Chapter - III

Self Similar Power Driven Isothermal Flow Behind Cylindrical Shock in Monochromatic Radiation with Gravitational force

INTRODUCTION

A self similar theoretical model of power driven isothermal expansion in a non homogeneous medium in the presence of monochromatic radiation with gravitational force is considered. The result discussed depends upon variation of the flow variables behind the shock which are displayed graphically. Gas is assume to be gray and opaque shock.


We have studied the propagation of cylindrical shock wave in a magnetogasdynamics rotating non uniform atmosphere in the presence of monochromatic radiation and gravitation. The shock is assumed to be propagating in a conducting
medium at rest with density varying as \( r^\beta \) \((-2 < \beta \leq 0\). The magnetic field distribution varies as \( r^\alpha \) \((\alpha < 0\) and is directed tangential to the advancing shock front. The radiation flux moves through the gas with a constant intensity in the direction opposite to that of the propagation of shock wave. Further, the rotating gas does not radiate itself and energy in absorbed only behind the shock wave. The radiation pressure and energy are very less hence neglected.

For isothermal expansion of the plasma the internal heat generation per unit mass is identical for all elements of the fluid behind surface of discontinuity and is only a function of time so that for isothermal case.

\[
\nu = \mu = \sigma = 0
\]  

(ii)

The medium in which expansion takes place is heterogeneous. The shock radius is a function of time. Thus density and shock radius have been assumed to obey the following power law.

\[
\rho_o = A t^\alpha
\]  

(iii)

\[
R = B t^\beta
\]  

(iv)

Where \( \rho_o \) is the density of the ambient medium and \( R \) is the shock radius. Here \( A, B, \alpha \) and \( \beta \) are constants.

EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

In accordance with the above assumption the motion of an inviscid perfect gas in a magnetogasdynamics rotating non-uniform medium in presence of monochromatic radiation and gravitation can be described by the following system of differential equation

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial t} + \frac{\rho u}{r} = 0
\]  

\[
(3.01)
\]

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + \frac{hu}{r} = 0
\]

\[
(3.02)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{h}{\rho} \frac{\partial h}{\partial r} + \frac{h^2}{pr} + \frac{Gm}{r} - \frac{v^2}{r} = 0
\]

\[
(3.03)
\]
\[
\frac{d}{dt} (v, r) = 0
\]  
\[
\frac{\partial m}{\partial r} = 2\pi pr
\]  
\[
\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{P}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r} \frac{\partial}{\partial r} (jr) = 0
\]  
\[
\frac{\partial j}{\partial t} = kj
\]  

Where \(u, P, \rho, h, m, v, j\) and \(e\) are radial component of velocity, pressure, density, magnetic field, mass per unit volume, azimuthal component of velocity monochromatic radiation flux, energy per unit mass at a radial distance \(r\) and time \(t\) respectively. \(G\) represents the gravitational constant and \(K\) is the absorption coefficient.

\[
e = \frac{P}{\rho(\gamma-1)}
\]

Treating the cylindrical shock front heading the expanding plasma as a surface of discontinuity. The jump conditions may written Whitam [6] as

\[
P_1 = \frac{2}{(\gamma+1)} \rho_0 \dot{R}^2
\]

\[
\rho_1 = \frac{\gamma + 1}{(\gamma - 1)} \rho_0
\]

\[
h_1 = \frac{\gamma + 1}{(\gamma - 1)} h_0
\]

\[
u_1 = \frac{2}{\gamma + 1} \dot{R}
\]

\[
v_1 = \frac{2}{\gamma + 1} \dot{R}
\]

\[
ej = \frac{2 \dot{R}^2}{(\gamma + 1)}
\]

\[
m_1 = m_0 = 2\pi \rho^* r^{2+\beta}
\]
Where suffixes ‘1’ denotes state jump behind shock surface and ‘0’ denotes state of flow variables jump ahead of the shock surface and the dot’s stands for differentiation with respect to time.

The Alfvén mach number and usual mach number are defined as

\[ M_A^2 = \frac{\rho_0 \dot{R}^2}{\dot{R}^2} \quad \text{and} \quad M^2 = \frac{\rho_0 \dot{R}^2}{\gamma P_0} \]

respectively where \( \dot{R} = \frac{dR}{dt} \) is the speed of shock.

The absorption co-efficient \( k \), is considered as

\[ k = k_o \rho^n P^m j^s r^t \]  

(3.18)

where the dimension of constant \( k_o \) is given by

\[ [K_o] = M^{-n-m-q} L^{3n+m-s} \quad T^{2m+3q-1} \]  

(3.19)

moreover the dimensions less constants \( J_0, P_0, \rho_0 \) are related as

\[ j_o = P_0^{1/2} \rho_0^{-1/2} \]  

(3.20)

under the equilibrium condition, we have from (3.03)

\[ G = - \left( \frac{1}{\gamma M^2} \right) + \frac{1}{(2M_A^2)} \left( 2 + \beta \right) \left( 1 + \beta \right) \left( \frac{dR}{dt} \right)^2 \pi \rho^* r^{\gamma+2} \]  

(3.21)

**SIMILARITY SOLUTIONS**

Let us consider the solution of the equation in the form

\[ u = \dot{R} \quad U (\eta) \]  

(3.22)

\[ \rho = \rho_0 \ g (\eta) \]  

(3.23)

\[ P = \rho_0 \dot{R}^2 \ X (\eta) \]  

(3.24)

\[ h = \sqrt{\rho_0} \dot{R} \ H (\eta) \]  

(3.25)

\[ v = \dot{R} \ V (\eta) \]  

(3.26)

\[ m = m_1 \ W (\eta) \]  

(3.27)

\[ J = J_o \ J (\eta) \]  

(3.28)

\[ e = \dot{R}^2 \ E (\eta) \]  

(3.29)

where \( \eta \) is a non dimensional parameter defined to be
\[ \eta = \frac{r}{R(t)} \]  

(3.30)

and thus \( \eta = 1 \) at shock front \( r = R \)

**SOLUTIONS OF EQUATIONS OF MOTION**

Equations (3.01) to (3.07) may also be transformed with the help of non-dimensional variables given in (3.22) – (3.30)

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \]  

(3.01)

By equation (3.23)

\[ \rho = \rho_0 \ g(\eta) \]

\[ \frac{\partial \rho}{\partial t} = \rho_0 \ g' \frac{\partial \eta}{\partial t} + g \frac{\partial \rho_0}{\partial t} \]

\[ \eta = \frac{r}{R} \]

\[ \frac{\partial \eta}{\partial t} = -\frac{r}{R^2} \dot{R} \]

\[ \frac{\partial \eta}{\partial t} = -\frac{\eta}{R} \dot{R} \]

By equation (ii)

\[ \rho_0 = A \ t^\alpha \quad \quad \quad R = B t^\beta \]

\[ \frac{\partial \rho_0}{\partial t} = \alpha A \ t^{\alpha-1} \]

\[ \frac{\partial \rho_0}{\partial t} = \alpha \frac{\rho_0}{t} \]

\[ \frac{\partial \rho}{\partial t} = \rho_0 \ g' \left( \frac{-\eta}{R} \right) \dot{R} + g \frac{\alpha \rho_0}{t} \]

\[ = \rho_0 \ g' \frac{-\eta \dot{R}}{R} + g \frac{\alpha}{\beta} \frac{\dot{R}}{R} \rho_0 \]

\[ \frac{\partial \rho}{\partial t} = \frac{\rho_0}{R} \dot{R} \left[ -g' \eta + g \frac{\alpha}{\beta} \right] \]
By equation (3.23)

$$\rho = \rho_0 \, g (\eta)$$

$$\frac{\partial \rho}{\partial \tau} = \rho_0 \, g' \frac{\partial \eta}{\partial \tau}$$

$$\frac{\partial \rho}{\partial \tau} = \frac{\rho_0 \, g'}{R}$$

By condition (3.22)

$$u = \dot{R} \, U (\eta)$$

$$\frac{\partial u}{\partial \tau} = \dot{R} \, U' \frac{\partial \eta}{\partial \tau}$$

$$\frac{\partial u}{\partial \tau} = \frac{\dot{R} \, U'}{R}$$

Substituting these values in equation (3.01)

$$-\rho_0 \, g' \eta \frac{\dot{R}}{R} + \frac{\alpha}{\beta} \frac{g}{g_0} \frac{\dot{R}}{R} + \dot{R} \, U \frac{\rho_0 \, g'}{R} + \frac{\rho_0 \, g \, \ddot{R} \, U'}{R} + \rho_0 \, g \frac{\dot{R} \, U'}{\eta R} = 0$$

$$-\frac{g'}{g} \, \eta - \frac{\alpha}{\beta} + \frac{U \, g'}{g} + U' + \frac{U}{\eta} = 0$$

$$-\frac{\alpha}{\beta} + \frac{g'}{g} (U - \eta) + U' + \frac{U}{\eta} = 0 \quad (3.31)$$

By equation (3.02)

$$\frac{\partial h}{\partial \tau} + u \frac{\partial h}{\partial \tau} + h \frac{\partial u}{\partial \tau} + hu = 0 \quad (3.02)$$

$$h = \sqrt{\rho_0} \, \dot{R} \, H (\eta)$$

$$\frac{\partial h}{\partial \tau} = \dot{R} \left[ H \frac{1}{2} \rho_0 \, \psi \frac{\partial \rho_0}{\partial \tau} + \frac{\partial H}{\partial \tau} \rho_0 \, \psi \frac{\partial \eta}{\partial \tau} \right] + \frac{\partial \dot{R}}{\partial \tau} \rho_0 \, H$$

By equation (iii)

$$R = B t^\beta$$

$$\dot{R} = \frac{\beta R}{t}$$

$$\ddot{R} = \frac{(\beta - 1)}{\beta} \frac{\dot{R}^2}{R}$$
\[
\frac{\partial \rho_0}{\partial t} = \rho_0 \frac{\alpha}{\beta} \frac{\dot{R}}{R}
\]

\[
\frac{\partial h}{\partial t} = \dot{R} \left[ \frac{H}{2} \sqrt{\rho_0} \frac{\alpha}{\beta} \frac{\dot{R}}{R} + H' \sqrt{\rho_0} \left( \frac{\eta}{R} \frac{\dot{R}}{R} \right) \right] + \frac{\partial \dot{R}}{\partial t} \sqrt{\rho_0} H
\]

\[
\frac{\partial h}{\partial t} = \frac{\dot{R}^2}{R} \sqrt{\rho_0} \left[ \frac{H\alpha}{2\beta} - H' \eta \right] + \frac{(\beta - 1)}{\beta} \frac{\dot{R}^2}{R} H
\]

By equation (3.25)

\[h = \sqrt{\rho_0} \dot{R} H' (\eta)\]

\[
\frac{\partial h}{\partial \tau} = \sqrt{\rho_0} \dot{R} H' \frac{\partial \eta}{\partial \tau}
\]

\[
\frac{\partial h}{\partial \tau} = \sqrt{\rho_0} \dot{R} H'\frac{R}{R}
\]

By equation (3.22)

\[u = \dot{R} U (\eta)\]

\[
\frac{\partial u}{\partial \tau} = \frac{\dot{R} U'}{R}
\]

substituting these values in equation (3.02) we get

\[
\frac{\dot{R}^2}{R} \sqrt{\rho_0} \left[ \frac{H\alpha}{2\beta} - H' \eta \right] + \dot{R} U \sqrt{\rho_0} \frac{\dot{R}}{R} H'
\]

\[
+ \frac{(\beta - 1)}{\beta} \frac{\dot{R}^2}{R} H + \sqrt{\rho_0} \dot{R} H \dot{U}' \frac{\dot{R}}{R'} + \sqrt{\rho_0} \dot{R} H \dot{U}' \frac{\dot{R}}{R R'} \frac{\eta R}{\eta R} = 0
\]

\[
\frac{H\alpha}{2\beta} - H' \eta + H' U + H U' + \frac{H U}{\eta} + \frac{(\beta - 1)}{\beta} H = 0
\]

\[
\frac{\alpha}{2} + \frac{(\beta - 1)}{\beta} H + H' (U - \eta) + H U' + \frac{H U}{\eta} = 0
\]

(3.32)

By equation (3.03)

\[
\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial \tau} + \frac{1}{\rho} \frac{\partial P}{\partial \tau} + h \frac{\partial h}{\partial \tau} + \frac{h^2}{\rho^2} + \frac{GM}{r} - \frac{v^2}{r} = 0
\]

By equation (3.22)

\[u = \dot{R} U (\eta)\]
\[
\frac{\partial u}{\partial t} = \dot{R} U' \frac{\partial \eta}{\partial t} + U \dot{R}
\]

\[
\frac{\partial u}{\partial t} = \frac{\dot{R}}{R} U' \frac{\partial \eta}{\partial \eta} \dot{R} + U \frac{(\beta - 1)}{\beta} \frac{\dot{R}^2}{R}
\]

and \[
\frac{\partial u}{\partial \tau} = \frac{\dot{R} U'}{R}
\]

By equation (3.24)

\[
P = \rho_o \dot{R}^2 X(\eta)
\]

\[
\frac{\partial P}{\partial r} = \rho_o \dot{R} X' \frac{\partial \eta}{\partial r}
\]

\[
\frac{\partial P}{\partial r} = \frac{\rho_o}{R} \dot{R} x
\]

By equation (3.25)

\[
h = \sqrt{\rho_o} \dot{R} H(\eta)
\]

\[
\frac{\partial h}{\partial \tau} = \frac{\dot{R} \sqrt{\rho_o} H'}{R}
\]

By equation (3.26)

\[
v = \dot{R} V(\eta)
\]

\[
v^2 = \dot{R}^2 V^2(\eta)
\]

\[
G = - \left[ \frac{1}{\gamma M^2} + \frac{1}{2M_A^2} \right] \frac{(2+\beta)(1+\beta)\dot{R}^2}{\pi \rho^* r \beta + 2}
\]

\[
m = m_c \ W
\]

\[
m = \frac{2\pi \rho^* r^{\beta+2}}{(2+\beta)} \ W
\]

\[
\frac{Gm}{r} = - \left[ \frac{1}{\gamma M^2} + \frac{1}{2M_A^2} \right] \frac{(2+\beta)(1+\beta)\dot{R}^2}{\pi \rho^* r^{\beta+2}} \cdot \frac{2\pi \rho^* r(\beta+2)}{(2+\beta) \eta R} \ W
\]

\[
\frac{Gm}{r} = - \left[ \frac{1}{\gamma M^2} + \frac{1}{2M_A^2} \right] \frac{(1+\beta)(2W \dot{R})}{\eta R}
\]

substituting these values in equation (3.03), we get
$$\frac{\dot{R}^2}{R} \left[ U'\eta + U \frac{(\beta - 1)}{\beta} \right] + \dot{R} U. \frac{\dot{R}}{R} U' + \frac{1}{\rho_0 g} \rho_0 \frac{\dot{R}^2}{R} + \sqrt{\rho_0} \frac{\dot{R}H}{R} \sqrt{\rho_0} H + \frac{\rho_0 \dot{R}^2 H^2}{\rho_0 g \eta R} \right]$$

$$\left[ \frac{1}{\eta M^2} + \frac{1}{2M^2_A} \right] \frac{2(1 + \beta)}{\eta} W - \frac{\nu^2 \dot{R}^2}{\eta R} = 0$$

$$(U - \eta) U' + \frac{X'}{g} + \frac{H}{\eta g} [H + \eta H'] - \left[ \frac{1}{\eta M^2} + \frac{1}{2M^2_A} \right] \frac{2(1 + \beta)}{\eta} W - \frac{\nu^2}{\eta} = 0 \quad (3.33)$$

By equation (3.04), we have

$$\frac{d}{dt} (\nu r) = 0$$

$$\left[ \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right] (\nu r) = 0$$

$$\frac{\partial (\nu r)}{\partial t} + u \frac{\partial}{\partial r} (\nu r) = 0$$

$$\nu \frac{\partial r}{\partial t} + r \frac{\partial v}{\partial t} + ur \frac{\partial v}{\partial r} + vu = 0 \quad (a)$$

By equation (3.26)

$$v = \dot{R} \frac{V}{(\eta)}$$

$$\frac{\partial v}{\partial t} = \dot{R} V' \frac{\partial \eta}{\partial t} + V \dot{R}$$

$$= - \dot{R} V' \frac{\eta \dot{R}}{R} + \frac{V(\beta - 1) \dot{R}^2}{R B}$$

$$\frac{\partial v}{\partial t} = \frac{\dot{R}^2}{R} \left[ -V' \eta + \frac{(\beta - 1)}{\beta} V \right]$$

again equation (3.26)

$$v = \dot{R} \frac{V}{(\eta)}$$

$$\frac{\partial v}{\partial t} = \dot{R} V' \frac{\partial \eta}{\partial t}$$

$$\frac{\partial v}{\partial r} = \frac{\dot{R} V'}{R}$$

$$\eta = \frac{r}{R(t)}$$
\[ r = \eta R(t) \]
\[ \frac{\partial \tau}{\partial t} = \eta \dot{R} \]

substituting these values in equation (a)

\[ \dot{R} V + \eta \dot{R} + \eta R \frac{\dot{R}^2}{R} \left[ -\eta V' + V \left( \frac{\beta - 1}{\beta} \right) \right] + \dot{R} U \eta R \frac{\dot{R} V'}{R} + \dot{R} V \dot{R} U = 0 \]

\[ V \eta + \eta \left[ -\eta V' + \frac{V (\beta - 1)}{\beta} \right] + \eta U V' + UV = 0 \]

\[ U \left[ V + \eta V' \right] - \eta \left[ \frac{(\beta - 1)}{\beta} V + \eta V' \right] = 0 \quad (3.34) \]

By equation (3.05)

\[ \frac{\partial m}{\partial t} = 2\pi \rho r \]

\[ m = m_1 W(\eta) \]

\[ \frac{\partial m}{\partial r} = m_1 W' \frac{\partial \eta}{\partial r} + W \frac{\partial m_1}{\partial r} \]

\[ = m_1 \frac{W'}{R} + W \frac{2\pi \rho^* (2 + \beta)}{r} r^{(2 + \beta)} \]

\[ = m_1 \left[ \frac{W'}{R} + \frac{W (2 + \beta)}{\eta R} \right] \]

\[ \frac{\partial m}{\partial t} = \frac{m_1}{R} \left[ W' + \frac{W (2 + \beta)}{\eta} \right] \]

Substituting putting these values in equation (3.05)

\[ \frac{m_1}{R} \left[ W' + \frac{W (2 + \beta)}{\eta} \right] = 2\pi \rho^* \eta R \]

\[ \frac{m_1}{R} \left[ W' + \frac{W (2 + \beta)}{\eta} \right] = g \frac{2\pi \rho^* r^{(\beta + 2) \eta} (\beta + 2)}{R (\beta + 2) \eta} \]

\[ \frac{m_1}{R} \left[ W' + \frac{W (2 + \beta)}{\eta} \right] = \frac{m_1 (2 + \beta)}{R \eta} g \]
\[ W' = \left[ \frac{(2+\beta)g}{\eta} \right] \left( \frac{(2+\beta)W}{\eta} \right) \]  

(3.35)

By equation (3.06)

\[ \frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{P}{\rho^2} \left[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right] + \frac{1}{\rho} \frac{\partial}{\partial r} (jr) = 0 \]  

(b)

By equation (3.29)

\[ e = \hat{R}^2 E (\eta) \]

\[ \frac{\partial e}{\partial t} = 2\hat{R} \hat{R} E + \hat{R}^2 E' \frac{\partial \eta}{\partial t} \]

\[ = 2\hat{R} \frac{(\beta-1)}{\beta} \frac{\hat{R}^2}{R} E + \hat{R}^2 E' \left( -\frac{\eta \hat{R}}{R} \right) \]

\[ \frac{\partial e}{\partial t} = \frac{\hat{R}^3}{R} \left[ \frac{2(\beta-1)}{\beta} E - \eta E' \right] \]

and

\[ \frac{\partial e}{\partial t} = \hat{R}^2 E' \frac{\partial \eta}{\partial r} \]

\[ \frac{\partial e}{\partial t} = E' R^2 \]

\[ \frac{P}{\rho} \left[ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right] = \frac{\hat{R}^3}{g^2} \frac{X}{R} \left[ -g' \eta + g \frac{\alpha}{\beta} \right] \]

\[ j = J_0 J(\eta) \]

\[ \frac{\partial j}{\partial r} = \frac{J_0 J'}{R} \]

Substituting these values in equation (b)

\[- \left[ \frac{(\beta-1)}{\beta} E + \eta E' \right] + UE - \frac{P}{g^2} \left[ \frac{\alpha}{\beta} g + (U-\eta) g' \right] + \frac{1}{Xg^2 \gamma^{3/2} M^3} [J + X J'] = 0 \]  

(3.36)

By equation (3.07)

\[ \frac{\partial j}{\partial t} = kj \]
\[ j = J_0 \, J (\eta) \]

\[ \frac{\partial j}{\partial r} = J_0 \, J' \frac{\partial \eta}{\partial r} \]

\[ \frac{\partial j}{\partial r} = \frac{J_0 \, J'}{R} \]

\[ \frac{J_0 \, J'}{R} = k_0 \, \rho^n \, j^u \, r^s \, t^l \]

\[ P_0^{* \frac{1}{2}} \rho^{\frac{1}{2}} \, J' = k_0 \, \rho^n \, P^{*m} \, j^q \, r^s \, t^l \]

\[ J' = \alpha_1 \, \eta^s \, g^n \, X^m \, J'^{q^l} \]

where \( \alpha_1 = k_0 \, \rho_0^{n+m-1} \, j^{q^l} \, \dot{r}^{2m-2s} \).

The boundary conditions are by equation (3.09) & (3.14)

\[ P_1 = \frac{2}{\gamma + 1} \, \rho_0 \, \dot{R}^2 \]

\[ \rho_0 \, \dot{R}^2 \, X (\eta) = \frac{2}{(\gamma + 1)} \, \rho_0 \, \dot{R}^2 \]

at \( \eta = 1 \)

\[ X (1) = \frac{2}{(\gamma + 1)} \]  

(3.38)

By condition (3.20) & (3.23)

\[ \rho_1 = \frac{\gamma + 1}{\gamma - 1} \, \rho_0 \]

\[ \rho_0 \, g (\eta) = \frac{(\gamma + 1)}{(\gamma - 1)} \, \rho_0 \]

at \( \eta = 1 \)

\[ g (1) = \frac{(\gamma + 1)}{(\gamma - 1)} \]  

(3.39)

By condition (3.11) & (3.25)

\[ h_1 = \frac{(\gamma + 1)}{(\gamma - 1)} \, h_0 \]
\[ \sqrt{\rho_0} \dot{R} H(\eta) = \frac{(\gamma + 1)}{(\gamma - 1)} \frac{\sqrt{\rho_0} \dot{R}}{mM_\lambda} \]

at \( \eta = 1 \) where \( M_\lambda^2 = \frac{\rho_0 \dot{R}^2}{h_0^2} \)

\[ H(1) = \frac{(\gamma + 1)}{(\gamma - 1)} \frac{1}{M_\lambda} \]

(3.40)

By condition (3.12) & (3.22)

\[ u_i = \frac{2}{(\gamma + 1)} \dot{R} \]

\[ \dot{R} U(\eta) = \frac{2}{(\gamma + 1)} \dot{R} \]

at \( \eta = 1 \)

\[ U(1) = \frac{2}{(\gamma + 1)} \]

By condition (3.13) & (3.26)

\[ v_i = \frac{2}{(\gamma + 1)} \dot{R} \]

\[ \dot{R} V(\eta) = \frac{2}{(\gamma + 1)} \dot{R} \]

at \( \eta = 1 \)

\[ V(1) = \frac{2}{(\gamma + 1)} \]

(3.42)

By condition (3.14) & (3.29)

\[ e_i = \frac{2 \dot{R}^2}{(\gamma + 1)^2} \]

\[ \dot{R}^2 E(\eta) = \frac{2 \dot{R}^2}{(\gamma + 1)^2} \]

at \( \eta = 1 \)

\[ E(1) = \frac{2}{(\gamma + 1)^2} \]

(3.43)
By condition (3.15)

\[ m_1 = \frac{2\pi \rho \rho^x \rho^{(2+\beta)}}{(2+\beta)} \]

\[ m_1 \ W(\eta) = m_1 \]

\[ \eta = 1 \]

\[ W(1) = 1 \]  

(3.44)

By condition (3.28)

\[ J_i = J_o \ J(\eta) \]

\[ J_{ii} = J_o \ J(\eta) \]

at \( \eta = 1 \)

\[ J(1) = 1 \]  

(3.45)

And appropriate transformed boundary conditions with shock front are

\[ X(1) = \frac{2}{(\gamma+1)} \]  

(3.38)

\[ g(1) = \frac{(\gamma+1)}{(\gamma-1)} \]  

(3.39)

\[ H(1) = \frac{(\gamma+1)}{(\gamma-1)} \frac{1}{M^2} \]  

(3.40)

\[ U(1) = \frac{2}{(\gamma+1)} \]  

(3.41)

\[ V(1) = \frac{2}{(\gamma+1)} \]  

(3.42)

\[ E(1) = \frac{2}{(\gamma+1)^2} \]  

(3.43)

\[ W(1) = 1 \]  

(3.44)

\[ J(1) = 1 \]  

(3.45)
RESULT AND DISCUSSION

The set of differential equation (3.31) – (3.37) have been integrated numerically with the help of boundary conditions (3.38) – (3.45) by well known Runge-kutta method for $\alpha_r = 2$ and $2.5 \beta = 1.5 & -2, \gamma = 4/3, 7/5; M^2 = 5, M^2_\infty = 10, \alpha = \frac{2(2-\gamma)}{\gamma+1}$ and $n = -1/2, M = 3/2, q = 0, s = 1$, we have plotted the graphs showing the variations of various flow parameters with distance for different values of $\gamma, \beta$ and $\alpha_r$ in presence and absence of gravitational field and in presence and absence of radiation. This helped us to study the importance of gravitation and radiation respectively on flow parameters.

From Fig. (3.11) to Fig. (3.18) we observe that the radial component of velocity, magnetic field, energy, rotational velocity, radiation flux decrease as we go towards the center of the explosion, while density and pressure increase as we go towards the center of the shock. In Fig. (3.18), it is surprising to note that the mass decreases for the case of $\gamma = 7/5$ for both presence and absence of gravitational field, it remains uniform for the case of $\gamma = 4/3$. In absence of gravitational field decrease in the radial velocity, magnetic field and energy is more prominent. In the presence of gravitational field we see that decrease in rotational velocity, radiation flux is more prominent while the increase in the value of density and pressure is more prominent.

To draw a comparison between gravitational effects vis-a-vis rotational effect, we have also observed the variations of flow parameters in the absence of rotation in fig. (3.19) to fig. (3.25) and compared it with the earlier drawn graphs for the absence of gravitation. We find that radial velocity, magnetic field, energy decrease more in presence of rotational velocity while radiation flux decreases more rapidly in the absence of rotational velocity. As expected, in the absence more rapidly in the absence of rotational velocity. As expected, in the absence of rotation, the value of pressure and density increase more as we go towards the center of the shock. However, the peculiar phenomenon to be observed is that the variation of mass remains uniform in case of $\gamma = 4/3, \alpha_r = 2$ and $\beta = 2$ irrespective of the presence or absence of the rotational
effect. We further note that the gravitational effect is important for the propagation of
shock waves in the present problem.

We have calculated our result in suitable form

\[ \frac{u}{u_1} = \frac{(\gamma + 1)}{2} \ U (\eta) \]

\[ \frac{P}{P_1} = \frac{(\gamma + 1)}{2} \ X (\eta) \]

\[ \frac{\rho}{\rho_1} = \frac{(\gamma - 1)}{(\gamma + 1)} \ g (\eta) \]

\[ \frac{h}{h_1} = \frac{(\gamma - 1)}{(\gamma + 1)} \ M_A \ H (\eta) \]

\[ \frac{e}{e_1} = \frac{(\gamma + 1)^2}{2} \ E (\eta) \]

\[ \frac{v}{v_1} = \frac{(\gamma + 1)}{2} \ V (\eta) \]

\[ \frac{m}{m_1} = W (\eta) \]
Variation of Radial velocity with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation ($\gamma=7/5, 4/3$)

Fig. 3.01
Variation of Density with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation ($\gamma=7/5, 4/3$)

Fig. 3.02
Variation of Magnetic field with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation ($\gamma=7/5, 4/3$)

Fig. 3.03
Variation of Magnetic field with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation ($\gamma=7/5, 4/3$)

Fig. 3.04
Variation of Pressure with Distance series 1&2 shows with Gravitation ($\gamma=7/5$, $4/3$) & series 3&4 without Gravitation ($\gamma=7/5$, $4/3$)

Fig. 3.05
Variation of Rotational with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation ($\gamma=7/5, 4/3$)

Fig. 3.06
Variation of Radiation flux with Distance series 1&2 shows with Gravitation (γ=7/5, 4/3) & series 3&4 without Gravitation (γ=7/5, 4/3)

Fig. 3.07
Variation of Mass with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation ($\gamma=7/5, 4/3$)

Fig. 3.08
Variation of Radial velocity with Distance

Fig. 3.09
Variation of Density with Distance

Fig. 3.10
References