Chapter - VIII

Similarity Solution of Cylindrical shock wave with radiation energy & material pressure in Magnetohydrodynamics

INTRODUCTION

The theory of shock wave and related flows are considerable physical interest. Shock waves conceivably driven by solar flares are observed to propagate into interplanetary medium. Rogers [1], Deb Ray [2] have obtained an exact analytic solution for the shock wave problem with an atmosphere of varying density. They have considered the problem taking effect of radiation energy and material pressure shock propagation at very high temperature in which the radiation effects might play a very important role through the coupling of radiation and magnetogasdynamics fields on account of the high temperature, gases are ionised over the entire region of interest in the shock and the medium behaves like a medium of very high electrical conductivity. The explosion along a line in a gas cloud has been discussed.

The propagation of shock waves has been studied by Greifigner and Cole [3] and Green span [4], Christer [5] and Ranga Rao & Ramana [6] without taking into account the radiation effects. Elliot [7], Wang [8] and Helliwell [9], have considered the effect of thermal radiation in their studies of gas dynamic using similarly method of sedov [10]. Singh [11] has studied the problem in ordinary gas dynamics. Theoretical and Experimental studied of radiative shock have been treated by C. Michaut & et al. [12].

In this chapter we consider the problem of cylindrical shock wave in magnetogasdynamics when the atmosphere is non-uniform and conducting taking counter pressure and radiation flux into account. The radiation pressure and radiation energy have been considered. The gas in the undisturbance field is assumed to be at rest and it is grey and opaque.

We suppose that the magnetic field $H_0$ and density $\rho_1$, distributions ahead of shock vary as an inverse power of radial distance from centre of symmetry i.e.
\[ H_{0i} = \frac{A}{r} \]

and

\[ \rho_t = \frac{B}{r^w} \quad (-2 \leq w \leq 2) \]

where \( A, B \) and \( w \) are constant.

**SELF SIMILAR FORMULATION**

The cylindrical polar coordinate, where \( r \) is the radial distance from axis of symmetry are used here. The equation of conservation of mass, momentum, energy and magnetic flux in the infinite conduction region behind the wave are,

\[
\frac{dp}{dt} + \rho \frac{\partial}{\partial r} (ru) = 0 \tag{8.01}
\]

\[
\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\mu H_{0i}}{r} \frac{\partial}{\partial r} (r H_{0i}) = 0
\]

\[
\frac{dH_{0i}}{dt} + H_{0i} \frac{\partial u}{\partial r} = 0 \tag{8.02}
\]

\[
\frac{dH_{0i}}{dt} + H_{0i} \frac{\partial u}{\partial r} = 0 \tag{8.03}
\]

\[
\frac{d}{dt} (e + e_r) + (p + p_r) \frac{d}{dt} \left( \frac{1}{\rho} \right) + \frac{1}{\rho r} \frac{\partial}{\partial r} (rq) = 0 \tag{8.04}
\]

\[
\rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial r} \left( p + p_r \right) + \frac{\mu H_{0i}}{r} \frac{\partial}{\partial r} (r H_{0i}) = 0
\]

where

\[
\frac{d}{dt} = \frac{\hat{t}}{\hat{t}} + u \frac{\hat{r}}{\hat{r}} \tag{8.05}
\]

and \( \rho \) is the density \( p \) the pressure, \( u \) the radial velocity, \( h \) the azimuthal magnetic field; \( q \) the heat flux, \( t \) the time and \( e \) is the material energy \( e_r \) radiation energy \( p_r \) radiation pressure. The magnetic permeability \( \mu \) is taken to be unity. For an ideal gas we have

\[
e = \frac{p \rho}{(\gamma - 1)}, \quad P = \Gamma \rho T \tag{8.06}
\]
Where \( \gamma \) is the adiabatic gas index, \( T \) the temperature and \( \gamma \) the gas constant. Assuming local thermodynamic equilibrium and taking Rosseland is diffusion approximation we have

\[
q = - \frac{cv}{3} \frac{\partial}{\partial r} \left( \sigma T^4 \right) \tag{8.07}
\]

where \( \frac{\sigma c}{4} \) is Stefan Boltzmann constant; \( C \) the velocity of light; and \( \nu \), the mean free path of radiation a function of density and temperature following Wang [13], we take

\[
\nu = \nu_0 \rho^\alpha T^\beta \tag{8.08}
\]

where \( \nu_0 \), \( \alpha \) and \( \beta \) are constant

Given \( P = \Gamma \rho T \)

By condition (8.07)

\[
q = - \frac{cv}{3} \frac{\partial}{\partial r} \left( \sigma T^4 \right)
\]

Substitute the value \( T \)

\[
q = - \frac{cv}{3} \frac{\partial}{\partial r} \left( \frac{\sigma P^4}{\Gamma^4 \rho^4} \right)
\]

\[
\nu = \nu_0 \rho^\alpha T^\beta
\]

substitute the value of \( \nu \) in above equation using

\[
q = \frac{A^2}{r^t} F(\eta)
\]

\[
\frac{A^2}{r^t} F(\eta) = \frac{c \sigma \nu_0}{3 \Gamma^4 \rho^4} \left\{ \frac{A^2 t^2}{r^4} G(\eta) \right\}^{\alpha - \beta} \left\{ \frac{A^2}{r^2} \frac{P(\eta)}{G(\eta)} \right\}^\beta \frac{\partial}{\partial r}
\]

\[
F = NG^{\alpha - \beta - 1} \rho^{\beta - 4} \left[ \frac{1}{P} \frac{dP}{d\eta} - \frac{1}{G} \frac{dG}{d\eta} \right]
\]

Where

\[
N = \frac{4 \sigma cv_0 \rho^{\alpha - 1}}{3 \gamma (\beta + 4)} \quad \alpha^1 = \frac{1}{2} \quad and \quad \beta^1 = -3 \tag{8.10}
\]

Following Singh & Vishwaka [14] the jump conditions at an isothermal shock front are taken to be
\[ \rho_2 = N \rho_1 \]
\[ P_2 = \frac{N \rho_1 \nu^2}{\gamma M^2} \] (8.12)
\[ H_{u_2} = N H_{u_1} \] (8.13)
\[ u_2 = \left(1 - \frac{1}{N}\right) \nu \] (8.14)
\[ P_{r2} = \frac{N \rho_1 \nu^2}{\gamma M^2} \] (8.15)
\[ q_2 = (N-1) \left[ \frac{1}{M^2_{\mu}} - \frac{N-1}{2N^2} \right] \rho_1 \nu^3 \] (8.16)

where

\[ N = \left( \alpha + \frac{1}{2} \right) + \left[ \left( \alpha + \frac{1}{2} \right)^2 + 2\gamma M^2 \alpha \right]^{1/2} \]

\[ \alpha = \frac{1}{\gamma} \left( \frac{M^2_{\mu}}{M} \right) \]

In which \( M \) and \( M_{\mu} \) denotes the mach number and Alfvén mach number respectively and

\[ M^2 = \frac{\rho_1 \nu^2}{\gamma P_1} \quad \text{and} \quad M^2_{\mu} = \frac{\rho_1 \nu^2}{\mu H_{u_1}^2} \] (8.17)

**SIMILARITY SOLUTION**

By the standard dimensional analysis of Sedov [10], the non dimensional variables \( \eta \) is defined by

\[ \eta = \frac{m}{\eta} \quad rt^{-6} \] (8.18)

where \( \delta = \frac{2}{(4-w)} \quad (w < 2) \) (8.18)
\[ \eta = \left( \frac{A^2}{B} \right)^{1/2} \] and the dimensionless constant \( m \) is defined such that \( \eta \) assumes the value one on the shock front. This choice enables us to write \( \eta = \frac{r}{R} \) and \( \dot{V} = \frac{dR}{dt} = \frac{\delta R}{t} \), where \( R \) is shock radius.

Then the field variables of the flow variables of the flow pattern in terms of a dimensionless function of \( \eta \) are

\[ u = \frac{r}{t} U(\eta) \quad (8.19) \]

\[ P = \frac{A^2}{r^2} P(\eta) \quad (8.20) \]

\[ H_0 = \frac{A}{r} H_0(\eta) \quad (8.21) \]

\[ \rho = \frac{A^2}{r^2} \rho(\eta) \quad (8.22) \]

\[ q = \frac{A^2}{rt^2} q(\eta) \quad (8.23) \]

\[ p_r = \frac{A^2}{r^2} p_r(\eta) \quad (8.24) \]

**Solutions of Equation of Motion**

By equation \( (8.01) \)

\[
\frac{d\rho}{dt} + \frac{\rho}{r} \frac{\partial}{\partial r} \left( ru \right) = 0 \quad (8.01)
\]

\[
\left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \rho + \frac{\rho}{r} \frac{\partial}{\partial r} \left( ru \right) = 0
\]

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{\rho}{r} \frac{\partial u}{\partial r} + \rho u = 0
\]

(a)

By equation \( (8.19) \)

\[ u = \frac{r}{t} U(\eta) \]

By equation \( (8.22) \)
\[ \rho = \frac{A^2 t^2}{r^4} \ G(\eta) \]

\[ \frac{\partial \rho}{\partial t} = \frac{A^2}{r^4} \left[ 2tG + t^2 G \ \frac{\partial \eta}{\partial t} \right] \]

\[ \eta = \left( \frac{m}{n} \right) r t^{-8} \]

\[ \frac{\partial \eta}{\partial t} = -\delta \left( \frac{m}{n} \right) r t^{-8-1} \]

\[ \frac{\partial \eta}{\partial t} = -\frac{\delta \eta}{t} \]

then

\[ \frac{\partial \rho}{\partial t} = \frac{A^2 t^2}{r^4} \left[ 2G - G' \ \delta \eta \right] \]

\[ \rho = \frac{A^2 t^2}{r^4} \ G(\eta) \]

\[ \frac{\partial \rho}{\partial r} = A^2 t^2 \left[ \frac{G'}{r^4} \ \frac{\partial \eta}{\partial r} - \frac{4}{r^5} \ G \right] \]

\[ \frac{\partial \rho}{\partial r} = \frac{A^2 t^2}{r^5} \left[ G' \ \eta - 4G \right] \]

By equation (8.19)

\[ u = \frac{r}{t} \ U(\eta) \]

\[ \frac{\partial u}{\partial r} = \frac{1}{t} \left[ r U' \ \frac{\partial \eta}{\partial r} + U.1 \right] \]

\[ = \frac{1}{t} \left[ r U' \ \eta + U \right] \]

\[ \frac{\partial u}{\partial r} = \frac{1}{t} \left[ U' \ \eta + U \right] \]

putting these values in equation (a), we get

\[ \frac{A^2 t}{r^2} \left[ 2G - G' \ \delta \eta \right] + \frac{r}{t} U \ \frac{t^2 A^2}{r^5} \left[ G' \ \eta - 4G \right] + \frac{A^2 t^2}{r^4} \ G \ \frac{1}{t} \ \left[ U + U^1 \ \eta \right] + \frac{A^2}{r^4} \ t^2 G \ \frac{1}{t} \ \frac{U}{r} = 0 \]

\[ 2G - G' \ \delta \eta + U G' \ \eta - 4G U + U G + U' G \eta + GU = 0 \]
\[
G' \eta (U - \delta) - 2GU + 2G + U' G \eta
\]

\[
\frac{dG}{d\eta} = \frac{G}{\eta(U - \delta)} \left[ 2(U - 1) - \eta \frac{dU}{d\eta} \right] \quad (8.25)
\]

Now by equation (8.02)

\[
\rho \frac{du}{dt} + \frac{\partial P}{\partial r} + \mu \frac{H_o}{r} \frac{\partial}{\partial r} (rH_o) = 0
\]

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial r} + \frac{\partial P}{\partial r} + \mu H_o \frac{\partial H_o}{\partial r} + \frac{\mu H_o^2}{r} = 0 \quad (b)
\]

By condition (8.21)

\[
H_o = \frac{A}{r} H_o (\eta)
\]

\[
\frac{\partial H_o}{\partial r} = \left[ \frac{A}{r} H_o \frac{\partial H_o}{\partial r} - \frac{\partial H_o}{r^2} \right]
\]

\[
\frac{\partial H_o}{\partial r} = \left[ \frac{A}{r} H_o \frac{\partial H_o}{\partial r} - \frac{A}{r^2} H_o \right]
\]

\[
\frac{\partial H_o}{\partial r} = \frac{A}{r^2} \left[ H_o \eta - H_o \right]
\]

By equation (8.20)

\[
P = \frac{A^2}{r^2} P(\eta)
\]

\[
\frac{\partial P}{\partial r} = A^2 \left[ \frac{1}{r^2} P \frac{\partial \eta}{\partial r} - \frac{2}{r^3} P \right]
\]

\[
= A^2 \left[ \frac{P^l}{r^2} \frac{\eta}{r} - \frac{2}{r^3} P \right]
\]

\[
\frac{\partial P}{\partial r} = \frac{A^2}{r^3} \left[ P^l \eta - 2P \right]
\]

&

\[
\frac{\partial u}{\partial r} = \frac{1}{f} \left[ U + U' \eta \right]
\]

By condition (8.19)

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\[ u = \frac{r}{t} U(\eta) \]

\[
\frac{\partial u}{\partial t} = r \left[ \frac{1}{t} U' \frac{\partial \eta}{\partial t} - \frac{1}{t^2} U \right] = r \left[ \frac{-U'}{t} \frac{\delta \eta}{t} - \frac{U}{t^2} \right]
\]

\[
\frac{\partial u}{\partial t} = \frac{r}{t^2} \left[ -U' \delta \eta - U \right]
\]

substituting these values in equation (b)

\[
\frac{A^2 t^2}{r^3} G \frac{r}{t^2} \left[ -U' \delta \eta - U \right] + \frac{A^2 t^3}{r^4} G \frac{r}{t} U \left( \frac{U + U' \eta}{t} \right) + \frac{A^2}{r^3} \left[ P' \eta - 2P \right] + \frac{\mu}{r} \frac{A^2}{r^3} H^3 = 0
\]

\[
+ \mu A H^2 \Omega \frac{A r^2}{r^2} \left[ H' \eta - H_\| \right] = 0
\]

\[
G \left[ -U - U' \delta \eta + U^2 + UU' \eta \right] + \left[ P' \eta - 2P \right] + \mu H_\| \Omega \eta = 0
\]

\[
\frac{dP}{d\eta} = \frac{G}{(U-\delta)} \left[ \frac{H^2}{G} - (U-\delta)^2 \right] \frac{dU}{d\eta} + \frac{G}{\eta} \left[ \frac{2P}{G} - U \right] (U-1) = 0 \quad (8.26)
\]

By equation (8.03)

\[
\frac{dH_\|}{dt} + H_\| \frac{\partial u}{\partial t} = 0 \quad (8.03)
\]

\[
\frac{\partial H_\|}{\partial t} + u \frac{\partial H_\|}{\partial t} + H_\| \frac{\partial u}{\partial t} = 0
\]

By condition (8.21)

\[
H_\| = \frac{A}{r} H_\| (\eta)
\]

\[
\frac{\partial H_\|}{\partial r} = \left[ A \frac{H_\|}{r} \frac{\partial \eta}{\partial r} - \frac{1}{r^2} \frac{\partial H_\|}{\partial r} \right]
\]

\[
\frac{\partial H_\|}{\partial r} = \frac{A}{r^2} \left[ H' \eta - H_\| \right]
\]

&

\[
\frac{\partial H_\|}{\partial t} = \frac{A}{r} \frac{\partial \eta}{\partial t}
\]

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\[ \frac{\partial H_o}{\partial t} = - \frac{AH_o}{r} \delta \eta \]  

By condition (8.19)

\[ u = \frac{r}{t} U(\eta) \]

\[ \frac{\partial u}{\partial r} = \left[ \frac{U}{t} + \frac{rU'}{t} \frac{\partial \eta}{\partial r} \right] = \left[ \frac{U}{t} + \frac{r}{t} \frac{U'}{r} \right] \]

\[ \frac{\partial u}{\partial r} = \frac{1}{t} \left[ U + U' \eta \right] \]

Putting these values in equation (c)

\[ - \frac{AH_o}{r} \delta \eta + \frac{r}{2} \frac{U}{t} \left[ H_o \eta - H_o \right] + \frac{H_o A}{r} \left[ \frac{U + U' \eta}{t} \right] = 0 \]

\[ H_o \eta (U - \delta) + U' H_o \eta = 0 \]

\[ \frac{dH_o}{d\eta} (u - \delta) + \frac{dU}{d\eta} H_o \eta = 0 \]

\[ \frac{dH_o}{d\eta} = -H_o \frac{dU}{d\eta} \]

By equation (8.04) we get

\[ \frac{d}{dt} \left( e + e_r \right) + \left( P + P_r \right) \frac{d}{dt} \left( \frac{1}{\rho} \right) + \frac{1}{\rho r} \frac{\partial}{\partial r} \left( rq \right) = 0 \]

\[ \left( \frac{\partial}{\partial t} + \frac{u \partial}{\partial r} \right) \left( e + e_r \right) + \left( P + P_r \right) \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \left( \frac{1}{\rho} \right) + \frac{1}{\rho r} \frac{\partial q}{\partial r} + \frac{q}{\rho r} = 0 \]

\[ \frac{\partial e}{\partial t} + \frac{\partial e_r}{\partial t} + u \frac{\partial e}{\partial r} + u \frac{\partial e_r}{\partial r} \left( P + P_r \right) \]
\[ \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho} \frac{\partial q}{\partial r} + \frac{q}{pr} = 0 \]  

(d)

\[ e = \frac{p_p}{(\gamma - 1)} \]

\[ \frac{\partial e}{\partial t} = \frac{1}{(\gamma - 1)} \frac{\partial}{\partial t} \left( p_p \rho \right) \]

\[ \frac{\partial e}{\partial t} = \frac{1}{(\gamma - 1)} \left[ \rho \frac{\partial \rho}{\partial t} + \rho \frac{\partial \rho}{\partial t} \right] \]

(i)

\[ \frac{\partial \rho}{\partial t} = \frac{A^2 t}{r^2} \left[ 2G - G' \delta \eta \right] \]

By condition (8.20)

\[ P = \frac{A^2}{r^2} P(\eta) \]

\[ \frac{\partial P}{\partial t} = \frac{A^2}{r^2} P \frac{\partial \eta}{\partial t} \]

\[ = \frac{A^2}{r^2} P' \left( - \frac{\delta \eta}{t} \right) \]

\[ \frac{\partial P}{\partial t} = - \frac{A^2}{r^2} P' \frac{\delta \eta}{t} \]

substituting these values in (i)

\[ \frac{\partial e}{\partial t} = \frac{1}{(\gamma - 1)} \left[ \frac{A^2}{r^2} P \frac{A^2 t}{r^2} \left( 2G - G' \delta \eta \right) - \frac{A^2}{r^4} t^2 G \frac{A^2}{r^2} P \frac{\delta \eta}{t} \right] \]

\[ \frac{\partial e}{\partial t} = \frac{A^2 t}{(\gamma - 1) r^6} \left[ 2PG - PG' \delta \eta - GP' \delta \eta \right] \]
By equation (8.06)

\[ e_r = \frac{(\rho \ p_r)}{(\gamma - 1)} \]

\[ \frac{\partial e_r}{\partial t} = \frac{1}{(\gamma - 1)} \frac{\partial}{\partial t} \left( \rho \ p \right) \]

\[ = \frac{1}{(\gamma - 1)} \left[ P_r \ \frac{\partial P_r}{\partial t} + \rho \ \frac{\partial P_r}{\partial t} \right] \]

By equation (8.24)

\[ \rho_r = \frac{A_2}{r^2} \ P_r \ (\eta) \]

\[ \frac{\partial P_r}{\partial t} = -\frac{A_2^2}{r^2} \ P_r \frac{\delta \eta}{t} \]

and

\[ \frac{\partial P}{\partial t} = \frac{A_2^2}{r^2} \left[ 2G - G^1 \delta \eta \right] \]

By equation (8.22)

\[ \rho = \frac{A_2^2 \ t^2}{r^2} \ G \ (\eta) \]

so

\[ \frac{\partial e_r}{\partial t} = \frac{A_4^4 \ r^6}{(\gamma - 1) r^6} \left[ 2P_r \ G - P_r \ G^1 \delta \eta - G \ P_r^1 \delta \eta \right] \]

\[ e = \frac{P}{(\gamma - 1) \rho} \]

\[ \frac{\partial e}{\partial \tau} = \frac{1}{(\gamma - 1)} \left[ \rho \ \frac{\partial P}{\partial \tau} + P \ \frac{\partial P}{\partial \tau} \right] \]

By equation (8.22)
\[ \rho = \frac{A^2 t^2}{r^3} G(\eta) \]

\[ \frac{\partial \rho}{\partial r} = A^2 t^2 \left[ \frac{G'}{r^4} \frac{\partial \eta}{\partial r} - \frac{4}{r^5} G \right] \]

\[ = A^2 t^2 \left[ \frac{G'}{r^4} \frac{\eta}{r} - \frac{4G}{r^5} \right] \]

\[ \frac{\partial \rho}{\partial r} = \frac{A^2 t^2}{r^5} \left[ G' \eta - 4G \right] \]

By equation (8.20)

\[ P = \frac{A^2}{r^3} P(\eta) \]

\[ \frac{\partial P}{\partial r} = A^2 \left[ \frac{P'}{r^2} \frac{\partial \eta}{\partial r} - \frac{2}{r^3} P \right] \]

\[ \frac{\partial P}{\partial r} = \frac{A^2}{r^3} \left[ P' \eta - 2P \right] \]

\[ \frac{\partial e}{\partial r} = \frac{1}{(\gamma - 1)} \left[ \frac{A^2}{r^4} t^2 G A^2}{r^3} \left(-2P + P' \eta\right) + \frac{A^2}{r^2} P \cdot \frac{t^2 A^2}{r^5} \left(G' \eta - 4G\right) \right] \]

\[ \frac{\partial e}{\partial r} = \frac{1}{(\gamma - 1)} \frac{A^2 t^2}{r^7} \left[-2GP + GP' \eta + GP \eta - 4GP\right] \]

\[ e_r = \frac{P_r - \rho}{(\gamma - 1)} \]

\[ \frac{\partial e_r}{\partial r} = \frac{1}{(\gamma - 1)} \left[ \rho \frac{\partial}{\partial r} P_r + P_r \frac{\partial \rho}{\partial r} \right] \]

By equation (8.24)

\[ P_r = \frac{A^2}{r^2} P_r(\eta) \]

\[ \frac{\partial P_r}{\partial r} = \frac{A^2}{r^3} \left[ P_r' \eta - 2 P_r \right] \]

and
\[
\frac{\partial p}{\partial r} = \frac{A^2 t^2}{r^5} \left[ G' \eta - 4G \right]
\]

\[
\frac{\partial e_r}{\partial r} = \frac{1}{(\gamma - 1)} \left[ \frac{A^2 t^2 G}{r^4} - \frac{A^2}{r^2} \left( P_r + P_r' \eta \right) + \frac{A^2}{r^2} P_r \frac{r^2 A^2}{r^5} \left( G' \eta - 4G \right) \right]
\]

\[
\frac{\partial e_r}{\partial \tau} = \frac{A^2 t^2}{r^7} \left[ -G P_r + G P_r' \eta + G P_r' \eta - 4G P_r \right]
\]

By equation (8.23)

\[
g = \frac{A^2}{r t} F' (\eta)
\]

\[
\frac{\partial q}{\partial \tau} = \frac{A^2}{t} \left[ \frac{F'}{r} \frac{\partial \eta}{\partial \tau} + F \left( \frac{-1}{r^2} \right) \right]
\]

\[
= \frac{A^2}{t} \left[ \frac{F'}{r} \frac{\eta}{r} - \frac{F}{r^2} \right]
\]

\[
\frac{\partial q}{\partial r} = \frac{A^2}{r^2} \left[ F' \eta - F \right]
\]

substituting these values in equation (d) we get

\[
\frac{A^4 t}{(\gamma - 1) r^5} \left[ 2(P + P_r) G - (P + P_r) G' \delta \eta - G P + (P + P_r) \delta \eta \right]
\]

\[+ \frac{A^4 t^2}{(\gamma - 1) r^7} \frac{r}{t} U \left[ -G (P + P_r) + G (P + P_r) \eta + G' (P + P_r) \eta - 4G (P + P_r) \right] = 0
\]

\[
\frac{A^2}{r^3} \left( P + P_r \right) \frac{r^8}{A^4 t^4 G^2} \left[ \frac{A^2 t^2}{r^4} \left( 2G - G' \delta \eta \right) + \frac{r}{t} U \frac{r^2 A^2}{r^5} \left( G' \eta - 4G \right) \right]
\]

\[+ \frac{r^4}{A^2 t^2 G} \left[ F' \eta - F \right] + \frac{A^2}{rt} \frac{F}{r \ A^2 G} = 0
\]

\[
\frac{dF}{d\eta} = \frac{G}{(\gamma - 1)} \left[ (U - \delta)^2 - \frac{H_0^2}{\gamma} + \frac{r (P + P_r)}{G} \right]
\]

\[- \frac{1}{\eta} \frac{G (U - \delta)}{(\gamma - 1)} \frac{2(P + P_r)}{G} - U (U - 1) + (2(P + P_r) U + F)
\]

Now by equation (8.05)
\[
\rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial r} \left( p + p_r \right) + \frac{\mu H_\theta}{r} \frac{\partial}{\partial r} \left( r H_\theta \right) = 0
\]

\[
\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\partial P_r}{\partial r} + \frac{\mu H_\theta}{r} \frac{\partial H_\theta}{\partial r} + \frac{\mu H_\theta^2}{r} = 0
\]

\[
\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial r} + \frac{\partial P_r}{\partial r} + \mu H_\theta \frac{\partial H_\theta}{\partial r} + \frac{\mu H_\theta^2}{r} = 0 \quad (e)
\]

By condition (8.19)

\[
u = \frac{r}{l} U (\eta)
\]

\[
\frac{\partial u}{\partial t} = \frac{r}{l^2} \left[ -U' \delta \eta - U \right]
\]

By condition (8.20)

\[
P = \frac{A^2}{r^2} P (\eta)
\]

\[
\frac{\partial P}{\partial r} = \frac{A^2}{r^3} \left[ P' \eta - 2 P \right]
\]

By condition (8.24)

\[
P_r = \frac{A^2}{r^2} P_r (\eta)
\]

\[
\frac{\partial P_r}{\partial r} = \frac{A^2}{r^3} \left[ P_r' \eta - 2 P_r \right]
\]

By condition (8.21)

\[
H_\theta = \frac{A}{r} H_\theta (\eta)
\]

\[
\frac{\partial H_\theta}{\partial r} = \frac{A}{r^2} \left[ H_\theta' \eta - H_\theta \right]
\]

By substituting these values in equation (e), we get

\[
\frac{A^2 l^2}{r^4} \left[ \frac{r}{r^3} \left( -U' \delta \eta - U \right) \right] + \frac{A^2}{r^3} \left[ P' + P_r' \right] \eta
\]

\[
-2 \left( p + p_r \right) + \mu \frac{A}{r} H_\theta \frac{A}{r^2} \left[ H_\theta' \eta - H_\theta \right] \right] + \mu \frac{A^2}{r^2} H_\theta \frac{2}{r} = 0
\]
\[ G \left( -U' \delta \eta - U \right) + \left( P' + P_r' \right) \eta - 2 \left( P + Pr \right) + \mu H_o^2 \left( H_0 \eta - H_o \right) + \mu H_o^2 = 0 \]

Putting of values \( \mu = 1 \) and \( H_o' \)

\[ \frac{dt}{d\eta} = \frac{2(P + P_r)}{G} \left( U - 2 \delta + 1 \right) - U \left( U - 1 \right) \left( U - \delta \right) + \frac{F'}{K} \frac{G^{\beta_{\alpha_{3,3}}}}{\left( P^{\beta_{4,3}} + P^{\beta_{4,3}} \right)} \left( U - \delta \right) \]  

\[ \left[ (U - \delta)^2 - \frac{H_o^2}{G} + \frac{(P + P_r)}{G} \right] \]

where \( k = \frac{4c_\sigma v_o}{3\Gamma A} \)

a non dimensional radiation parameter.

The transformed shock conditions are, using equation (8.11-8.16) & (8.19-8.24) & (i), (ii) By condition (8.14) & (8.19)

\[ U_2 = \left( 1 - \frac{1}{N} \right) V \]

\[ \frac{R}{t} U \left( \eta \right) = \left( 1 - \frac{1}{N} \right) \frac{\delta R}{t} \]

\( r = R \) at \( \eta = 1 \)

\[ \frac{R}{t} U \left( i \right) = \left( 1 - \frac{1}{N} \right) \frac{\delta R}{t} \]

\[ U\left( i \right) = \left( 1 - \frac{1}{N} \right) \delta \]  

By condition (i), (8.13) & (8.21)

\[ H_{\alpha_2} = NH_{\alpha_1} \]

\[ \frac{A}{r} H_o \left( \eta \right) = N, \frac{A}{r} \]

at \( \eta = 1 \)

\[ H_o \left( i \right) = N \]  

By condition (8.11) & (8.22)

\[ \rho_2 = N \rho_1 \]
\[
\frac{A^2 t^2}{r^4} G (\eta) = N \rho_i
\]

\[
\frac{A^2 t^2}{r^4} G (\eta) = N, \frac{M h^2}{v^2} H_0^2, \mu
\]

\[
\frac{A^2}{r^4} t^2 G (\eta) = N.M_h^2 \frac{A^2}{r^2} \frac{t^2}{\delta^2 R^2}
\]

\[R = r, \eta = 1\]

\[G (t) = \frac{N M_h^2}{\delta^2}\]  \hspace{1cm} (8.32)

By condition (8.12) & (8.20)

\[P_r = N \rho_i \frac{v^2}{\gamma M^2}\]

\[\frac{A^2}{r^2} P (\eta) = \frac{N.M_h^2}{\gamma M^2} \mu H_\theta^2\]

\[= \frac{N.M_h^2}{\gamma M^2} \mu \frac{A^2}{r^2}\]

\[\mu = 1 \& \eta = 1\]

\[P (t) = \frac{N.M_h^2}{\gamma M^2}\]  \hspace{1cm} (8.33)

By condition (8.15) & (8.24)

\[P_r = N \rho_i \frac{v^2}{\gamma m^2}\]

\[\frac{A^2}{r^2} P_r (\eta) = \frac{N.M \mu H_\theta^2}{\gamma M^2}\]

\[\eta = 1, \& \mu = 1\]

\[P_r (t) = \frac{N M_h^2}{M^2}\]

By condition (8.16) & (8.23)
\[ q_2 = (N-1) \left[ \frac{1}{M_h^2} - \frac{N-1}{2N^2} \right] \rho, \ v^2 \]

\[ \frac{A^2}{r \cdot t} F' (\eta) = (N-1) \left[ \frac{1}{M_h^2} - \frac{N-1}{2N^2} \right] M_h^2 \mu H_o \cdot v \]

at \( \eta = 1, \mu = 1 \)

\[ F' (1) = L M_h^2 \delta \] \hspace{1cm} (8.35)

where

\[ L (N-1) \left[ \frac{1}{M_h^2} - \frac{N-1}{2N^2} \right] \]

as the initial values for our numerical calculation

**RESULTS AND DISCUSSION**

In order to exhibit the numerical solution it is convenient to write the field variables in the non dimensional form as

\[ \frac{u}{u_2} = \eta \frac{U(\eta)}{1 - \frac{1}{N}} \delta \] \hspace{1cm} (8.36)

\[ \frac{\rho}{\rho_2} = \frac{\delta^2}{\eta^2 \frac{N}{M_h^2}} G (\eta) \] \hspace{1cm} (8.37)

\[ \frac{P}{P_2} = \frac{1}{\eta^2} \frac{\gamma M^2}{N M_h^2} P (\eta) \] \hspace{1cm} (8.38)

\[ \frac{Pr}{Pr_2} = \frac{1}{\eta^2} \frac{\gamma M^2}{N M_h^2} Pr (\eta) \] \hspace{1cm} (8.39)

\[ \frac{H_o}{H_{o_2}} = \frac{1}{\eta} \frac{1}{N} H (\eta) \]

\[ \frac{q}{q_2} = \frac{1}{\eta M_h^2 \delta} \eta^2 \frac{P(\eta)}{G(\eta)} \] \hspace{1cm} (8.40)
\[
\frac{T}{T_2} = \gamma \left( \frac{M}{\delta} \right)^2 \eta^2 \frac{P(\eta)}{G(\eta)}
\]  

(8.41)

By using equation. We have calculated our result for the following set of parameters:

\[\gamma = 1.4 \quad M_k^2 = 20, \quad M^2 = 10 \quad \delta = 5, \quad \zeta = 10\]  

(8.42)

The numerical results for certain choice of parameter are reproduced in tabular form. The nature of field variables is illustrated through table 8.01 and 8.02.

We can easily see that the radiation parameters has an important effect on the flow variables magnetoradiative effects are prominent on the field variables when we compare our results with the results of ordinary gas dynamics.
### Table 8.01

$K = 10$, $\delta = 2$

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<th>$\eta$</th>
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175
$K = 10, \delta = 5$

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References

BIO-DATA

Born in Uttar Pradesh, Jhansi on 10th July 1979 obtained his B.Sc. and M.Sc. in Mathematics from Bundelkhand University, Jhansi India and B.Ed. from Indira Gandhi National Open University, Delhi (IGNOU). Joined as a Research Scholar Topic is “Self Similarity solutions for spherical and cylindrical shock waves in magnetodynamics” under the supervision of Dr. Kishore Kumar Srivastava, Head Department of Mathematics, Bipin Bihari (P.G.) College, Jhansi, U.P. Indian on 20 June 2007.
PRESENTED IN CONFERENCES


2. National Seminar on “Application of Mathematics in Engineering & Technology”, Kishore Kumar Srivastava and Jitendra Kumar, held in Department of Applied Science in Madhav Institute of Technology and Science, Gwalior (M.P.)
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