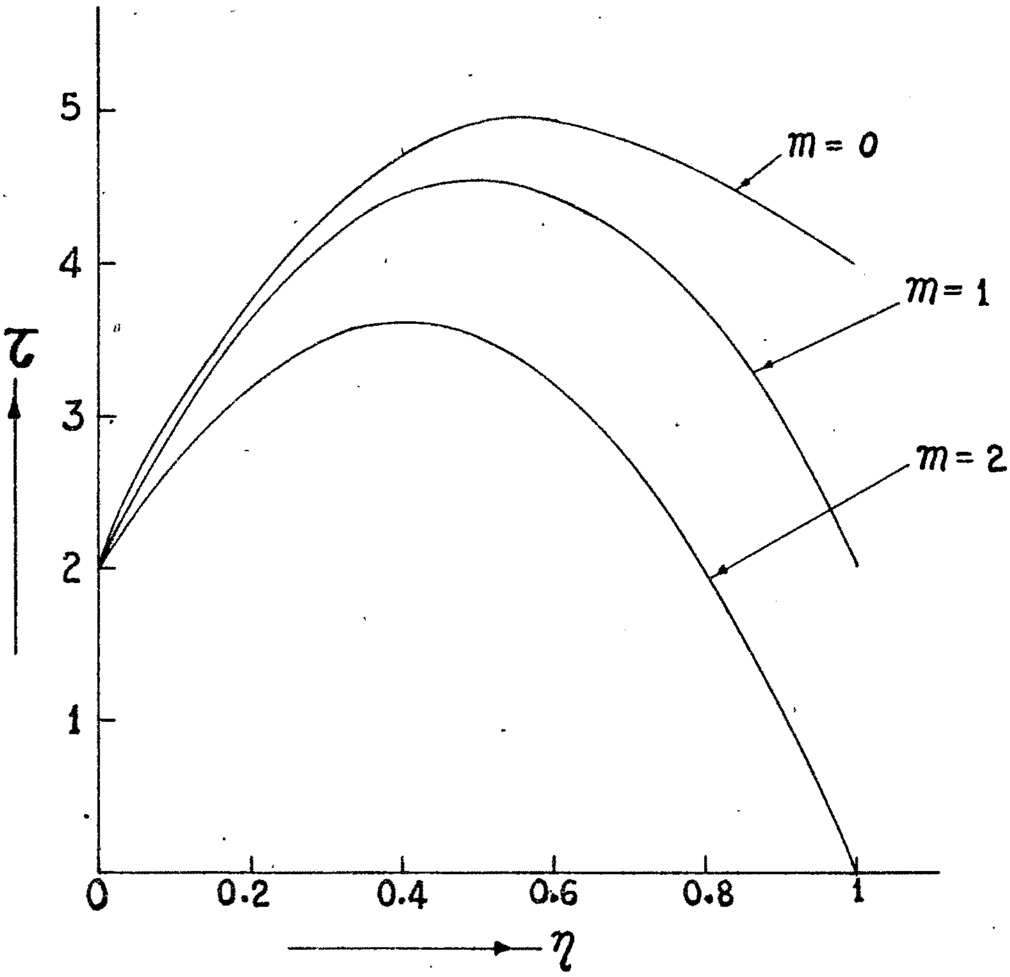

FIGURE: 7.8

TEMPERATURE DISTRIBUTION FOR DIFFERENT VALUES OF m

M = 2 , ξ = 10 , R = 0.5