Chapter II
Basics and modeling aspects of mixed layer
2.1 Definition

Action of wind, wave and radiation induces vertical mixing in the upper columnar water of the ocean. Mixed layer is that portion of the upper ocean where significant physical, chemical and biological activities are taking place. This layer is well mixed by wind and waves and hence it possess homogeneous/quasi-homogeneous properties like temperature, salinity and density and is known as mixed layer depth and it is confined to a depth of 20m – 150m in the Indian Ocean.

2.2 Upper Ocean Structure

The salient feature of the oceanic thermal structure is a remarkably shallow thermocline, especially in the tropics and subtropics. Which factors determine its depth? The oceanic circulation that maintains this thermal structure has two main components, a shallow wind-driven circulation and a deep thermohaline circulation. Theories for the deep thermohaline circulation provide an answer that depends on oceanic diffusivity (but they deny the surface winds an explicit role). Theories for the shallow ventilated thermocline take into account the influence of the wind explicitly. To complete and marry the existing theories, for the oceanic thermal structure Giulio et al., 2004, proposed a balanced heat budget for the ocean. According to his theory, oceanic heat gain occurs primarily in the upwelling zones of the tropics and subtropics and depends strongly on oceanic conditions, specifically the depth of the thermocline. The heat gain is large when the thermocline is shallow but is small when the thermocline is deep. Therefore,
an increase in heat loss in high latitudes can result deepening the thermocline and decrease in heat loss can cause shoaling of the tropical thermocline.

Based on thermal property of ocean, it is divided into three zones. There is an upper zone from surface to 200m depth, and a zone below this extending to 500m (thermocline) and a deep zone. The temperature in the upper layer shows seasonal variations, particularly in middle latitudes. Over the mid-latitude oceans, at winter season sea surface temperature is low and the mixed layer is deep and may extend to the main thermocline (~150m from surface) and during summer season surface temperature rises and a seasonal thermocline often develops in the upper ocean. Figure 2.1 shows the thermal structure of the ocean at low, mid, and high latitudes.

**Fig. 2.1:** Typical mean temperature/depth profiles for the open ocean. (Adapted from Pickard and Emery, 1982)
2.3 Forcing on marine Mixed Layer

The different physical processes that control the change in physical and chemical properties of the upper ocean are called the atmosphere-ocean exchange processes and which are divided into four parts:

1. Radiative forcing
2. Wind forcing
3. Penetration of solar radiation
4. Evapo-precipitation

The radiative forcing plays a vital role in vertical convective processes rather than horizontal advection in the upper ocean. As a result the vertical process has significant role than the horizontal one in changing the upper ocean properties at any given time. For example, the gradient in the seawater temperature has a significant variation of nearly 30m depth whereas on horizontal direction, similar variation can be observed over a distance of 1000m or so. Also, convection is aided by wind forcing, in part because winds help to disrupt the viscous sub-layers at the sea surface, permitting more rapid transport of heat through the surface. However the action of wind on the surface layer enhances advection. To quantify the different energy fluxes exchanged through the ocean-atmosphere interface, an empirical formulae based on available observations were established (Coantic, 1974).
2.3.1 Radiative forcing

Radiative forcing over the sea surface comprises of incoming shortwave radiation $Q_s$, and outgoing longwave radiation $Q_L$. $Q_L$ consists of infrared radiation (Black body radiation), $Q_B$, sensible heat $Q_H$, and latent heat $Q_E$.

Shortwave radiation $Q_s$

The largest term in the heat balance and the only one of global consequence that is positive is $Q_s$, the incoming shortwave radiation. The direct solar energy reaching the sea surface is centered in the band of wavelength from $0.5 \mu m - 1 \mu m$, between the visible and infrared portions of the electromagnetic spectrum. The actual amount of solar energy $E_0$, integrated over the entire electromagnetic spectrum, that falls on a disk centered at the mean Earth-Sun distance and having a radius to the radius of the Earth is about 1390Wm$^{-2}$, and is called the solar constant. Since, the disk has an area of $\pi R^2$, where $R$ is the radius of the Earth, and the sphere has a surface area $4\pi R^2$, then the globally averaged value of solar energy falling on the Earth at the top of the atmosphere is just $S_0 = \frac{E_0}{4}$, or about 347.5Wm$^{-2}$.

Since the Earth is not a perfect absorber, the solar energy $S_0$ reaching the top of the atmosphere is immediately reflected back into space. A measure of the amount of energy reflected back into space is called the albedo ($\alpha$). The amount of solar energy entering the top of the atmosphere is then $S = S_0 (1 - \alpha) = 222$Wm$^{-2}$, where Earth’s average albedo $\alpha$ is about 0.3. From this total amount, the clouds absorb a fraction of the energy before reaching the sea surface, and the remaining
is absorbed by atmospheric dust, ozone, and water vapor in the atmosphere. The result is a global average of about 150Wm$^{-2}$ of solar energy that actually reaches the sea surface.

**Infrared radiation $Q_B$**

Earth radiates energy in the infrared portion of the spectrum, which results in a globally averaged loss of heat ($Q_B$) of about 50Wm$^{-2}$ over the oceans. The amount of heat radiates from the sea surface in this fashion can be estimated from the Stefan-Boltzmann relation;

$$Q_B = e_m \sigma_B (T_s - T_{ref})^4$$

where, $e_m$ is emissivity of the sea surface, $\sigma_B$ is Stefan-Boltzmann constant, $T_s$ is Surface temperature, and $T_{ref}$ is characteristic radiative temperature at cloud bottoms.

More often we express this temperature $T_{ref}$ based on the air temperature $T_a$ by using the relation:

$$T_{ref} = T_a - \Delta T_{at}$$

where, $\Delta T_{at}$ is the difference in temperature between the cloud bottom and the oceanic atmospheric layer.

**Sensible heat flux $Q_H$**

Normally the ocean and atmosphere are at different temperatures, there will be a flux of heat between them in order to attain a state of thermal equilibrium. This
heat flux is called sensible heat, or conductive heat flux \((Q_H)\). This heat flux corresponds to a loss or gain of energy by the sea, depending on the sign of the temperature difference \((T_s - T_a)\). The physical method by which the heat it transferred between the atmosphere and ocean is quite complicated and depends on the temperature difference, relative humidity, wind speed, and a number of other environmental parameters. It is possible to gain a simple understanding of heat transfer process between ocean and atmosphere by considering the temperature gradient denoted by \(\frac{\partial T}{\partial z}\), where \(T\) is temperature. We can anticipate that, in the absence of other factors, a gradient such as this would result in diffusion of the property \(T\) in the directions of decreasing \(T\). As the gradient increases or decreases, it is expected the diffusive flux of \(T\) would increase or decrease correspondingly. We can state in this case the flux of \(T\) is proportional to the gradient of \(T\), or \(F = -\kappa \frac{\partial T}{\partial z}\), where the minus sign denotes the fact that the flux is in the opposite direction to the gradient, and the constant of proportionality \(\kappa\) known as the diffusivity. Diverse formulae used operationally are derived from different meteorological parameters. For example, citing the semi-empirical form of Coantic, (1974):

\[
Q_H = \rho_a C_p C_H (T_a - T_s)
\]

where, \(\rho_a\) is air density, \(C_p\) is the specific heat capacity at constant pressure, and \(C_H\) is heat transfer coefficient.
Latent heat flux $Q_L$.

Since the atmosphere directly above the ocean is usually not saturated with moisture (i.e., the relative humidity <100%), there will be a tendency for evaporation from the sea surface in order to increase the moisture content of air. The energy required for evaporation is called latent heat of evaporation. Normally this energy is taken from the ocean surface and thus it consequently cools the sea surface; this amount of heat energy can be calculated by using the formula given below.

$$Q_L = \rho_v L C_e (P_s - P_a)/P_a$$

where, $\rho_v$ is density of water vapor, $L$ is latent heat of vaporization, $C_e$ is evaporation coefficient, $P_s$ and $P_a$ are vapor pressures at temperature $T_s$ and $T_a$.

Advective heat $Q_V$

Ocean cannot transport any net heat on a globally averaged basis, but advective heat fluxes between ocean basins or within a basin are quite possible. If the ocean is truly in thermal equilibrium and we neglect the geothermal heating (benthic heating), then we can write that

$$Q_{SURF} + Q_V = 0,$$

where, $Q_V$ is the advective heat transport by the ocean. Since $Q_{SURF}$ represents a flux of heat through the sea surface, and $Q_V$ represents an internal flux of heat through the ocean, integral over a region of ocean to find that

$$\int_{\Phi_0}^{\Phi_1} \int_{\lambda_0}^{\lambda_1} Q_{SURF} \, d\lambda \, d\Phi = -\int_{\Phi_0}^{\Phi_1} \int_{-H}^{0} Q_V \, d\lambda \, dz$$
This equation implies that if there is a net loss of heat through the surface of the ocean in some region, then there must be some net advection of heat into that region by the ocean circulation in order to maintain thermal equilibrium.

To generalize, the advective heat flux \( Q_v \) is proportional to both the velocity of the ocean circulation \( V \) and the temperature of the water; \( T \). Thus we can parameterize \( Q_v \) as
\[
Q_v \approx VT = \rho CVT
\]
where, \( \rho \) is seawater density and \( C \) is the heat capacity of seawater. Advective flux is a vector quantity, since it depends on both the magnitude and direction of \( V \).

**Heat balance \( Q_T \)**

Shortwave radiation and latent heat loss are the dominant terms in the heat balance (Oberhuber, 1988; da Silva et al., 1994). The conservation of heat in a medium is called heat balance or heat budget. This quantifies source and sink terms of the heat in all directions. Generally, the heat budget of the oceanic component of the Earth can be written as
\[
\downarrow Q_T = \downarrow Q_s + \uparrow Q_B + \uparrow Q_H + \uparrow Q_E + \uparrow Q_v + \uparrow Q_G
\]
where, \( Q_T \) is net heat flux into the Earth (0Wm\(^{-2}\)), \( Q_s \) is direct solar input (150Wm\(^{-2}\)), \( Q_B \) is black body radiation (-50Wm\(^{-2}\)), \( Q_H \) is sensible heat loss (-10Wm\(^{-2}\)), \( Q_E \) is evaporative heat transfer (-90Wm\(^{-2}\)), \( Q_v \) is advective heat transfer (0Wm\(^{-2}\)), and \( Q_G \) is geothermal heating (~10\(^{-2}\)Wm\(^{-2}\)).
These are the major terms with approximate global average values are given inside the parentheses. It is assumed in this formulation that the Earth is presently in thermal equilibrium: the heat lost equals the heat gained. If these are indeed a global warming (or global cooling) underway, then $Q_T$ is not zero; however, nearly all contemporary attempts to deduce a value for $Q_T$ yields a result that smaller than the errors in the determination. Thus, now $Q_T = 0$ to within our ability to measure it. The terms in the right side are individually somewhat known on a global average; with arrow headed upward, implying a net heat gain by the Earth and down headed arrow denotes a net heat loss. The advective heat transfer, $Q_v$, must equal to zero on a global average basis since the ocean cannot, on a net basis, create any heat by advection. The geothermal heating term, $Q_G$, is small enough to be considered and negligible with respect of the other terms, but it may have a major impact on the circulation of the deep sea, where the direct heat flux from the sea surface is small.

2.3.2. Wind forcing

The momentum transferred by sea surface winds to ocean surface influence the upper ocean properties such as currents, temperature/salinity to change via a mechanism called wind stirring, this impact also felt on the evolution of biology and bio-geochemical processes. Generally surface winds are represented by wind stress and empirically given as;

$$\tau^* = \rho_g C_D |w| w$$
where, $\rho_a$ is air density, $C_D$ the drag coefficient and $W$ sea surface wind represented as vector.

**Breaking waves**

Large scale breaking of waves evidenced at the surface by white-capping and surface foam disrupts the ocean's cool skin. Small scale breaking, which has no visible signature, also disrupts the ocean's cool skin. Turbulence observations in the surface layer under a variety of conditions have indicated that at times (generally lower winds and simpler wave states) the turbulence dissipation rate (and presumably other turbulence quantities including fluxes) behave in accord with simple wall-layer scaling and is in this way similar to the atmospheric surface layer. However, under higher winds, and perhaps more complicated wave states, turbulence dissipation rates greatly exceed those predicted by wall-layer scaling. This is a problem of great importance in determining both transfer rates across the air-sea interface to the mixed layer below and the evolution of the mixed layer itself. It is at times when turbulence is most intense that most of the air-sea transfers and most of the mixed layer modification occur.

**Langmuir circulation**

The Langmuir circulations are coherent structure within the mixed layer that produces counter-rotating vortices with axes aligned parallel to the wind. Their surface signature is familiar as windrows: lines of bubbles and surface debris aligned with the wind that marks the convergence zones between the vortices.
These convergence zones enhance gas exchange rates with the atmosphere. Langmuir circulations appear to be intimately related to the Stokes drift, a small net current parallel to the direction of wave propagation, generated by wave motions. Stokes drift is concentrated at the surface and is thus vertically sheared. Small perturbations in the wind-driven surface current generate vertical vorticity, which is tilted toward the horizontal (downwind) direction by the shear of the Stokes drift. It is the convergence associated with these vortices that concentrates the wind-driven surface current into jets. Langmuir cells thus grow by a process of positive feedback. Ongoing acceleration of the surface current by the wind, together with convergence of the surface current by the Langmuir cells, provides a continuous source of coherent vertical vorticity. Maximum observed velocities are located well below the sea surface but also well above the mixed layer base. Langmuir circulations are capable of rapidly moving fluid vertically, thereby enhancing and advecting the turbulence necessary to mix the weak, near-surface stratification, which forms in response to daytime heating. However, this mechanism does not seem to contribute significantly to mixing the base of the deeper mixed layer, which is influenced more by storms and strong cooling events. In contrast, penetration of the deep mixed layer base during convection (driven by the conversion of potential energy of dense fluid plumes created by surface cooling/evaporation to kinetic energy and turbulence) is believed to be an important means of deepening the mixed layer.
Wind-Driven shear

Wind-driven shear erodes the thermocline at the mixed layer base. Wind-driven currents often veer with depth due to planetary rotation. Fluctuations in wind speed and direction result in persistent oscillations at near-inertial frequencies. Such oscillation are observed almost everywhere in the upper ocean, and dominate the horizontal velocity component of the inertial wave field. Because near-inertial waves dominate the vertical shear, they are believed to be important sources of mixing at the base of the mixed layer. In the upper ocean, near-inertial waves are generally assumed to be the result of wind forcing. Rapid diffusion of momentum through the mixed layer tends to concentrate shear at the mixed layer base. This concentration increases the probability of small-scale instability. The tendency toward instability is quantified by the Richardson number, $R_i = \frac{N^2}{S^2}$, where $N^2 = -\left(\frac{g}{\rho}\right)\rho_z$, represents the stability of the water column, and shear, $S$, represents an energy source for instability. Small values of $R_i (<1/4)$ are associated with Kelvin-Helmholtz instability. Through this instability, the inertial shear is concentrated into discrete vortices (Kelvin-Helmholtz billows) with axes aligned horizontally and perpendicular to the current. Ultimately, the billows overturn and generate small-scale turbulence and mixing. Some of the energy released by the instabilities propagates along the stratified layer as high frequency internal gravity waves. The mixing of fluid from below the mixed layer by inertial shear contributes to increasing the density of the mixed layer and to mixed layer deepening.
Temperature ramps

Another form of coherent structure in the upper ocean has been observed in both stable and unstable conditions. In the upper few meters temperature ramps, aligned with the wind and marked by horizontal temperature changes of 0.1K in 0.1m, indicate the upward transport of cool/warm fluid during stable/unstable conditions. This transport is driven by instability triggered by the wind and perhaps similar to the Kelvin-Helmholtz instability discussed above.

2.3.3. Solar penetrative profile.

Seawater properties will change under the accumulation of hydrosols (suspended particles, and yellow sediments), which attenuates the penetrated solar radiation. Normally the long-wave component of the solar radiation of the visible spectrum (300nm - 700nm) being absorbed in the top few centimeters and the remaining component of the radiation penetrates up to a depth of 20m – 40m. This characteristic depth of penetration may change depending on the constituents in the ocean waters.

At the depth $h$, the effect of solar radiation having penetrated the mass of water above is generally expresses as a function of radiative flux through the surface $\Phi_0$ and of the extinction coefficient $\kappa_s(h)$. So, we write the solar radiation penetration to the depth $h$;

$$S(h) = \Phi_0 \exp \left\{ - \int_0^h k_s(z) \, dz \right\}$$
The extinction coefficient is often considered as constant. Then we simply have:

\[ S(h) = \Phi_0 \exp\left(-\kappa_s h\right) \]

so, at depth \( h \), the energy acquired from solar radiation is written:

\[ R(h) = \frac{\partial S}{\partial z} \bigg|_h = \kappa, \Phi_0 \exp -\kappa, h \]

Attenuation of solar radiation in the water bodies is of great interest for both marine optics and biology. A reduction of solar radiation occurs by penetrating in the ocean water, caused by a mixture of particle of different composition, increases with decreasing wavelength (Kirth, 1985). Though the scattering depends weakly (for large particles) on wavelength, the effect of selectivity is largely caused by absorption.

It is known that, the useful wave band of light for photosynthesis is from 350nm to 700nm (Dera, 1992). As far as the suitability of light for photosynthesis is concerned, the end of the useful wave band is not irreversibly fixed in the literature, as the influence of light on photosynthesis is complicated. Often the useful range of light is taken to be the visible wave band, 400-700nm. Since ultraviolet is strongly observed in the water, the energy differences in the water depth ensuing from such shifts in the endpoints of the wave band concerned are small, around 1% (Dera, 1992). Accordingly, most of the Photo-synthetically Active Radiation (PAR) wave band sensors measure in the range 360nm-700nm.
2.3.4. Evapo-precipitation

Evaporation minus precipitation is the mass transfer quantity at the air-sea interface. The increase in one quantity will lead to decrease the other quantity. Here the evaporative term is an outgoing mass quantity, and the other is the condensation term, which is entering as fresh water at the air-sea interface. Outgoing mass term (evaporation) induces instability in the upper ocean by increasing the density. Thereby it makes a shift in halocline profile. If this discontinuity is not balanced by thermal shift, a diurnal pycnocline starts forming between the upper layer and the permanent thermocline or halocline. On the other hand the incoming fresh water mass makes the upper ocean to a less dens and hence it becomes more stable. A temporary thermocline can be seen in the form of an inversion to the normal profile. Rainfall on the sea surface can catalyze several important processes that act to both accentuate and reduce upper ocean mixing. Drops falling on the surface disrupt the viscous boundary layer, and may carry air into the water by forming bubbles. Rain is commonly said to ‘knock down the seas.’ The evidence for this is the reduction in breaking wave intensity and white capping at the sea surface. Smaller waves (<20cm wavelength) may be damped by surface turbulence as heavy rainfall acts to transport momentum vertically, causing drag on the waves. The reduced roughness of the small-scale waves reduces the probability of the waves exciting flow separation on the crests of the long waves, and hence reduces the tendency of the long waves to break. While storm winds generate intense turbulence near the surface, associated rainfall can confine this turbulence to the upper few meters, effectively insulating the water
below from surface forcing. This is due to the low density of fresh rainwater relative to the saltier ocean water due to evaporation. Turbulence must work against gravity to mix the surface water downward, and turbulence mixing is therefore suppressed. So long as vertical mixing is inhibited, fluid heated during the day will be trapped near the sea surface. Preexisting turbulence below the surface will continue to mix fluid in the absence of direct surface forcing, until it decays due to viscous dissipation plus mixing, typically over the time scale of a buoyancy period, $N'$. 

2.4. Basic Mixing Mechanism

Two types of physical mechanisms are governing the evolution of marine mixed layer; namely the radiative mixing and the wind mixing.

2.4.1 Radiative mixing

Solar heating has a profound effect on the depth of convection because it is concentrated close to the surface. Half of the solar energy is absorbed in the top meter of the ocean. For several hours centered at noon each day at most locations around the world, the oceanic heat gain from the sun is more than double the loss to the atmosphere. Late evening and before noon of the next day sea surface cools and this cold dense fluid, which later sink to a depth determined by the local stratification in a convective process. Cooling occurs almost every night and sometimes during daytime in association with local weather systems such as cold air outbreaks from continental landmasses. Convection may also be caused by an
excess of evaporation over precipitation, which increases salinity, and hence
density, at the surface. Winds aid convection by a variety of mechanism, which
agitates the sea surface, thereby disrupting the viscous sub-layer and permitting
rapid transfer of heat through the surface. Convection in the ocean is analogous to
that found in the daytime atmosphere boundary layers, which are heated from
below. Retrospective studies of atmospheric convection have helped in
understanding the ocean's behavior. The depth of convection is then less than one
meter, regardless, of the depth of the turbocline (where the vertical gradient of
turbulence is large), the later normally being much more than a-meter. At night,
the heat loss to the atmosphere is supplied convectively from heat storage during
the day and, in the winter season, during earlier days. The depth of convection
then nearly reaches the turbocline defining the bottom of the mixed layer. Surface
tension and viscous forces initially prevent dense, surface fluid parcels from
sinking. Once the fluid becomes sufficiently dense, however, these forces are
overcome and fluid parcel sinks - this process is called convective plumes. The
relative motions of the plumes help to generate small-scale turbulence, resulting in
a turbulent field encompassing a range of scales from the depth of the mixed
layer. The depth of convection is limited by the local thermocline. Mixing due to
penetrative convection in to the thermocline represents another source of cooling
of the mixed layer above. Within the convective layer, there is an approximate
balance between buoyant production of turbulent kinetic energy and viscous
dissipation.
At the top of the convective layer in which the fluid parcel/sea water vertical motion takes place due to density variation (Fig. 2.2). Convective process takes place when the seawater instability occurs due to solar heating and action of wind at the surface. It is embedded in the mixed layer, which is conventionally turbulent. The bottom of the mixed layer is defined by the turbocline (the layer in which the turbulent process occurs) below which turbulence is intermittent and an average very much weaker. The diurnal (the hourly variation) pycnocline lies between the turbocline and the top of the seasonal pycnocline that extends down to the top of the permanent pycnocline. The diurnal pycnocline disappear every night and the seasonal pycnocline every night.

Fig. 2.2. A typical structure of the layers in the upper ocean defined based on density profile. The mixed layer defined here with uniform density values. Pycnocline region is divided according to the nature of persistence, namely diurnal pycnocline, seasonal pycnocline, and permanent pycnocline. (Adopted from Woods and Barkmann, 1986).
winter; the permanent pycnocline is always present. The water column is statically unstable (Brunt-Vaisala frequency $N$ is negative) in the convective layer ($0 < Z < C$) and statically stable ($N^2 > 0$) in the rest of the mixed layer ($C < Z < H$). In the diurnal pycnocline ($H < Z < H_{max}$), the seasonal pycnocline ($H_{max} < Z < D$) and the permanent pycnocline ($Z > D$) the water masses are statically stable ($N^2 > 0$).

2.4.2. Wind mixing

Convection is aided by wind forcing because winds help to disrupt the viscous sub-layer at the sea surface by permitting rapid transport of heat. Atmospheric forcing of the upper-Ocean Boundary Layer (OBL) plays a fundamental role in regulating the sea surface temperature of the world's ocean (Eric et al., 2000). The most direct influence of the atmosphere is through surface fluxes of latent and sensible heat. During conditions of strong surface heat loss and weak forcing, the Ocean boundary layer behaves much like a well-mixed, convective boundary layer with turbulent fluxes that are in agreement with Monin-Obukhov similarity theory near the surface and a mixed layer structure that scales with the surface buoyancy flux (Shay and Gregg, 1986; Lombardo and Gregg, 1989). Often however, upper Ocean mixing is driven by wind and surface wave forcing, with entrainment flux at the mixed layer base dominating the Ocean boundary layer heat budget and the surface heat flux having a secondary role. Therefore, the Ocean boundary layer behaves more like a stratified boundary layer and cannot be easily described through classical boundary layer theory (Mahrt, 1999). Build up of momentum in the Ocean boundary layer through inertial resonance has been
well documented through observation and one-dimensional modeling studies (Crawford and Large, 1996; Large and Crawford, 1995). What has been thoroughly explained is how the upper ocean currents create and interact with turbulence and the stratified pycnocline at the boundary layer base. A schematic diagram, figure 2.3, shows the pathways that wind energy follows in driving inertial currents and turbulence. In the Ocean boundary layer, energy provided by the wind is partitioned between the mean current and turbulence generated by shear production and wave-current interaction (Stokes production). Some fraction of this energy is removed through turbulent dissipation, $\varepsilon$. Another portion goes into vertical mixing of thermocline water, thereby reducing the Ocean boundary layer temperature and increasing wave energy is generated at the mixed layer base through the shear of the mean inertial current. Energy from this process is also

![Schematic diagram of wind force mixing in marine mixed layer](After Eric et al., 2000)
used to mix thermocline water and is dissipated through friction and internal wave propagation.

2.5 Mixed-Layer Models

Given the inputs of heat, fresh water, and momentum from the atmosphere, a one-dimensional mixed layer model predicts the depth of the mixed layer and in some models the shape of the mixed layer base as a function of time. There are three basic families of mixed layer models: the “bulk Turbulent Kinetic Energy (TKE)” models, the “shear instability” models, and the “turbulent closure” models (Martin, 1985, 1986; Niiler and Kraus, 1977). The assumed physical mechanisms by which entrainment occurs differ fundamentally among the three families, but all are reasonably successful at predicting the observed depth of the mixed layer.

Integrated TKE, or “Bulk”, models

The so-called bulk turbulent kinetic energy model, initially formulated by Kraus and Turner, (1967), treats mixing based on a budget for the integrated turbulent kinetic energy of the surface ocean. A fundamental assumption of this model is that the mixed layer is completely homogeneous in the various state variables (temperature, salinity, surface current, turbulent kinetic energy, solutes, etc.). This assumption appears to be well founded in most parts of the surface ocean (Price et al., 1986; Martin, 1986). A heat balance is constructed which accounts for exchange with the atmosphere and fluxes by radiative transfer, and cooling caused
by entrainment of colder water from below (as the mixed layer depth, $h$, changes with time):

$$ h \frac{dT_s}{dt} = \lambda \frac{dh}{dt} \Delta T - \frac{1}{\rho C_p} \left( Q_f - Q_{blue} - e^{-\beta t} - Q_h - L_f - E - H \right) $$

(1)

where,

$\lambda = 0$, if $f(dh, dt) < 0$

and

$\lambda = 1$, if $f(dh, dt) > 0$

$T_s$ is the sea surface temperature, and $\Delta T$ is the temperature difference across the mixed layer base.

The two-degree of freedom in equation (1) are sea surface temperature and the depth of the mixed layer; another independent equation is necessary to close the system. The requisite constraint is based on a kinetic energy budget. The source of turbulent kinetic energy is the wind stress (a scaling coefficient, $m$, times the "friction velocity" of the wind, $U^3$) and the sink is dissipation ($D$). The residual kinetic energy (the difference between input and dissipation) is transformed into potential energy by mechanical mixing of cold water into the mixed layer (also, the average $TKE$ of the mixed layer decreases as quieter water is entrained):

$$ \frac{1}{2} \frac{dT_s}{dt} h^2 + \lambda \Delta Th \frac{dh}{dt} = m U^3 - D $$

Since (Kraus and Turner, 1967), many other workers have used models based on this formulation, and much of the work has centered on the dissipation of
turbulence (Stevenson, 1979). The generation and dissipation of turbulence is the greatest weakness of this type of model; in the case of dissipation in particular, the link between the model and the actual physics is week. Some authors scale turbulent kinetic energy dissipation as a constant loss rate times the mixed layer depth (Kim, 1976; Niiler and Kraus, 1977). Others calculate the dissipation rate as a function of the total amount of mixed layer turbulent kinetic energy, the Coriolis parameter (Grawood, 1977) and/or the Monin-Obukhov length scale for turbulence (Gasper, 1988). To simulate the mixed layer response to hurricanes, (Elsberry et al., 1976) used a parameterization for dissipation as a fraction of surface input of turbulent kinetic energy that increases with increasing mixed layer depth.

The other weakness of the bulk turbulent kinetic energy family of models is the generation of turbulent kinetic energy (calculated as a constant, $m$, times the wind stress). The value of $m$ is "tuned" to fit the model predictions to observed data, and the best value for $m$ tends to vary with location and conditions (ranging from 0.1 to 0.39 (Martin, 1985), 0.3 to 0.9 (Price et al., 1978) and 0.4 to 0.5 (Davis et al., 1981)). The instability of $m$ and the uncertainty about turbulent kinetic energy dissipation certainly detract from the predictive ability of the bulk turbulent kinetic energy type model.
Shear Instability models

Another potential source of turbulent kinetic energy, not considered in the original Kraus-Turner formulation, is the generation of turbulence by current shear. Although the only source of energy to the mixed layer is input by wind, in the shear models this energy is assumed to generate mean flow rather than turbulent energy, and the flow is converted to turbulence at the base of the mixed layer (Gargett et al., 1979).

Formulation

Mixing in a stratified fluid is governed by the gradient of velocity with depth (shear) and the density stratification (Ellison and Turner, 1959). The relevant non-dimensional parameter is calculated as the ratio so these quantities (from Price et al., 1986),

\[
Rg = \frac{g \frac{\partial \rho}{\partial z}}{\frac{1}{\rho_0} \left( \frac{\partial u}{\partial z} \right)^2}
\]

and is called the gradient Richardson number.

The Richardson number used in most mixed layer models is defined somewhat differently. The mixed layer is viewed as a slab, with uniform velocity and density. Instead of the differential quantities \( \frac{\partial \rho}{\partial z} \) and \( \frac{\partial u}{\partial z} \), the differences \( \frac{\Delta \rho}{h} \) and \( \frac{\Delta u}{h} \) are used, where \( \Delta \rho \) and \( \Delta u \) are the changes in density and velocity across the mixed layer base. The length scale is the thickness of the mixed layer, \( h \), rather than a scale associated with the thickness of the mixed layer base. The
A dimensionless number thus defined, \( R_b = \frac{g \Delta \rho h}{\rho_o (\Delta u)^2} \) is called the bulk Richardson number (Pollard et al., 1973; Price et al., 1986). In the shear models, mixing is assumed to begin when \( R_b \) falls below a critical value, and water is entrained until \( R_b \) reaches the critical value once again. Models based on the bulk Richardson number predict mixed layer variations quite well, and the only potential "tunable" parameter is the critical Richardson number, which appears to be stable throughout a variety of locations and chemical conditions (Price et al., 1978; Price et al., 1986). Theoretical justification for use of bulk, rather than the gradient, Richardson number for these models is still a matter of discussion (Pollard et al., 1973; Price et al., 1978).

The shear instability model of (Price et al., 1986) uses both \( R_b \) and \( R_g \) to determine mixing. The model is used to simulate detailed diurnal fluctuations in the mixed layer depth and current profiles from the subtropical Pacific. In the model, the heat fluxes (except penetrating solar radiation) and the wind stress are applied to the mixed layer. The bulk Richardson number at the base of the surface box is calculated, and if the shear is greater than the density stratification necessary to support it \( (R_b < 0.65) \), then the properties of the two adjacent boxes are averaged (the boxes are mixed). This process continues downward until \( R_b > 0.65 \). The mixed layer is considered homogeneous for momentum and density, as it is in the bulk turbulent kinetic energy formulations considered above. The density transition at the mixed layer base is smoothed using the gradient Richardson number. If \( R_g \) between two adjacent boxes is smaller than a critical value \( (R_g < \)
0.25), the adjacent boxes are partially mixed until the shear becomes sub-critical again, and the process is repeated until $R_x$ is greater than or equal to the critical value throughout the water column. Thus, there is a region of partial mixing below the completely mixed zone. The model also includes convective entrainment driven by surface cooling.

**Behaviour**

A model in which the generation of turbulence scales with the Richardson number behaves differently from a model, which scales the turbulence directly with wind stress (Price et al., 1978). This is largely because of the rotation of an inertial current to the surface of the earth (the Coriolis effect). In the presence of steady, non-rotating wind forcing, a natural limit is imposed on the current velocity by the "inertial" rotational frequency (the value for which varies as a function of latitude). The advantage of scaling entrainment to the current velocity, as opposed to scaling with the wind stress directly, is that the current velocity is limited by the rotational frequency, and no artificial dissipation term is required to limit the steady-state mixed layer. (With constant wind forcing and no dissipation term, a turbulent kinetic energy model would entrain forever). Treatment of dissipation is one of the major weaknesses in the turbulent kinetic energy model formulation. Also, the value of the critical $R_b$ required to simulate oceanographic data using a shear instability mixed layer model is fairly constant throughout a range of climatic conditions (Price et al., 1978; Pollard et al., 1973; Price et al., 1986). This can be contrasted with the wind-scaling coefficient used in the turbulent kinetic
energy model, which must be tuned to simulate data from different locations or climatic regimes.

**Turbulence closure models**

The third family of upper ocean mixing models is the most general, the most complicated, and the most stringently founded in the theoretical and empirical properties of fluid turbulence. These are the “turbulence closure” models, first introduced into the mainstream oceanography literature by Mellor, 1973 (also Mellor and Yamada, 1974, 1982; Mellor and Durbin, 1975). Turbulence closure models were originally constructed for use in the atmospheric boundary layer. In the oceans, Mellor’s model is general enough to be applied to special cases like the equator, where the equatorial undercurrent produces regions of extremely high shear and to a system of estuarine circulation, spanning the benthic boundary layer, the highly stratified shear flow between the saline and fresh waters, and the surface boundary layer.

**Formulations**

The physical foundation for the turbulence closure models begins with the Reynolds’ equations for momentum and heat (from Mellor and Yamada, 1974):

\[
\frac{\partial U}{\partial t} + U \nabla U + f \times U = \nabla p - g \beta \theta + \nu \nabla^2 U \quad \text{and} \quad \frac{\partial \theta}{\partial t} + U \nabla \theta = \alpha \nabla^2 \theta
\]
where, $U$ is velocity, $\theta$ is temperature, $f$ is Coriolis parameter, $P$ is pressure, $g$ is the acceleration due to gravity, $\nu$ is the molecular viscosity, $\alpha$ is the thermal diffusivity, and $\beta$ is the thermal expansion coefficient. The equations for the mean flow and temperature can be found by decomposing the total velocity and temperature fields into mean and fluctuating components and averaging:

\[
\frac{\partial U}{\partial t} + U \cdot \nabla U + f \times U = \nabla P - g \beta \theta + \nu \nabla^2 U - \nabla \cdot \overline{uu} \quad \text{and}
\]

\[
\frac{\partial \theta}{\partial t} + \nabla U \theta = \alpha \nabla^2 \theta - \nabla \cdot \overline{u\theta}
\]

the terms $\overline{uu}$ and $\overline{u\theta}$ represent the effect of the turbulent fluctuations on the mean velocity and temperature (the Reynolds stress and "eddy diffusion"). Finding analytical expressions for these terms is the problem at the heart of the turbulence closure models.

Derivation of expressions for these terms begins with these two sets of equations, and incorporates several empirical simplifying relationships based on laboratory data (Mellor and Yamada, 1974). The salient point to understand about the derivation is that the solutions contain several empirical constants, which are fit to laboratory data from neutral (de-stratified) flows. Mellor and Yamada, (1982) showed the surprising result that the model, using these constants, is also able to predict laboratory data for stratified flow, flow in a pipe, and flow over a curved surface. Some of the terms in the equations also require a constant with units of length to maintain dimensional homogeneity. Mellor assumes that the length scales are all proportional to a single "mixing length scale" ($l$). This assumption,
and the derivation of a number for $l$, is considered a weak link in the turbulent closure scheme. The choice of $l$ is crucial; the levels 2 and 2.5 models supply eddy diffusion coefficients that scale directly to $l$.

The mixing coefficient predicted by the model turns out to be a function of the local gradient Richardson number (Niiler and Kraus, 1977; Mellor and Durbin, 1975); above $R_g = 0.23$, no mixing at all is predicted. This is similar to the Price model, in which mixing takes place until a critical $R_g > 0.25$ is attained. The level 2.5 model has been used in a global ocean model that focuses on the surface ocean and the sea surface temperature (Rosati and Miyakoda, 1988).

**Behaviour**

The turbulence closure models are more difficult to understand in an intuitive way than the other models. Insight can be gained by comparing the behavior of the model with those of the bulk turbulent kinetic energy and shear models described above.

One obvious distinction is the degree of mixing in the turbulence closure model mixed layer. Whereas the other formulations assume homogeneity of all properties within the mixed layer, the turbulence models predict high but finite mixing coefficients within this zone. Under conditions of deep convective mixing, the "bulk" assumption (that the mixed layer is completely homogeneous in all properties) is probably inadequate, but under conditions of shallower, wind-driven
mixing, the "bulk" approximation may be closer to reality than the output of the turbulence models (Martin, 1986).

The fundamental distinction between the bulk and shear models presented earlier is the source of the turbulent energy for entrainment: from wind stress directly or from current shear. The main source of entrainment energy in the turbulence closure models is difficult to judge from first principles. For the primary source of turbulence to be generation at the surface, the rate of transport of turbulent energy through the mixed layer must be greater than the rate of generation of turbulence by shear at the mixed layer base. In the level-2 model, transport of turbulence does not occur; all turbulence is dissipated locally. Numerical experiments using level-2.5 and higher have shown that the transport flux of turbulent energy from the surface zone in these models is also small relative to the generation of turbulence by shear (Klein and Coantic, 1981; Martin, 1986). Thus the turbulence closure models appear to function primarily as shear instability models.

**Computational efficiency**

Martin, 1986 also compared the computation time required by the different types of models. As written, the bulk model is the fastest. The Price shear-driven model is significantly slower, mainly because of calculations involved in maintenance of the partially mixed zone beneath the mixed layer (based on $R_g$, explained above), which can be eliminated (Archer et al., 1993). The turbulence models are the slowest, because of their complexity and the higher temporal and spatial
resolution required. Turbulence models are the most suitable for use in a vectorized or parallel computer architecture, which would reduce somewhat the computation disparity between the model types.

2.6 Summary

Mixed layer is a region of stable stratification that partially insulates the upper ocean from the ocean interior. Heat, momentum, and chemical species exchanged between the atmosphere and the ocean interior must traverse the centimeters thick cool skin at the very surface, the surface layer, and the mixed layer to modify the stable layer below. These vertical transports are governed by a combination of processes, including those that affect only the surface itself (rainfall, breaking surface gravity waves), those that communicate directly from the surface throughout the entire mixed layer (convective plumes) or a good portion of it (Langmuir circulations) and also those processes that are forced at the surface but have effects concentrated at the mixed-layer base (inertia shear, Kelvin-Helmholtz instability, propagating internal gravity waves). Several of these processes are represented in schematic form in figure 2.4.

All of the models can adequately predict observed fluctuations in the mixed layer depth of the ocean, under most conditions. There may be situations (for example, the equatorial undercurrent, or regions near a western boundary current), which are more complicated and these will probably be better handled by the turbulence closure scheme. A criterion for comparison of the model formulations is the
quantity and stability of the empirical parameters required for oceanic simulation. The bulk turbulent kinetic energy models appear to be weakest in this respect in that the coefficient which predicts turbulent kinetic energy flux into

Fig. 2.4: Contributors to mixing the upper ocean, courtesy to Moun and Symth, 2001: Diagram showing processes that have been identified by a wide range of observational techniques as important contributors to mixing the upper ocean in association with surface cooling and winds. The temperature ($\theta$) profiles shown here have the adiabatic temperature (that due to compression of fluid parcels with depth) removed; this is termed potential temperature. The profile of velocity shear (vertical gradient of horizontal velocity) indicates no shear in the mixed layer and nonzero shear above. The form of the shear in the surface layer is a current area of research. Shear-induced turbulence near the surface may be responsible for temperature ramps observed from highly resolved horizontal measurements. Convective plumes and Langmuir circulations both act to redistribute fluid parcels vertically; during convection, they tend to move cool fluid downward. Wind-driven shear concentrated at the mixed-layer base (thermocline) may be sufficient to allow instabilities to grow, from which internal gravity waves propagate and turbulence is generated. At the surface, breaking waves inject bubbles and highly energetic turbulence beneath the sea surface and disrupt the ocean's cool skin, clearing a pathway for more rapid heat transfer into the ocean.

the mixed layer based on the wind stress seems to vary as a function of climatic conditions. The empirical parameters used by the shear instability models are the critical Richardson numbers $R_b$ and $R_g$, which do not vary in this way (Price et al.,
The constants in the turbulence closure models were determined from laboratory experiments, and appear to be adequate for oceanic use (Rosati and Miyakoda, 1988). The detailed comparisons of each model in terms its ability, to simulate the ocean state, is given by Martin, (1985; 1986). The latter study is an extension of the former, with the inclusion of the (Price et al., 1986) model and a turbulence closure scheme other than Mellor’s (Therry and Lacarrere, 1983).

The conclusions are the following. First, the turbulence closure schemes are essentially Richardson number driven models, and the main differences among them (and the Price et al., 1986 model) are the exact value of the critical Richardson number at which mixing commences. Second, the bulk turbulent kinetic energy models do not attempt to simulate partial mixing at the base of the mixed layer, as do the Price model and the turbulence closure schemes. For this reason, the bulk turbulent kinetic energy model mixed layer base is unrealistic. Third, the turbulence models predict a greater gradient of momentum and density through the mixed layer than is observed (Price et al., 1986); apparently, some mechanism for mixing exists (such as Langmuir circulation) that is not included in the 1-Dimensional turbulence closure model formulation. Finally, under some circumstances, each of the models predicts mixed layer depths that deviate somewhat form the observed data.