4.1 Introduction

The European Union is bound by international and domestic laws to reduce its GHG emissions to 20% below 1990 levels by 2020. As of 2009, aviation was the largest source of GHG emissions not yet covered by measures designed to reduce GHG emissions. As the EU did not foresee effective global measures emerging from cooperation under ICAO, as mandated under the 1997 Kyoto Protocol, the EU extended its ETS to control aviation emissions effective January 1, 2012 to all domestic and international flights – from or to anywhere in the world – that arrive at or depart from an EU airport (Leggett et al., 2012). Few other countries like Australia and New Zealand have also included aviation in their domestic programs, but not international flights to and from their countries, which is the unique and probably most debated issue of the aviation sector in EUETS. Some economists are afraid of the emergence of artificial stops outside EU territory (e.g. Switzerland, Turkey), or even the complete redirection of traffic flows, due to this policy (Vespermann and Wald, 2010). This probable route configuration might even result into an environmental phenomenon commonly known as carbon leakage\(^1\) through bypassing off the EU sky by international flights (both passenger and cargo) (The Ernst & Young & York Aviation study, 2008).

In the context of the inclusion of both domestic and international flights in the EUETS, this chapter extends the theoretical model of domestic network structure of aviation by Brueckner and Zhang (2010) in an international network structure scenario. Airlines can either operate a hub-and-spoke (HS) network or a fully connected / point to point (FC) network. In a three equidistant-node symmetric city framework of domestic aviation network structure Brueckner and Zhang (2010) found that it cannot be explicitly identified which network type

\(^{1}\)Carbon leakage is said to happen if due to any carbon emission reducing effort, CO2 emission increase as a result. Since CO2 is a global pollutant, so it does not matter where the emission is increasing finally.
is optimal. Moreover, they found the effects of emission charges on the profit maximising network structure (HS vs. FC) to be ambiguous. We extend their model set up in an international scenario where the two spoke points of the HS model for airlines lie outside EU whereas the hub lies inside EU to see the conditions for the shift of the existing preference for the HS to FC network in this context. The importance of this simple extension is that, if two spoke points lie outside the EU then a preference shift towards a FC network from a HS network would imply reduction of emission within the EU but simultaneous increase in the emission outside the EU.

Before going to the theoretical model, the next section presents the basic features of the aviation sector in EUETS.

4.2. Basic features of aviation in EUETS

In the year 2012, the total quantity of CO₂ allowances allocated to aircraft operators is planned to be 97% of their historical emissions in the years 2004-2006. This so called overall ‘cap’, however, is expected to be lowered by another 2% in 2013 (Scheelhaase, 2010). However, flights by aircraft with a maximum take-off weight (MTOW) of less than 5700 kg (Scheelhaase and Grimme, 2007), operators which have less than 243 flights per year or produce emission less than 10,000 tonnes per year, police, military, rescue, humanitarian flights will be excluded from the scheme (Hanus and Vittek, 2011). Also, exemptions will be available for operators with very low traffic levels on routes to, from and within EU. Under this mechanism many operators from developing countries with only limited air traffic links with the EU will be exempted (Ares, 2012).

Allowances will be distributed to individual airlines in proportion to tonne-kilometres flown within the reference years (the benchmark method)². The allocation methodology is to be same (i.e. harmonised) across all Member States. About 82% of the cap will be issued in the form of EU Aviation Allowances and allocated to airlines free of charge. A further 15% of the cap will be in the form of allowances that are to be auctioned, while the remaining 3% will be held for new entrants as a special reserve (Clements et al., 2011). Certified emissions

² benchmark = \frac{(1 - \text{quota of auctioned allowances}) \times (1 - \text{reduced quota}) \times \sum_{i=1}^{n} \text{average annual emission}_{2004-2006}}{\sum_{i=1}^{n} \text{revenue tonne kilometers of the monitoring year}} (Scheelhaase et al., 2010)
reductions (CERs) and emission reduction units (ERUs) from the Clean Development Mechanism and the Joint Implementation of the Kyoto Protocol may be used up to an amount equalling 15% of an airline’s EUETS allocation in 2012. From 2013 onwards the usage of these credits is however unclear (Anger, 2010).

An open trading system is proposed – i.e. the airline sector can trade with all other sectors covered by the EUETS. However, this open trading is open on one side, i.e. only for buying from other sectors, but airlines cannot sell their allowances to the trading sectors other than the air transport sector itself. This is because the allowances that are issued for airlines under the EUETS are not considered within the Kyoto allowances nor included in the Kyoto targets (Anger, 2010; Anger and Kohler, 2010).

Allowances will be allowed to bank for the next accounting period. Excess emissions penalty shall be EUR 100 per tonne of emitted CO₂ equivalent, for which the operator has not surrendered allowances (Hanus and Vittek, 2011). Finally, to ensure consistent and robust enforcement throughout EU, as a last resort, Member States could ban an operator in the EU if it consistently fails to comply with the scheme (Ares, 2012).

4.3. Model

The model set up is same as Brueckner and Zhang (2010), however we have some modifications in the set up. While the three cities/airports in Brueckner and Zhang (2010) are assumed to be within EU, in our set up only one airport (the hub in HS network) is assumed to be located within EU whereas the two other airports are located outside EU. Otherwise, our analysis is also conducted in a three-node symmetric city layout with all the links equidistant and the three city pair markets assumed to have the same demand. The three cities (one in EU and the other two outside) are served by two competing international airlines denoted by $i, (i = 1, 2)$, where one is a non-EU airline (airline 1) and the other is an EU airline (airline 2). Each of the airlines either use FC network to serve the three airports/cities or both use a HS network (asymmetric network choices are ruled out). Under the FC network, passengers in the three airports are carried by direct (non-stop) flights on three routes. For a HS network, with one airport serving as the hub (the airport located inside EU in our case), there are just two spoke routes, which connect the two non-hub airports to the hub. While
spoke passengers still take direct flights, passengers travelling between the two non-hub airports must take two flights and connect at the hub. The two non-hub airports located outside EU implies the direct route between them is outside the scope of permit costs.

4.3.1 Cost

Aircraft operating cost has three components – fuel cost, leasing cost and permit cost. The fuel cost depends on aircraft fuel efficiency, which is measured as fuel consumption per seat per flight hour and denoted by $e_i$ and aircraft seating capacity $s_i$ for airline. Let $r$ denote the unit price, then the fuel cost is simply $re_i s_i$. Leasing cost per hour flown, denoted by $g_i(e_i, s_i) = \frac{\beta + es_i}{e_i}$, depends on $s_i$ and $e_i$, where $\beta$ and $e$ are positive parameters. Since a lower $e$ implies a more fuel efficient plane which is more costly and a larger plane is also costlier, therefore we have $\frac{\partial g_i}{\partial e_i} < 0$ and $\frac{\partial g_i}{\partial s_i} > 0$. Economies of larger aircraft implies that $\frac{g_i}{s_i}$ is decreasing in $s_i$. Fuel efficiency is increasingly costly i.e. $g_i$ is convex in $e_i$. Since fuel efficiency is difficult to get in larger plane, we have $\frac{\partial g_i/\partial e_i}{\partial s_i} < 0$. Let $f_i$ be the flight frequency and $k$ be the hours per flight, which is an increasing function of distance travelled $d$.

Let $z$ denote the unit permit price and $x$ be the number of permits required per unit of fuel consumed, then the total permit cost is $zxe_i s_i f_i k(d)$. In this case the total cost becomes

$$TC_i = \left[ re_i s_i + \frac{\beta + es_i}{e_i} + zxe_i s_i \right] f_i k(d) \quad (1)$$

However, before the imposition of the permit cost or in the case when permits are distributed on a free (grandfathering) basis the total cost is:

$$TC_i = \left[ re_i s_i + \frac{\beta + es_i}{e_i} \right] f_i k(d) \quad (2)^3$$

---

3 This is the total cost when permits are distributed on a free (grandfathering) basis.
### 4.3.2. Demand

The two airlines are assumed to carry passenger volumes of $q_1$ and $q_2$. Each passenger makes one trip on either airline 1 or airline 2, so that the total number of passengers are fixed and normalised to 1 \( (i.e. q_1 + q_2 = 1) \). ‘No travel’ option is ignored. \( q_i \) depends on fares charged by airlines denoted by \( p_i \) and on service qualities they provide. One element of service quality is flight frequency, which determines ‘frequency delay’ experienced by a passenger (the difference between passenger’s preferred time of departure and the nearest flight time). A passenger’s expected frequency delay is inversely proportional to the airlines flight frequency \( f_i \). The passenger’s cost of frequency delay on airline \( i \) can be written as \( \frac{Y}{f_i} \), where \( Y \) is a cost parameter common to all passengers. Another type of ‘stochastic delay’ arises through excess demand. Stochastic delay is denoted by \( l_i \) and mainly affected by airline’s load factor. The cost of this delay can be written as \( \lambda l_i \), where \( \lambda \) is a common cost parameter. Therefore, the cost of flying with airline \( i \) is the sum of fare and two delay costs \[ p_i + \frac{Y}{f_i} + \lambda l_i \].

There is another important factor which acts towards the airline’s demand which is the brand loyalty. Brand loyalty appears as a negative cost for passengers preferring that airline and a positive cost for passengers preferring the other airline. Assuming that, this brand loyalty is uniformly distributed over \([-\alpha, \alpha]\), the number of passengers preferring airline \( i \) can be written as:

\[
q_i = \frac{1}{2} - \frac{1}{\alpha} \left( p_i - p_j + \frac{Y}{f_i} - \frac{Y}{f_j} + \lambda l_i - \lambda l_j \right), \quad i \neq j
\]  

(3)

Thus, an increase in flight frequency by any airline increases its demand while reducing the demand of the other airline. An increase in the load factor or the fare will have opposite effects. When all these variables are equal across two airlines, each airline faces a demand of \( \frac{1}{2} \).
4.3.3. HS vs. FC profits

Profit is denoted by $\pi$, which is the difference between the revenue and the total cost, where revenue is the product of price and quantity. Thus for a single route the profit is:

$$\pi_i = p_i q_i - TC_i$$

Where, cost function will be either (1) or (2) depending on the presence or absence of permit-costs. When we talk about a network, the profit is some composition of this single route profit. Under the FC network, passengers in the three airports are carried by direct (non-stop) flights on three routes. Thus, In FC network occupied seats to the passengers total $(f_i s_i l_i)$ is equal to the demand $(q_i)$. With three equidistant points, the profit for a FC network is simply:

$$\pi_{FCi} = 3p_i q_i - 3TC_i$$

For a HS network, with one airport serving as the hub (the airport located inside EU in our case), there are just two spoke routes, which connect the two non-hub airports to the hub. While spoke passengers still take direct flights, passengers travelling between the two non-hub airports must take two flights and connect at the hub. Thus, occupied seats to the passengers total $(f_i s_i l_i)$ is equal to twice of the demand $(2q_i)$. Therefore although revenue is three times, cost is only two times. i.e.: 

$$\pi_{HSi} = 3p_i q_i - 2TC_i$$

In the next sections we compare the route networks (HS vs. FC) between the initial situation without permit and the situation with permit.

4.4. Scenario 1: without emission permits

In this scenario, cost function (2) will be valid.

4.4.1. Hub and spoke (HS) network

The HS network catering passengers for three equidistant airports has 3 times revenue of a single route but cost will be two times only of the cost function (2):
\[ \pi_{HSi} = 3p_{HSi}q_{HSi} - 2 \left[ r_{HSi}s_{HSi} + \frac{\beta + \varepsilon s_{HSi}}{e_{HSi}} \right] f_{FSi} k(d) \]

Since both the spoke routes of the HS network carry both local and connecting passengers, thus \( f_{HSi} s_{HSi} l_{HSi} = 2q_{HSi} \) i.e. total number of seats (flight frequency multiplied by seating capacity multiplied by stochastic delay) equals twice the demand. Substituting for \( s_{HSi} = \frac{2q_{HSi}}{f_{HSi} l_{HSi}} \) and then collecting the terms involving \( q_{HSWi} \) and putting the value for \( q_{HSi} \) we have-

\[
\pi_{HSi} = \left[ 3p_{HSi} - \frac{4r e_{HSi} k(d)}{l_{HSi}} - \frac{4\varepsilon k(d)}{e_{HSi} l_{HSi}} \right] \left[ \frac{1}{2} \right]
- \frac{1}{\alpha} \left( p_{HSi} - p_{HSj} + \frac{\gamma}{f_{HSi}} + \lambda l_{HSi} - \frac{\gamma}{f_{HSj}} - \lambda l_{HSj} \right)
- \frac{2f_{HSi} k(d) \beta}{e_{HSi}}
\]

(4) implies that part of profit depends on the demand and the other part is independent of it.

Airline \( i \) chooses \( p_{HSi}, e_{HSi}, l_{HSi}, f_{HSi} \) to maximise \( \pi_{HSi} \).

\[
\frac{\partial \pi_{HSi}}{\partial p_{HSi}} = 3q_{HSi} + \left( -\frac{1}{\alpha} \right) \left[ 3p_{HSi} - \frac{4r e_{HSi} k(d)}{l_{HSi}} - \frac{4\varepsilon k(d)}{e_{HSi} l_{HSi}} \right] = 0
\]

(5)

\[
\frac{\partial \pi_{HSi}}{\partial e_{HSi}} = \left[ -\frac{4r k(d)}{l_{HSi}} + \frac{4\varepsilon k(d)}{e_{HSi} l_{HSi}} \right] q_{HSi} + \frac{2f_{HSi} k(d) \beta}{e_{HSi}}
\]

(6)

\[
\frac{\partial \pi_{HSi}}{\partial f_{HSi}} = \left[ 3p_{HSi} - \frac{4r e_{HSi} k(d)}{l_{HSi}} - \frac{4\varepsilon k(d)}{e_{HSi} l_{HSi}} \right] \left( \frac{\gamma}{\alpha f_{HSi}^2} \right) - \frac{2k(d) \beta}{e_{HSi}} = 0
\]

(7)

\[
\frac{\partial \pi_{HSi}}{\partial l_{HSi}} = \left[ \frac{4r e_{HSi} k(d)}{l_{HSi}^2} + \frac{4\varepsilon k(d)}{e_{HSi} l_{HSi}} \right] q_{HSi} + \left[ 3p_{HSi} - \frac{4r e_{HSi} k(d)}{l_{HSi}} - \frac{4\varepsilon k(d)}{e_{HSi} l_{HSi}} \right] \left( -\frac{\lambda}{\alpha} \right) = 0
\]

(8)

The above FOCs imply that the marginal effects of the variables on profit are via marginal revenues and marginal costs\(^4\). Given the symmetry of the model the equilibrium values of the choice variables will be symmetric across carriers. Thus, each airline’s equilibrium traffic will be equal \( \frac{1}{2} \), which provide the following results:

\(^4\) The SOC i.e. the positive definiteness of the Hessian matrix of profit is assumed to hold.
From (5) we have:

\[ p_{HSi} = \frac{\alpha}{2} + \frac{4k(d)}{3l_{HSi}} \left[ re_{HSi} + \frac{\varepsilon}{e_{HSi}} \right] \]  

(9)

From (6) we have:

\[ e^2_{HSi} = \frac{\varepsilon + \beta f_{HSi} l_{HSi}}{r} \]  

(10)

From (5) and (7) we have:

\[ f^2_{HSi} = \frac{3e_{HSi} \gamma}{4k(d)\beta} \]  

(11)

From (5) and (8) we have:

\[ l^2_{HSi} = \frac{4k(d)}{3\lambda} \left[ r e_{HSi} + \frac{\varepsilon}{e_{HSi}} \right] \]  

(12)

The equilibrium denoted by \((p_{HSi}^*, e_{HSi}^*, f_{HSi}^*, l_{HSi}^*)\) will be the solution to the equations (9) to (12). To solve the four simultaneous equations for the equilibrium values of the choice variables, we square both the sides of (10), put values of \(f^2_{HSi}\) and \(l^2_{HSi}\) from (11) and (12) and obtain the following:

\[ e_{HSi}^* = \left[ \frac{\left( 2\varepsilon + \frac{\beta \gamma}{\lambda} \right) + \sqrt{\frac{\beta \gamma}{\lambda} \left( 8\varepsilon + \frac{\beta \gamma}{\lambda} \right)}}{2r} \right]^{1/2} \]

Now, we get two positive values if \( \left( 2\varepsilon + \frac{\beta \gamma}{\lambda} \right)^2 > \frac{\beta \gamma}{\lambda} \left( 8\varepsilon + \frac{\beta \gamma}{\lambda} \right) \), which boils down to \(4\varepsilon(\varepsilon - \frac{\beta \gamma}{\lambda}) > 0\). Thus to ensure one unique real solution we assume \(\varepsilon < \frac{\beta \gamma}{\lambda}\) to hold and get the solution as:

\[ e_{HSi}^* = \left[ \frac{\left( 2\varepsilon + \frac{\beta \gamma}{\lambda} \right) + \sqrt{\frac{\beta \gamma}{\lambda} \left( 8\varepsilon + \frac{\beta \gamma}{\lambda} \right)}}{2r} \right]^{1/2} \]  

(13)
4.4.2. Fully connected (FC) network

In this network system passengers travel all the routes directly. Thus revenue and cost both are three times that of any single route:

\[
\pi_{FCi} = 3p_{FCi}q_{FCi} - 3\left[re_{FCi}s_{FCi} + \frac{\beta + \epsilon s_{FCi}}{e_{FCi}}\right]f_{FCi}k(d)
\]

Using \( f_{FCi}s_{FCi}l_{FCi} = q_{FCi} \), i.e. total seats occupied is simply the demand of that route and then collecting the terms including \( q_{FCi} \) we get:

\[
\pi_{FCi} = 3\left[p_i - \frac{re_{FCi}k(d)}{l_{FCi}} - \frac{ek(d)}{e_{FCi}l_{FCi}}\right]\left[\frac{1}{2} - \frac{1}{a}\left(p_{FCi} - p_{FCJ} + \frac{\gamma}{f_{FCi}} + \lambda l_{FCi} - \frac{\gamma}{f_{FCJ}} - \lambda l_{FCJ}\right)\right] - 3\frac{f_{FCi}k(d)\beta}{e_{FCi}} \tag{4'}
\]

Airline \( i \) chooses \( p_{FCi}, e_{FCi}, l_{FCi}, f_{FCi} \) to maximise \( \pi_{FCi} \). Proceeding in the similar manner we derive the following values of the choice variables:

\[
p_{FCi} = \frac{\alpha}{2} + \frac{k(d)}{l_{FCi}}\left[re_{FCi} + \frac{\epsilon}{e_{FCi}}\right] \tag{9'}
\]

\[
e_{FCi}^2 = \frac{\epsilon + 2\beta f_{FCi}l_{FCi}}{r} \tag{10'}
\]

\[
f_{FCi}^2 = \frac{e_{FCi}\gamma}{2k(d)\beta} \tag{11'}
\]

\[
l_{FCi}^2 = \frac{k(d)}{\lambda}\left[re_{FCi} + \frac{\epsilon}{e_{FCi}}\right] \tag{12'}
\]

In this case, the equilibrium denoted by \( (p_{FCi}^*, e_{FCi}^*, f_{FCi}^*, l_{FCi}^*) \) will be the solution to the equations (9') to (12'). To solve for all the equilibrium values of the choice variable we derive \( e_{FCi}^* \) from (10') and proceeding in the similar manner as done in the HS case, and assuming \( \epsilon < \frac{2\beta\gamma}{\lambda} \) to ensure one unique real solution we get the solution as:

\[
e_{FCi}^* = \left[\left(2\epsilon + \frac{2\beta\gamma}{\lambda}\right) + \sqrt{\frac{2\beta\gamma}{\lambda} \left(8\epsilon + \frac{2\beta\gamma}{\lambda}\right)}\right]^{1/2} \tag{13'}
\]
From (13) and (13’) it is seen that, $e$ rises by less than a factor of $\sqrt{2}$ from HS to FC network. $\frac{e_{FCWPi^*}}{e_{HSWPi^*}} = \sqrt{2}$ provided $\varepsilon = 0$. Otherwise we have:

$$e_{HSi^*} < e_{FCi^*} < \sqrt{2} e_{HSi^*}$$

(14)

i.e. HS network has more fuel-efficient network than FC network. From (11), (11’) and (14) we have

$$\frac{f_{FCi}^2}{f_{HSi}^2} = \frac{2 e_{FCi^*}}{3 e_{HSi^*}} < \frac{2}{3} \sqrt{2} < 1 $$

(15)

i.e. flight frequency is higher in HS network compared to FC network. These results are as in Brueckner and Zhang (2010).

### 4.4.3. Profit differential

Comparing (4) and (4’) we see that the profit differential is

$$\pi_{HSi} - \pi_{FCi} = k(d) \left[ r \left( \frac{3 e_{FCi}}{2 l_{FCi}} - 2 \frac{e_{HSi}}{l_{HSi}} \right) + \varepsilon \left( \frac{3}{2} \frac{1}{e_{FCi} l_{FCi}} - 2 \frac{1}{e_{HSi} l_{HSi}} \right) \right] + \beta \left( 3 \frac{1}{e_{FCi}} - 2 \frac{1}{e_{HSi}} \right)$$

(16)

The sign of the profit differential depends on the magnitudes of the individual choice variables. Therefore, from the above interactions between the choice variables of two types of network we cannot conclude unambiguously which network is preferred to the other.

### 4.5. Scenario 2: With emission permits

In this scenario, cost function (1) will be valid. We write $P$ in the subscript of the variables to denote that these pertain to the different network equilibria with emission permit.
4.5.1. Hub and spoke (HS) network

In this scenario both the spoke routes (through the hub in EU) are included in the EUETS, thus we have the cost function (1):

\[ \pi_{HSP_i} = 3p_{HSP_i}q_{HSP_i} - 2\left[ re_{HSP_i}q_i + \frac{\beta + \varepsilon s_{HSP_i}}{e_{HSP_i}} + zx e_{HSP_i} s_{HSP_i} \right] f_{HSP_i} k(d) \]

Again substituting for \( s_{HSP_i} = \frac{2q_{HSP_i}}{f_{HSP_i} l_{HSP_i}} \) and then collecting the terms involving \( q_i \) we have -

\[ \pi_{HSP_i} = \left[ 3p_{HSP_i} - \frac{4r e_{HSP_i} k(d)}{l_{HSP_i}} - \frac{4\varepsilon k(d)}{e_{HSP_i} l_{HSP_i}} - \frac{4z x e_{HSP_i} k(d)}{l_{HSP_i}} \right] q_{HSP_i} - \frac{2f_{HSP_i} k(d) \beta}{e_{HSP_i}} \]

Maximising (4a) with respect to \( p_{HSP_i}, e_{HSP_i}, l_{HSP_i}, f_{HSP_i} \) similar to the previous case we get the following results:

\[ p_{HSP_i} = \frac{\alpha}{2} + \frac{4k(d)}{3l_{HSP_i}} \left[ re_{HSP_i} + \frac{\varepsilon}{e_{HSP_i}} + zxe_{HSP_i} \right] \]  

(9a)

\[ e_{HSP_i}^2 = \frac{\varepsilon + \beta f_{HSP_i} l_{HSP_i}}{2(r + zx)} \]  

(10a)

\[ f_{HSP_i}^2 = \frac{3e_{HSP_i} \gamma}{4k(d) \beta} \]  

(11a)

\[ l_{HSP_i}^2 = \frac{4k(d)}{3\lambda} \left[ re_{HSP_i} + \frac{\varepsilon}{e_{HSP_i}} + zxe_{HSP_i} \right] \]  

(12a)

The equilibrium denoted by \((p_{HSP_i}^*, e_{HSP_i}^*, f_{HSP_i}^*, l_{HSP_i}^*)\) will be the solution to the equations (9a) to (12a). To solve the four simultaneous equations for the equilibrium values of the choice variables, we square both the sides of (10a), put values of \( f_{HSP_i}^2 \) and \( l_{HSP_i}^2 \) from (11a) and (12a) and obtain the following:
Now, we get two positive values if \((4\varepsilon + \frac{\beta \gamma}{\lambda})^2 > \frac{\beta \gamma}{\lambda} (24\varepsilon + \frac{\beta \gamma}{\lambda})\), which boils down to \(\varepsilon > \frac{\beta \gamma}{\lambda}\). As before, to ensure one unique real solution we assume \(\varepsilon < \frac{\beta \gamma}{\lambda}\) to hold and get the solution as:

\[
e_{HSPi}^* = \left[\frac{4\varepsilon + \frac{\beta \gamma}{\lambda} + \sqrt{\frac{\beta \gamma}{\lambda} (24\varepsilon + \frac{\beta \gamma}{\lambda})}}{8(r + zx)}\right]^{1/2}
\]

4.5.2. Fully connected (FC) network

In this scenario two of the three direct routes which touches EU are included in EUETS, whereas, there is no permit cost for the third direct route. Thus, the total cost is

\[
2 \left[ r_{FCPi} e_{FCPi} s_{FCPi} + \frac{\beta + \varepsilon s_{FCPi}}{e_{FCPi}} + zx e_{FCPi} s_{FCPi} \right] f_{FCPi} k(d) + \left[ r e_{FCPi} s_{FCPi} + \frac{\beta + \varepsilon s_{FCPi}}{e_{FCPi}} \right] f_{FCPi} k(d).
\]

The profit function in the FC model is the following:

\[
\pi_{FCPi} = 3p_{FCPi} q_{FCPi} - 2 \left[ r_{FCPi} e_{FCPi} s_{FCPi} + \frac{\beta + \varepsilon s_{FCPi}}{e_{FCPi}} + zx e_{FCPi} s_{FCPi} \right] f_{FCPi} k(d) - \left[ r e_{FCPi} s_{FCPi} + \frac{\beta + \varepsilon s_{FCPi}}{e_{FCPi}} \right] f_{FCPi} k(d)
\]

Again substituting \(s_{FCPi} = \frac{q_{FCPi}}{f_{FCPi} l_{FCPi}}\) and collecting all the terms with \(q_i\) we have:

\[
\Rightarrow \pi_{FCPi} = \left[3p_{FCPi} - \frac{3 r e_{FCPi} k(d)}{l_{FCPi}} - \frac{3 \varepsilon k(d)}{e_{FCPi} l_{FCPi}} - \frac{2 zx e_{FCPi} k(d)}{l_{FCPi}} \right] \frac{1}{2} l_{FCPi} k(d)
\]

\[
- \frac{1}{\lambda} \left( p_{FCPi} - p_{FCPj} + \frac{\gamma}{f_{FCPi}} + \lambda l_{FCPi} - \frac{\gamma}{f_{FCPj}} - \lambda l_{FCPj} \right) \frac{3 f_{FCPi} k(d)}{e_{FCPi}}
\]

Maximising \((4a')\) with respect to \(p_{FCPi}, e_{FCPi}, l_{FCPi}, f_{FCPi}\) similar to the previous case we get:
In this case, the equilibrium denoted by \((p_{FCPl}^*, e_{FCPl}^*, f_{FCPl}^*, l_{FCPl}^*)\) will be the solution to the equilibriums (9a') to (12a').

To solve for all the equilibrium values of the choice variable we derive \(e_{FCPl}^*\) from (10a') and proceeding in the similar manner as done in the HS case, and assuming \(\varepsilon < \frac{2\beta Y}{\lambda}\) to ensure one unique real solution we get the solution as:

\[
e_{FCPl}^* = \left[ \frac{2\varepsilon + \frac{2\beta Y}{\lambda}}{2 \left( r + \frac{2xz}{3} \right)} + \sqrt{\frac{2\beta Y}{\lambda} \left( 8\varepsilon + \frac{2\beta Y}{\lambda} \right)} \right]^{1/2}
\]

Comparing (13a) and (13a') we see that the ranking of the fuel efficiency is unclear for the two networks with the permits. This gives rise to the following proposition:

**Proposition 1:** Although the fuel efficiency is higher in hub and spoke network compared to the fully connected network without the permits, however, the ranking is unclear in the presence of the permits.
4.5.3. Profit differential

Comparing (4a) and (4a’) with the equilibrium values we see that the profit differential is

\[
\pi_{HSPi} - \pi_{FCPi} = k(d) \left[ r \left( \frac{3}{2} \frac{e_{FCPi}^*}{l_{FCPi}^*} - \frac{2}{l_{HSPi}^*} \right) + \varepsilon \left( \frac{3}{2} \frac{1}{e_{FCPi}^* l_{FCPi}^*} - \frac{2}{e_{HSPi}^* l_{HSPi}^*} \right) \right] \\
- \beta \left( \frac{3f_{FCPi}^*}{e_{FCPi}^*} - \frac{2f_{HSPi}^*}{e_{HSPi}^*} \right) \\
- \left( \frac{zxe_{FCPi}^*}{l_{FCPi}^*} - \frac{2zxe_{HSPi}^*}{l_{HSPi}^*} \right) 
\]  

(16a)

In this case also sign of the profit differential depends on the magnitudes of the individual choice variables and from the above interactions between the choice variables of two types of network we cannot conclude unambiguously that which network is preferred to the other. However, we compare the profit differential without permit (\(\pi_{diff}\)) and the profit differential permit (\(\pi_{diffP}\)) from (16) and (16a):

\[
\pi_{diff} - \pi_{diffP} \\
= k(d) \left[ r \left( \frac{3}{2} \left( \frac{e_{FCPi}^*}{e_{FCPi}^*} - \frac{e_{FCPi}^*}{l_{FCPi}^*} \right) - 2 \left( \frac{e_{HSPi}^*}{l_{HSPi}^*} - \frac{e_{HSPi}^*}{l_{HSPi}^*} \right) \right) \\
+ \varepsilon \left( \frac{3}{2} \left( \frac{1}{e_{FCPi}^* l_{FCPi}^*} - \frac{1}{l_{FCPi}^*} \right) - 2 \left( \frac{1}{l_{HSPi}^*} - \frac{1}{l_{HSPi}^*} \right) \right) \right] \\
+ \beta \left[ 3 \left( \frac{f_{FCPi}^*}{e_{FCPi}^*} - \frac{f_{FCPi}^*}{l_{FCPi}^*} \right) - 2 \left( \frac{f_{HSPi}^*}{e_{HSPi}^*} - \frac{f_{HSPi}^*}{e_{HSPi}^*} \right) \right] \\
+ zx \left( \frac{e_{FCPi}^*}{l_{FCPi}^*} - \frac{2e_{HSPi}^*}{l_{HSPi}^*} \right) 
\]  

(17)

If the above final expression (17) is positive (negative), which implies a (an) reduction (increase) in the profit differential after imposition of permit price then we find a bias towards the FC (HS) network.

With an increase in \(z\), the change in the profit differential \(\frac{\partial(\pi_{diff} - \pi_{diffP})}{\partial z}\) will depend upon the sign of \(\frac{e_{FCPi}^*}{l_{FCPi}^*} - \frac{2e_{HSPi}^*}{l_{HSPi}^*}\). since \(\varepsilon\) is given a positive parameter. Thus, with an increase in the permit price the condition for a bias towards a FC network is:
The above condition implies that if the ratio of the fuel efficiency in the FC network to the HS network is higher than twice the ratio of the load factor (FC to HS), then there will be a bias towards the FC network with an increase in the permit price. This leads to the following proposition:

Proposition 2: With an increase in the permit price there will be a bias towards the fully connected network if the ratio of the fuel efficiency in the FC network to the HS network is higher than twice the ratio of the load factor (FC to HS).

4.6. Conclusions

In the context of the inclusion of both domestic and international flights in the EUETS, this chapter extends the theoretical model of domestic network structure of aviation by Brueckner and Zhang (2010) in an international network structure scenario. Airlines can either operate a hub-and-spoke (HS) network or a fully connected / point to point (FC) network. In a three equidistant-node symmetric city framework of domestic aviation network structure Brueckner and Zhang (2010) found that it cannot be explicitly identified which network type is optimal. Moreover, they found the effects of emission charges on the profit maximising network structure (HS vs. FC) to be ambiguous. We extend their model set up in an international scenario where the two spoke points of the HS model for airlines lie outside EU whereas the hub lies inside EU.

As in Brueckner and Zhang (2010) we also found that the ranking the profits of the two networks is not unique. However, we found that in contrast to the higher fuel efficiency in the HS network compared to the FC network without the permits, the ranking is unclear in the presence of the permits. Although in our model we have assumed the permit price to be given, but in reality it is decided in the market and is expected to increase over the time. Thus
we checked the effect of an increase in the permit price on the profit differential and found that there will be a bias towards the fully connected network if the ratio of the fuel efficiency in the FC network to the HS network is higher than twice the ratio of the load factor (FC to HS). The way we have set up our model, the two spoke points are outside EU, so a direct route (under FC network) has no permit charges applied to it. Although HS network has its own incentive to economise through higher aircraft fuel efficiency, in our set up both the connecting routes (under HS network) has to incur permit charges. Thus, there is a trade off between the permit cost and efficiency gain under HS network when the two spoke points lie outside the EU.

The result derived from a simple extension of the network structure model of Brueckner and Zhang (2010) is important because, if two spoke points lie outside the EU then a preference shift towards a FC network from a HS network would imply reduction of emission within the EU but simultaneous increase in the emission outside the EU. Although our model set up assumes away any possibility of asymmetric routes of the two airlines, however, if the two spoke points lie outside EU, then in reality it might happen that only the non-EU airline will have direct flights between the two spoke points and the EU airline continue to have HS network. In that case the profit comparison of the HS and FC network will be only valid for the non-EU airline. In the extreme case, if the cost of travelling in the EU sky exceeds the revenue due to a high permit cost, then it might happen that only EU airline will be operating in the HS network and the non-EU airline will only have direct flights between the two spoke airports. This would lead to a carbon leakage via alternative routes and diversion of traffic, as indicated in the literature.