Chapter 4  Measurement and analysis of abrasion wear

4.1  Introduction

Quenched and tempered low alloy steels are extensively being used in earthmoving, mining and mineral processing industries which involve frequent and often severe mechanical interactions between metals, between metals and abrasives, and nonmetallic and metallic materials. These interactions results in the removal of large amounts of material from the wear surfaces of scraping, digging and ore processing equipments, and expenditure of large amount of energy to overcome frictional forces.

Since, most of the equipments used in these industries makes use of easily replaceable or repairable wear components which are either screwed, riveted or fabricated using welding processes. The welded region of these components consists of various zones with significant microstructural variations that consequently affect the wear behavior. Thus it becomes of utmost importance to investigate the wear–microstructure relationship and to identify the workable range of such fabricated structures (Hawk & Wilson, 2000).

To make effective use of wear components, it is essential to identify the wear parameters that would ensure the minimum wear rate conditions. To find the desired balance, the combined effect of different variables on the abrasion wear must be determined, and the best combination of the variables be selected which would ensure the minimum wear rate conditions. Since, different welded joint were fabricated using various filler materials, thus, a common set of parameters were to be identified that would result in minimum wear loss conditions. In order to pursue with the goal, the
optimum wear conditions were identified for the base material which were further used to make comparison in wear loss behavior among different welded joints.

Mathematical modeling that can be applied in a variety of fields for investigations was adopted to accomplish the aim of predicting the effect of wear parameters on the wear behavior, because using this method the research becomes more economical, fast and versatile. The statistical technique of central composite rotatable design has been employed to conduct the experiments, on the basis of which such mathematical models or equations could be developed, which can be further utilized for determining the most optimal working conditions. The design of experiments for developing mathematical models for wear loss prediction, and the procedures for testing the significance of coefficients and adequacies of such mathematical models are covered in this chapter. After testing the adequacies of the models and dropping the insignificant coefficients, the final mathematical models were used to show graphically the relationships between the wear variables and the wear rate. Finally, the optimized wear parameters were obtained using RSM, which were used to achieve minimum and maximum wear rate conditions.

Further, the obtained optimum wear conditions were used to identify the wear performance of the welded joints by extracting the specimens from the different zones of the weldments.

4.2 Wear

Wear, fatigue and corrosion are the three most commonly encountered industrial problems leading to the replacement of components and assemblies in engineering. Wear is rarely catastrophic, but it reduces operating efficiency by increasing the power losses, oil consumption and the rate of component replacement.
4.2.1 Classification of wear

The wear can be classified in the following types (Kato & Adachi, 2000):

1. Adhesive wear—Oxidative wear, metallic wear, galling
2. Abrasive wear—Low stress scratching abrasion, high stress grinding abrasion, gouging abrasion
3. Erosion—Particle impingement, cavitation
4. Fretting

4.2.1.1 Adhesive wear

Adhesive wear generally describes wear due to the sliding action between two metallic components, where no abrasives are intended to be present. When the applied load is sufficiently low, an oxide film is usually generated as a result of frictional heating accompanied by sliding. The oxide film prevents the formation of metallic bond between the sliding surfaces, resulting in low wear rates. This form of wear is called the oxidative or mild wear. If the applied load is high, formation of metallic bond occurs between the surfaces of mating materials. The resulting wear rates are extremely high. This form of wear is called severe or metallic wear. Galling is a special form of severe adhesive wear. It occurs if the wear debris is larger than the clearance and if seizure of the moving components occurs. Generally, lubrication is used to prevent the adhesive wear.

4.2.1.2 Abrasive wear

Low stress scratching abrasion occurs from a cutting action by sliding abrasives stressed below their crushing strength. In this kind of abrasive wear, worn surfaces contain scratches and the amount of subsurface deformation is minimal. High stress grinding abrasion occurs under conditions where the stress is high enough to crush the abrasives. In this type of wear, stresses are high enough to cause gross plastic
deformation of the ductile constituents. Gouging abrasion describes high stress abrasion where gouges or deep grooves are created on the wearing surfaces.

4.2.1.3 Erosion

Impingement erosion occurs under the cutting action of a fluid-borne moving particle. The contact stress arises from the kinetic energy of a particle flowing in an air or liquid stream as it encounters a surface. Impingement erosion varies with the angle of impingement. Cavitation involves the collapse of air bubbles caused by the turbulence. When these bubbles collapse on the material surface, wear occurs by the resultant shock waves. This type of wear is called cavitation erosion.

4.2.1.4 Fretting

This wear phenomenon occurs between two surfaces having oscillatory relative motion of small amplitude. It is a combination of abrasive and oxidative wear.

4.2.2 Abrasion wear

Abrasion wear is the displacement of materials caused by the presence of hard particles, between or embedded in one or both the two surfaces in relative motion, or by the presence of hard protuberances on one or both of the relatively moving surfaces. The nature of abrasive wear is determined by the way which particles traverse the worn surfaces i.e. the particles may roll and/or slide over the surface. Therefore, two basic modes have been identified namely; two-body and three-body abrasion wear.

4.2.2.1 Two-body and three-body abrasion wear

Abrasion wear can be classified as two-body or three body abrasion and are shown in Figure 4.1. The two-body abrasive wear involves the removal of material by abrasive particles which are held fixed (as in abrasive paper) while being moved across a surface. This form of abrasive wear results in grooving form of wear.
In three-body abrasion wear phenomenon the abrasive particles are trapped between two surfaces which may rotate as well as slide as they contact the wearing surface. This type of wear is much more common but more complicated than two-body abrasion. Furthermore, in three-body abrasion, the movement patterns of abrasives are more complicated than in two-body abrasion, since the abrasives not only slide, but also roll. Thus, a relatively wide range of wear rates have been previously reported for three-body abrasion conditions, which depend not only on the material being tested, but also on the testing apparatus. In three-body abrasion of metals, cutting wear and plastic deformation wear coexist (Fang et al., 1998). It has been very well established that the two-body systems experiences 10 to 1000 times as much loss as three-body systems for a given load and path length of wear. This is because in three-body abrasion a small proportion of the abrasives cause wear, due to variations in the angles of attack and movement patterns of abrasives (Czichos, 1992).

![Figure 4.1: Abrasion wear modes: (a) two-body abrasion; (b) three-body abrasion (Rabinowicz, 1965)](image)

4.2.2.2 Mechanisms of abrasion wear

Various wear mechanisms have been proposed to explain the material removal process during two-body and three body abrasion wear of the material. Broadly, the material removal processes have been categorized as ploughing, cutting and spalling (Figure...
4.2. However, the underlying mechanisms include plastic deformation, fracture, fatigue, grain pull-out and corrosion. In simpler terms, these mechanisms can be separated into two types, i.e. plastic deformation and fracture. Under some circumstances, plastic flow may occur alone, but because of the complexity of abrasion, rarely does one mechanism completely account for all the loss. Although the plastic deformation mechanism is often linked with ductile materials and the fracture mechanism is linked with brittle materials, both can occur together.

![Figure 4.2: Schematic illustration of the mechanisms of material removal during abrasion: (a) ploughing; (b) cutting; (c) cracking](image)

**4.2.2.2.1 Plastic deformation**

The abrasive particles that come in contact with the surface of a relatively ductile material results in two major processes (Moore, 1979):

1. The formation of grooves which do not involve direct material removal (Ploughing).
2. The separation of material in the form of primary wear debris or microchips (microcutting).

Ploughing occurs as a result of considerable plastic deformation within the wear path and causes material to be heaped up on either side of the wear groove (Figure 4.2(a)). Since this process involves fairly ductile plastic deformation at the surface, the abraded material is not detached, so that there is virtually no volume loss during a
single pass of abrasive particle. Volume loss does of course occurs as a consequence of
the action of many abrasive particles, since the material can only tolerate a finite
amount of deformation before fracture. In microcutting, the material displaced from the
groove by fracture results in the formation of machining chips, shaving, fragments etc.
(Figure 4.2(b)), and there is an obvious material loss corresponding to the volume of
wear grooves.

Rabinowicz suggested a simple model for the abrasive wear process by
plastic deformation (Rabinowicz & Mutis, 1965). This model is based on an
abrasive particle, idealized as a sharp cone of semi angle $\theta$, being dragged across
the surface of a ductile material which flows under an indentation load $F$. It
forms a groove in the material with hardness $H$, and wear is assumed to occur by
removal of some proportion of the material which is displaced by the particle
from the groove. The volume of groove $V$ per unit length can be obtained:

$$ \frac{V}{L} = \frac{2}{\pi \tan \theta} \frac{F}{H} $$  \hspace{1cm} (4.1)

Therefore, the wear rate $Q$ is defined as:

$$ Q = K \frac{F}{H} $$  \hspace{1cm} (4.2)

This is the well-known Rabinowicz equation. $K$ is the wear coefficient and defined as:

$$ K = \frac{2\eta}{\pi \tan \theta} $$  \hspace{1cm} (4.3)

where, $\eta$ is the fraction of material displaced from the groove.

According to the Rabinowicz model (Equation 4.2), wear in homogeneous
materials only depends on the attack angle $\theta$, the normal load $F$, the hardness $H$ of the
material and the geometry of the indenter. This simple model suggests that the wear rate
per unit length of sliding will be directly proportional to the load, and will vary
inversely as the hardness of the surface.
4.2.2.2 Fracture
In brittle materials the local surface strain exceeds a critical value and the wear occurs by relatively brittle fracture known as spalling. This occurs when forces applied by the abrasive grain exceed the fracture toughness of the material, and it can sometimes be observed locally in the harder phase (e.g. carbides, intermetallic compounds) of wear resistant alloys. Although plastic deformation occurs during abrasive wear of brittle materials, fracture is often the rate controlling mechanism. Even during the wear of ductile materials, fracture may occur. For a ductile material, fracture is most likely to occur just behind a contacting abrasive particle since this region is subject to a tensile stress. The material removal in brittle materials is likely to be controlled by fracture rather than by plastic deformation, except during wear by very lightly loaded blunt abrasive particles (S. V. Prasad & Kosel, 1984).

4.2.2.3 Parameters influencing abrasion wear
A variety of parameters influence all wear mechanisms and thus influence the abrasive wear behavior of materials. To a certain extent, abrasive wear tests have been designed to emphasize one or more of these parameters, especially, the dominant one in an application. These parameters can be categorized and are discussed in the following paragraphs (Hawk, 2000).

4.2.2.3.1 Effect of abrasive properties
This part of the section discusses the different properties of the abrasives used to simulate different wear testing conditions.

1. The abrasive hardness: The hardness of the abrasive particles is important to the rate of abrasion of the subject material. As the hardness of the abrasive exceeds that of the wear material, abrasive wear typically becomes much worse as shown in Figure 4.3. As the abrasive hardness exceeds the hardness of the
material, it is able to penetrate the surface and cut/remove material without having its cutting edges broken or rounded (Tylczak & Oregon, 1992).

![Graph showing the variation of wear rate with material hardness](image)

**Figure 4.3:** Graph showing the variation of wear rate with material hardness (Tylczak & Oregon, 1992)

2. **The abrasive shape:** It is now appreciated that sharper (angular) abrasive particles give rise to a higher weight loss when compared with rounded particles. The wear rate with the angular crashed quartz abrasives was found to be between 2 to 5.5 times greater than rounded Ottawa sand due to deteriorating behavior of angular abrasives. The cross-sectional area of a groove depends on the particle shape, so that the ratio of cross-sectional to projected area of contacts is higher for pyramidal or conical contacts than for spherical contacts. The higher deteriorating effect of angular abrasives is important particularly in carbide containing alloys, since preferential abrasion of a matrix phase may lead to unsupported carbides. In addition to that, the sharp edges of the abrasives can cause extensive carbide cracking which is not observed with round abrasives. (Swanson & Klann, 1981) have reported a factor of ten in the volume loss of the plain carbon and low alloy steels examined using the wet and dry sand rubber wheel abrasion tests with round and angular silica abrasives.

3. **The abrasive size:** In a wide range of materials, the wear rate has been shown to increase with the abrasive particle size. The effect is highest for non-metals. The
predominant changes in material removal mechanisms are suggested to be responsible for this consequence. A number of possible explanations have been made to clarify the lower wear rate associated with a decrease in the abrasive size. It has been suggested that only a lower fraction of the load is carried with the fine abrasives (Thakare et al., 2012). The other probability is that loose wear debris prevents some abrasives from contacting the material surface. It is quite likely that the worn surface is clogged by wear debris. The probability for clogging is higher with fine abrasives, and therefore the number of contacts between the abrasives and the worn surfaces is less than the coarse abrasives.

4. **The effect of abrasive particles strength:** Plastic deformation of the surface will occur as the normal load on the particle is increased only if the particle can sustain this contact pressure without deforming or crushing. Thus, the strength and toughness of an abrasive particle are important factors in abrasive wear. (Gåhlin & Jacobson, 1999) described abrasive with quartz and chert, which have similar hardness, but with chert having a greater resistance to fracture than quartz; wear generated by chert was between two and three times that quartz.

### 4.2.2.3.2 Properties of the material

The metallurgical structure of the specimen and its hardness has a profound effect on the wear performance of the material. These aspects are discussed in the following paragraphs.

1. **Microstructure of the specimen:** The microstructural constituents present in the specimen have profound influence on the wear performance and characteristics of the materials. Various factors such as carbide content, grain size, retained austenite; heat treatment regime and alloy content were found to exhibit a significant effect on the wear properties of the ferrous materials and their
fabricated structures. Parameters such as volume fraction and distribution of a dispersed phase, its coherency and hardness, all affect abrasive particle indentation, strain hardening, strain distribution, fracture, and recovery processes particularly in heterogeneous materials like composites and thereby affecting the wear behaviour of the material (I. M. Hutchings, 1994).

2. **Hardness of specimen:** Generally, it is recognized that the materials with higher hardness result in lower rates of abrasive wear, however, the relationship between material hardness and wear resistance is much more complicated. It has been suggested in Equation (4.2), that the wear rate varies inversely with the hardness of the material. Many pure metals do behave in this way, although alloys often exhibit more complex behaviour (Khruschov, 1974). This loss of proportionality between hardness and the relative wear rate for hardened metals is the result of defining the wear resistance in terms of the un-deformed hardness of the metal. The materials at the worn surface gets strain-hardened by plastic flow, and that hardness will generally be greater than that of the bulk. Moreover, the microcutting mechanism becomes more dominant as the hardness of the material increases (Stachowiak & Batchelor, 2005).

### 4.2.2.3.3 Operational parameters

Operational parameters characterize the functional conditions of a tribosystem. They can be considered (with the exception of friction-induced temperatures) as independent variables that can be varied during tribological testing to obtain friction and wear data experimentally. The basic operational parameters involved during abrasion wear are described below (Czichos, 1992).

1. **Type of motion:** The kinematics of triboelements can be classified in terms of sliding, rolling, spin and impact, and their possible superpositions.
2. **Load (F):** It is defined as the total force (including weight) that acts perpendicular to the contact area between the triboelements.

3. **Velocity (v):** It is specified with respect to the vector components and the absolute values of the individual motions of triboelements. Distinctions must be made among the relative velocity \( v_r \) (relevant to friction-induced temperature rises), the sum velocity \( v_s \) (relevant to lubricated tribosystems to the formation of an elasto-hydrodynamic film), and the slide-to-roll ratios.

4. **Temperature (T):** Temperature of the structural components at stated location and time, i.e. the initial (steady-state) temperature and the friction-induced temperature rise (average temperature rise and flash temperatures) to be estimated on the basis of friction heating calculations.

5. **Time:** Time dependence of the set of operational parameters (load, velocity, temperature). For example, load cycles and heating or cooling intervals etc.).

6. **Duration (t):** Duration of operation, performance of test.

### 4.2.2.4 Abrasion wear measurement

Abrasion wear measurement tests have been standardized and been divided into following types (Hawk & Wilson, 2000; Hawk et al., 1999):

- **G 56**—Abrasiveness of ink-impregnated fabric printer ribbons
- **G 76**—Conducting erosion tests by solid particle impingement using gas jets
- **G 65**—Measuring abrasion using the dry sand/rubber wheel apparatus
- **G 132**—Pin abrasion testing
- **G 75**—Slurry abrasivity and slurry abrasion response of materials
- **G 105**—Conducting wet sand/rubber wheel abrasion test
- **G 81**—Jaw crusher gouging abrasion test
4.2.2.4.1 Two-body abrasion wear test using pin-on-disc abrasion tribometer (POD)

Dry abrasion against fixed particles is typically performed using a pin abrasion wear tester. Pin abrasion wear testing simulates high-stress, quasi-two-body abrasive wear. The wear test is high stress because the abrasive grains are frequently fractured during the test. It is quasi-two-body because at the onset of the test, the abrasive particles are fixed to the cloth/paper backing with an adhesive. As the test progresses, fractured bits of the abrasive and wear debris become trapped in the interfacial region between the end of the pin and the abrasive cloth/paper, thus adding a third body to the system. Many different pin abrasion wear testers have been developed over the years (viz. Pin-on-disk, Pin-on-belt, Pin-on-drum and Pin-on-table) with various geometries of wear track (viz. Circular, spiral, linear, rectilinear and helical). All these devices expose the specimen to an environment where the abrasive grains are initially fixed to a substrate. ASTM G132-96 provides a useful guide to the general features of pin abrasion wear testing (ASTM, 2013a).

The Pin-on-disc abrasion tester (Figure 4.4) has been used to study the wear of tillage equipment materials, where, depending on the load pressing a test specimen against a bonded abrasive paper, either low stress abrasion or high stress abrasion may be produced. Early results obtained from this type of apparatus were reported by Khruschov and Babichev. It was found that the wear resistances of annealed pure metals and steels were substantially in direct proportion to their indentation hardness. The relevance of their work to the wear of agricultural implements has been discussed by Richardson. It was reported that in field tests of ground engaging tools for a wide range of soil conditions the relative wear rates of a number of metals were in general agreement with the results obtained by Khruschov and Babichev (Ruff & Bayer, 1993).
Apart from heat treated and untreated base materials, this test has extensively been used to study the wear behaviour of hardfaced and thermally sprayed components fitted in aircraft, automobiles, constructions machinery, gas exploration and chemical processing equipments that are subjected to sliding and abrasive wear conditions (Thakur & Arora, 2013).

**Figure 4.4:** Schematic diagram of the pin-on-disc test for sliding abrasion: FT–transverse feed

### 4.2.2.4.2 Three-body abrasion wear test using dry sand rubber wheel tribometer (DSRW)

The most commonly used test configuration for three-body abrasion is that of a specimen loaded against a rotating wheel with abrasive particles being entrained into the contact zone. This is the basic principle of the test described by ASTM-G65 (ASTM, 2010a) and the test schematic is shown in Figure 4.5. The rubber wheel tester, which was standardized by ASTM, has been said to produce low stress three-body abrasion (Hawk et al., 1999). Even before the test became an ASTM standard, it had been used by a number of laboratories for many years. The test is widely used to rank materials for components that will be subjected to low stress abrasion in service like agricultural tools, chutes and hoppers in ore processing plant, and construction equipment (Swanson & Klann, 1981). This test has a longer history, and has generated more data than other types of abrasion testing machines. However, this test
configuration has some limitations. For example, the abrasive particles may get embedded in the rubber wheel and scratch the test specimen in a manner similar to two-body abrasion. In the test, a plane specimen is loaded against the rim of a rotating rubber wheel; sand is fed into the gap between the wheel and specimen and is carried past the specimen and thus abrades it. The behaviour of a material in a rotating wheel abrasion test depends not only on the intrinsic properties of the test sample itself, but also on the conditions of the test, such as nature of the abrasive particle type, size, shape, brittleness, the wheel hardness, its stiffness and the nature of the environment.

![Figure 4.5: Schematic diagram of dry sand rubber wheel abrasion test apparatus (ASTM, 2010a)](image)

4.2.2.5 Wear measurement

Wear is defined as the progressive loss of material from the operating surfaces of the mechanically interacting elements of tribosystems which may be measured both qualitatively and quantitatively in terms of the following parameters (Ruff, 1992):

1. **Mass loss measurement:** It is a quantitative approach of wear measurement in which an original specimen is weighed and that the weight of the object after wear exposure be determined and subtracted from the original to determine the difference in weight (i.e. mass change). As the parts involved become smaller
and lighter, or the wear loss becomes smaller, it becomes necessary to use highly sensitive weighing equipment.

2. *Linear measurement:* It is one-dimensional changes in the geometry of interacting tribo-elements perpendicular to their common contact area. Examples include the wear of bushings or shafts, ball-bearing retainers, piston-cylinder wall contacts etc.

3. *Area measurement:* It is the two-dimensional changes of cross sections of interacting triboelements perpendicular to their common contact area. Certain wear contact geometries produce material loss over a localized area on the two surfaces which can be measured and is proportional to the amount of wear. Examples include worn areas on gear teeth, on bearing retainers and on sliding pads with contoured surfaces.

4. *Volume measure of wear:* It is the three-dimensional changes of geometric regions of interacting triboelements adjacent to their common contact area. These measurements are generally made when the worn region is very irregular or unsymmetrical in shape, or when high accuracy in the result is needed. In addition to these quantities, other common wear parameters which are being used, includes, wear-time ratio (defined as wear velocity), wear rate (defined as wear volume per unit of sliding distance), and the wear coefficient, k, which is expressed in Equation (4.4).

\[ K = \frac{W}{F \times S} \]  

(4.4)

where, \(W\) is the wear volume (in mm\(^3\)), \(F\) is the applied load (in Newton), and \(S\) is the sliding distance (in meters).
4.3 Plan of investigation

A detailed study to investigate the abrasion wear performance of the welded joints was carried out in two stages:

1. In the first stage, the mathematical modeling and optimization of the abrasion wear behaviour of the Q&T low alloy abrasion resistant steel under two-body and three-body abrasion wear conditions was carried out.

2. In the second stage, the wear performance of the welded joints laid down using different filler material compositions was carried out under the optimum working conditions obtained in the first stage.

4.4 Design of experiments

Design of experiment procedure is a powerful approach to improve product design or improve process performance. This procedure constitutes a systematic method concerning the planning of experiments, collection and analysis of data with near-optimum use of available resources. It is used to systematically investigate process variables that influence the quality of products. It is possible to identify the process conditions that influence product quality and costs, which in turn enhance the product manufacturability, quality, reliability, productivity and working life (Myers et al., 2009).

4.4.1 Identification, selection and establishment of operating ranges of wear variables

This was a prerequisite step which had to be taken for later parts of the investigation to follow. The important independent controllable variables such as load, number of revolutions, abrasives grain size etc. which affect the wear performance of the steels were identified and selected. Apart from taking the large number of trial runs, the
literature was reviewed in detail to establish the limits or operating ranges of these factors or variables. Other than above factors, the working limitations of the machine were given the main consideration for establishing the working ranges of the variables. To have better understanding of the concepts, it becomes imperative to know the technical vocabulary of the technique as listed below:

1. **Factor**: One of the independent variables that can be set to a desired value is called a factor. Factors are under the direct control of the experimenter during an experimental run.

2. **Level**: It is the numerical value or qualitative feature of each factor. The level of a factor is its value or setting during the experimental run.

3. **Treatment**: The specific combination of the levels, on from each factor which is tested is termed as treatment.

4. **Response**: It is the numerical result of an observation made with a particular treatment combination.

The various factors at their different levels under varied categorizations for two-body and three-body abrasion tests are shown in Table 4.1 and Table 4.2 respectively. On the basis of the planning of experiments, design of experiments includes number of techniques like factorial design, Taguchi method, response surface methodology etc.

### Table 4.1: Factors and levels of independent variables for two-body abrasion wear test (POD)

<table>
<thead>
<tr>
<th>Factors</th>
<th>Symbol</th>
<th>Type</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-α</td>
</tr>
<tr>
<td>Load (N)</td>
<td>A</td>
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<td>7.93</td>
</tr>
<tr>
<td>Revolutions (Rev.)</td>
<td>B</td>
<td>Numeric</td>
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</tr>
<tr>
<td>Grit size</td>
<td>C</td>
<td>Categorical</td>
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Table 4.2: Factors and levels of independent variables for three-body abrasion wear test (DSRW)

<table>
<thead>
<tr>
<th>Factors</th>
<th>Symbol</th>
<th>Type</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-α-1 0 1+α</td>
</tr>
<tr>
<td>Load (N)</td>
<td>A</td>
<td>Numeric</td>
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<tr>
<td>Speed (m/s)</td>
<td>B</td>
<td>Numeric</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>No. of revolutions (Rev.)</td>
<td>C</td>
<td>Numeric</td>
<td>500 1000 1500 2000 2500</td>
</tr>
<tr>
<td>Abrasive flow rate (gm/min)</td>
<td>D</td>
<td>Numeric</td>
<td>150 200 250 300 350</td>
</tr>
</tbody>
</table>

In the present work, response surface methodology based on the center composite rotatable design was used to plan the experiments and subsequent analysis of the data collected.

4.4.2 Response surface methodology

Response surface methodology (RSM) is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and objective is to optimize this response (Myers et al., 2009).

In real life problems, the form of the relationship between the response and the input variables is unknown. Thus RSM provides a suitable approximation for the true functional relationship between response ‘y’ and a set of independent variables \(x_1, x_2, \ldots, x_k\) as given in Equation (4.5).

\[
y = f(x_1, x_2, \ldots, x_k) + \varepsilon \tag{4.5}
\]

where, \(\varepsilon\) represents the error observed in the response.

If the response is well modeled by a linear function of the independent variables, the approximating function is the first order model (Equation 4.6).

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k + \varepsilon \tag{4.6}
\]

If there is curvature in the system, a polynomial of higher degree must be used, such as the second order model expressed by Equation (4.7).

135
\[ y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i < j} \beta_{ij} x_i x_j + \epsilon \quad (4.7) \]

where, \( \beta \)'s are regression coefficients and \( \epsilon \) is the error observed in the response.

Almost all RSM problems use one or both of these models. A polynomial model can be a reasonable approximation of the true functional relationship over the entire space of the independent variables, but for a relatively small region they usually work quite well (Montgomery, 2007).

In the present research, RSM has been applied to develop the mathematical models in the form of multiple regression equations for the wear loss of the pins and rectangular specimens extracted from the base material. Second order models have been developed which contain linear, squared and cross product terms of independent variables.

In order to estimate the regression coefficients, a number of experimental design techniques are available. Cochran and Cox developed the new designs, specifically, for fitting the second order response surfaces called central composite rotatable designs, which are constructed by adding further treatment combinations to those obtained from a 2k factorial (Myers et al., 2009).

In the present research, central composite rotatable design has been used to develop second order prediction models for the prediction of wear loss during the two-body and three-body abrasion test of quenched and tempered low alloy abrasion resistant steel.

**4.4.2.1 Center composite rotatable design**

Rotatable design indicates that the standard error of the predicted response at any point on the fitted surface will be same for all the points that are at the same distance from the centre of the region. This criterion of rotatability can be explained as: Let the point (0,
0,---, 0) represent the centre of the region in which the relation between dependent variable Y and independent variable X is under investigation. From the results of any experiment, the standard error $\varepsilon$ of Y can be computed at any point on the fitted surface. Because of rotatability condition, this standard error will remain same at all equidistant points with the distance $\delta$ from the centre of region i.e. for all points, which satisfy the Equation (4.8),

$$x_1^2 + x_2^2 + x_3^2 + \ldots + x_k^2 = \delta^2 = \text{constant} \quad (4.8)$$

Central composite rotatable designs for any number of k variables can be built-up from the following three components:

1. The points that constitute the $2^k$ factorial.
2. The extra $2^k$ points to form a central composite design with $\alpha$. The value of $\alpha$ must be $2^{k/4}$ in order to make the design rotatable. These are called the star or axial points. With five or more variables, the size of the experiment is reduced by using a half replicate in the $2^k$ factorial. With the half replicate $\alpha$ becomes $2^{(k-1)/4}$. The additional factor combinations introduced by the extra $2^k$ points are formed by keeping one of the factors at $-\alpha$ level, while keeping the remaining of the factors at 0 level, and again keeping that factor at $+\alpha$ level while keeping the rest of the factors at the same 0 level till all the factors have been designed $-\alpha$ and $+\alpha$.
3. A few points are added at the centre. These are called centre points. All factors are kept at their 0 level on these points (Myers et al., 2009). By replicating the centre points, no replication is needed to find mean square error (Akhanazarova & Kafarov, 1982).

Once the operating ranges of the factors are established, the upper and the lower limits are coded as $+\alpha$ and $-\alpha$ level respectively. The ranges from $+\alpha$ and $-\alpha$ level are
divided into four equal parts forming the intermediate levels of -1, 0, +1 with 0 level being the centre point. This is done to facilitate the formation of the design matrix, which shows the different combinations of the factors, according to which the experimental runs are to be carried out randomly to avoid systematic errors. The components of central composite rotatable design for different number of variables are given in Table 4.3 and a pictorial representation of different points for the case of three independent variables is shown in Figure 4.6.

**Table 4.3: Components of CCRD**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Factorial Points</th>
<th>Star point</th>
<th>Center points</th>
<th>Total number of experiments (N)</th>
<th>α-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numeric (k)</td>
<td>Categoric (n)</td>
<td>(2&lt;sup&gt;k&lt;/sup&gt;)(n+1)</td>
<td>(2&lt;sup&gt;k&lt;/sup&gt;)&lt;sup&gt;(n+1)&lt;/sup&gt;</td>
<td>≈ [6*(n+1)]</td>
<td>α&lt;sub&gt;Value&lt;/sub&gt;</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>8</td>
<td>8</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>40</td>
</tr>
</tbody>
</table>

**Figure 4.6: Central composite rotatable design for three variables: red circles—factorial points; yellow circles—star points; blue circles—center points**
4.4.3 Development of design matrix

The design matrix developed with the help of the statistical technique of central composite rotatable design was used to conduct the experiments for two-body and three-body abrasion wear conditions, which are shown in Table 4.4 and Table 4.5 respectively, along with the wear loss values measured as response.

Table 4.4: Design matrix for two-body abrasion test in actual form

<table>
<thead>
<tr>
<th>Experiments order</th>
<th>A:Load (N)</th>
<th>B:Revolutions (Rev.)</th>
<th>C:Grit size (mesh)</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Run</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 1</td>
<td>15</td>
<td>200</td>
<td>80</td>
<td>70.8</td>
</tr>
<tr>
<td>16 2</td>
<td>10</td>
<td>300</td>
<td>180</td>
<td>26.9</td>
</tr>
<tr>
<td>24 3</td>
<td>15</td>
<td>200</td>
<td>180</td>
<td>26.6</td>
</tr>
<tr>
<td>17 4</td>
<td>20</td>
<td>300</td>
<td>180</td>
<td>35.5</td>
</tr>
<tr>
<td>4 5</td>
<td>20</td>
<td>300</td>
<td>80</td>
<td>103.9</td>
</tr>
<tr>
<td>6 6</td>
<td>22.07</td>
<td>200</td>
<td>80</td>
<td>84.3</td>
</tr>
<tr>
<td>26 7</td>
<td>15</td>
<td>200</td>
<td>180</td>
<td>28.6</td>
</tr>
<tr>
<td>1 8</td>
<td>10</td>
<td>100</td>
<td>80</td>
<td>32.1</td>
</tr>
<tr>
<td>23 9</td>
<td>15</td>
<td>200</td>
<td>180</td>
<td>25.7</td>
</tr>
<tr>
<td>20 10</td>
<td>15</td>
<td>58.58</td>
<td>180</td>
<td>10.4</td>
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</tr>
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<td>21 12</td>
<td>15</td>
<td>341.42</td>
<td>180</td>
<td>34.0</td>
</tr>
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<td>11 13</td>
<td>15</td>
<td>200</td>
<td>80</td>
<td>69.9</td>
</tr>
<tr>
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<td>22.07</td>
<td>200</td>
<td>180</td>
<td>27.3</td>
</tr>
<tr>
<td>14 15</td>
<td>10</td>
<td>100</td>
<td>180</td>
<td>13.8</td>
</tr>
<tr>
<td>8 16</td>
<td>15</td>
<td>341.42</td>
<td>80</td>
<td>92.4</td>
</tr>
<tr>
<td>18 17</td>
<td>7.93</td>
<td>200</td>
<td>180</td>
<td>19.0</td>
</tr>
<tr>
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<td>15</td>
<td>200</td>
<td>180</td>
<td>25.2</td>
</tr>
<tr>
<td>10 19</td>
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<td>80</td>
<td>70.0</td>
</tr>
<tr>
<td>25 20</td>
<td>15</td>
<td>200</td>
<td>180</td>
<td>29.4</td>
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<td>7.93</td>
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<td>46.4</td>
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<td>100</td>
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<td>47.2</td>
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<td>9 23</td>
<td>15</td>
<td>200</td>
<td>80</td>
<td>69.2</td>
</tr>
<tr>
<td>3 24</td>
<td>10</td>
<td>300</td>
<td>80</td>
<td>73.4</td>
</tr>
<tr>
<td>13 25</td>
<td>15</td>
<td>200</td>
<td>80</td>
<td>68.1</td>
</tr>
<tr>
<td>7 26</td>
<td>15</td>
<td>58.58</td>
<td>80</td>
<td>29.0</td>
</tr>
</tbody>
</table>
Table 4.5: Design matrix for three-body abrasion test in actual form

<table>
<thead>
<tr>
<th>Experiments order</th>
<th>Input variables</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std</td>
<td>Run</td>
<td>A:Load (N)</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>27</td>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>9</td>
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<td>22</td>
<td>11</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>60</td>
</tr>
<tr>
<td>26</td>
<td>13</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
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</tr>
<tr>
<td>16</td>
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<td>100</td>
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<td>4</td>
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<td>100</td>
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<td>24</td>
<td>21</td>
<td>80</td>
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<tr>
<td>10</td>
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</tr>
<tr>
<td>20</td>
<td>23</td>
<td>80</td>
</tr>
<tr>
<td>21</td>
<td>24</td>
<td>80</td>
</tr>
<tr>
<td>11</td>
<td>25</td>
<td>60</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>80</td>
</tr>
<tr>
<td>28</td>
<td>27</td>
<td>80</td>
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<tr>
<td>29</td>
<td>28</td>
<td>80</td>
</tr>
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<td>29</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>

4.4.4 Experimental procedure

Wear test specimens for two-body and three-body abrasion tests were machined from 15 mm thick plate of abrasion resistant steel (JFE-EH400). For two-body abrasion, 26
number of cylindrical pins having a diameter of 6 mm and length 25 mm, thus resulting into cross-sectional area of 28.26 mm², were extracted from the base metal. Similarly, for three-body abrasion, 30 numbers of rectangular specimens of size (76.2 × 25.4 × 12.7) mm were machined out from the base metal. The detailed experimental procedure followed to perform the experiments has been outlined in Section 3.7.1. The measured values of response (weight loss) were used to calculate the coefficients of the regression equation or model.

4.4.5 Estimation of the coefficients

The regression equation representing second order response surface has been given in Equation (4.7). The same may be rewritten in matrix form as in Equation (4.9).

This equation may be written as in matrix form:

\[ Y = X\beta + \varepsilon \]  \hspace{1cm} (4.9)

where,

\[ Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{k1} & x_{11}^2 & \cdots & x_{11}x_{21} & \cdots \\ 1 & x_{12} & x_{22} & \cdots & x_{k2} & x_{12}^2 & \cdots & x_{12}x_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots \\ 1 & x_{1N} & x_{2N} & \cdots & x_{kN} & x_{1N}^2 & \cdots & x_{1N}x_{2N} & \cdots \end{bmatrix}, \beta = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_p \end{bmatrix}, \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix} \]

N: Total number of experiments,

P: Total number of coefficients,

Y: (N × 1) vector of the observations,

X: (N × P) matrix of the levels of the independent variables,

β: (P × 1) vector of the regression coefficients,

ε: (N × 1) vector of random errors.

The regression coefficients are obtained using the method of least squares. The least squares method minimizes the residual error between actual and estimated response. The least square estimator can be expressed as Equations (4.10) or Equation (4.11).
\( L = \sum_{i=1}^{N} \varepsilon_i^2 = \varepsilon' \varepsilon = (Y - X\beta)'(Y - X\beta) \) \hfill (4.10)

\( L = Y'Y - \beta'X'Y - Y'X\beta + \beta'X'X\beta \) \hfill (4.11)

Since, \( \beta'X'Y \) is a \((1 \times 1)\) matrix and its transpose will also be a \((1 \times 1)\) matrix. Thus

\[
(\beta'X'Y)' = \beta XY'
\] \hfill (4.12)

Hence the Equation (4.11) has been rewritten as Equation (4.13):

\[
L = Y'Y - 2\beta'X'Y + \beta'X'X\beta
\] \hfill (4.13)

Also, the least square estimates must satisfy as Equation (4.14):

\[
\frac{\partial L}{\partial \beta} \bigg|_{\beta} = -2X'Y + 2X'X\beta = 0
\] \hfill (4.14)

This simplifies to;

\[
X'X\beta = X'Y
\] \hfill (4.15)

Thus, the least squares estimator of \( \beta \) is

\[
\beta = (X'X)^{-1}X'Y
\] \hfill (4.16)

The Equation (4.16) is used to calculate the regression coefficients (Montgomery, 2007).

**4.4.6 Checking the adequacy of the model, model coefficients and test for lack of fit**

Once the coefficients have been estimated, the calculated coefficients or the model equation need to be tested for statistical significance. An analysis of variance (ANOVA) is commonly used to perform test for (i) significance of the regression model, (ii) significance of individual model coefficients (iii) lack-of-fit of model. This analysis is based on two assumptions; (a) the variables are normally distributed, and (b) homogeneity of variance. Significant violation of either assumption can increase the chances of committing either a Type-I or Type-II error, depending on the nature of analysis and violation of the assumption.
The normal probability plot of the residuals is used to examine the viability of assumptions of ANOVA for residuals. The normal probability plot indicates that whether the residuals follow a normal distribution or not. To improve the normality of residuals, the Box-Cox transformation is employed. The Box-Cox provides a family of transformations that will normalize the data which are not normally distributed by identifying an appropriate exponent (Lambda=\(\lambda\)). The Lambda value indicates the power to which all data should be raised. Box and Cox originally envisioned this transformation as a panacea for simultaneously correcting normality, linearity and homogeneity (Osborne, 2010).

4.4.6.1 Test for significance of the regression model

The test for the significance of the regression model is performed using analysis of variance (ANOVA) procedure by calculating the ratio between the regression sum of squares and error sum of squares, and comparing the result to the F-ratio with the appropriate degrees of freedom at a given significance level. Usually, significance levels of 0.10, 0.05, and 0.01 are used to determine the value of F-ratio to identify a significant model as desired.

4.4.6.2 Test of significance on individual regression coefficients

Another test needs to be performed in order to check the significance of the individual coefficients. This test forms the basis for model optimization by adding or deleting the coefficients through backward elimination, forward addition or stepwise elimination/addition/exchange. It involves the determination of p-value or probability value, usually relating to the risk of falsely rejecting a given hypothesis. The determined “Prob.>F” value can be compared with the desired probability. If the value of “Prob.>F” for individual regression coefficient is less than the desired probability, then this term will be significant model term.
4.4.6.3 Test for lack of fit

If replicate measurements are present, a test can be performed which provide the significance of the replicate error in comparison to the model dependent error. The test splits the residual or error sum of squares into a contribution of the pure error which is based on the replicate measurements, and a fraction of which is due to the lack of fit based on the model performance. The test statistic for lack of fit is given by the ratio between the lack of fit mean square and the pure error mean square. This F-test statistic can be used to determine whether the lack of fit error is significant or otherwise at the desired significance level. Insignificant lack of fit is desired, as significant lack of fit indicates that there might be contributions in the regression response relationship which is not accounted for by the model.

The additional checks need to be performed which includes the finding of various coefficient of determination ($R^2$). These $R^2$ coefficients have values between zeros to one. In this term, adjusted $R^2$ and predicted $R^2$ should be within approximately 0.20 of each other to be in reasonable agreement (Aggarwal et al., 2008).

4.4.7 Important terminology in ANOVA

Several terms are used during analysis of variance. A brief description of these terms is given in this section (Design Expert 8.0.4.1).

4.4.7.1 Terminology for regression model

1. **Sum of squares**: It is the total of the sum of squares for the terms in the model. It helps in estimating the coefficient of each factor.

   $$SS_{Total} = Y'Y - \left(\frac{\sum_{i=1}^{n} Y_i^2}{n}\right)^2$$

   (4.17)

   where,

   $$Y'Y = (\sum_{i=1}^{n} Y_i^2)^2$$

   $$Y = \text{response matrix}; \ Y' = \text{transpose matrix}$$
2. **Degrees of freedom (DF):** It is the degrees of freedom for the model. It is the number of model terms, including the intercept minus one.

   \[ DF_{\text{Total}} = \text{Total number of runs} - 1 \]  

   \( (4.18) \)

3. **Mean square:** It estimates the model variance, which is calculated by the model sum of squares divided by model degrees of freedom.

   \[ MS_{\text{Total}} = \frac{SS_{\text{Total}}}{DF_{\text{Total}}} \]  

   \( (4.19) \)

4. **F–value:** It is the test for comparing model variance with residual (error) variance. If the variances are close to the same, the ratio will be close to one and it is less likely that any of the factors have a significant effect on the response. The Equation (4.20) is used to calculate F value.

   \[ F-\text{value} = \frac{\text{Model mean square}}{\text{Residual mean square}} \]  

   \( (4.20) \)

5. **Prob.\( > \)F:** It is the probability of seeing the observed F value, if the null hypothesis is true (there is no factor effect). Small probability values call for rejection of the null hypothesis. The probability equals the proportion of the area under the curve of the F-distribution that lies beyond the observed F value. The F distribution itself is determined by the degrees of freedom associated with the variances being compared. If the “Prob.\( > \)F” value is very small (less than 0.05), then the terms in the model have a significant effect on the response at 95% confidence level.

### 4.4.7.2 Terminology for residuals

The difference between the actual and predicted response values is known as residuals. These are the following terms used in analysis of variance to estimate residuals.

1. **Sum of squares:** This equals the sum of squares for all the terms not included in the model. It is a measure of the discrepancy between the experimental data and
the estimated data from the model. A small residual sum of square indicates a tight fit of the model to the data.

\[ SS_{Residuals} = SS_{Total} - SS_{Regression} \]  
\[ SS_{Regression} = b'XY - \frac{(\sum_{i=1}^{n} Y_i^2)^2}{n} \]  

where,  
b=Predicted response matrix; X’=Transpose of input matrix; Y=Response matrix

2. **Degrees of freedom (DF):** It is the corrected degree of freedom for residual or error. Degree of freedom of residual is estimated by subtracting degrees of freedom of main effects, quadratic effects and interaction effects from total degrees of freedom.

\[ DF_{Residual} = DF_{Total} - DF_{Model} \]  

3. **Mean square:** It is the estimate of process variance. The square root of this provides an estimate of the process standard deviation.

\[ MS_{Residuals} = \frac{SS_{Residual}}{DF_{Residual}} \]

4.4.7.3 **Terminology for lack of fit (LOF)**

The lack of fit is the variation of the data around the fitted model. If the model does not fit the data well, this will be significant. The non-significant lack of fit is invariably desirable.

1. **Sum of squares:** It is the residual sum of squares after removing the pure error sum of squares.

\[ SS_{Residual} = SS_{Lack of fit} + SS_{pure error} \]

2. **Degrees of freedom (DF):** It gives the amount of information available after accounting for blocking, model terms, curvature and pure error.
3. **Mean square:** The mean square or variance is associated with the residual of lack of fit. This is the variation about the fitted model.

\[ MS_{\text{Lack of fit}} = \frac{SS_{\text{Lack of fit}}}{DF_{\text{Lack of fit}}} \]  

(4.26)

4. **F-value:** It is the test for comparing lack of fit variance with pure error variance. If the variances are close to the same, the ratio will be close to one and it is less likely that lack of fit is significant.

\[ F - \text{value} = \frac{SS_{\text{Lack of fit}}}{MS_{\text{Pure error}}} \]  

(4.27)

5. **Prob.>F:** Probability of seeing the observed F value, if the null hypothesis is true. Small probability values call for rejection of the null hypothesis that lack of fit is not significant. If the “Prob.>F” value is very small (less than 0.05) then lack of fit is significant at 95% confidence level. In other words the variation in the model points significantly differs from the variation in the replicated points. Thus it is desired that “Prob.>F” value for lack of fit to be greater than 0.05.

### 4.4.7.4 Terminology for pure error

The amount of variation in the response in replicated design points is called pure error.

1. **Sum of squares:** It is the pure error sum of squares from replicated points.

2. **Degrees of freedom (DF):** It shows the amount of information available from replicated points.

3. **Mean square:** It estimates the pure error variance.

### 4.4.7.5 Terminology for R-squared (R²) values

1. **Ordinary R-square:** It is the measure of the amount of variation around the mean explained by the model.

\[ R^2 = 1 - \frac{SS_{\text{residual}}}{SS_{\text{model}} + SS_{\text{residual}}} \]  

(4.28)
2. **Adj R-squared**: It is the measure of the amount of variation around the mean explained by the model, adjusted for the number of terms in the model. The adjusted R-squared value decreases as the number of terms in the model increases, if those additional terms do not add value to the model.

\[
Adj \ R^2 = 1 - \frac{SS \ residual / DF \ residual}{(SS \ model + SS \ residual) / (DF \ model + DF \ residual)} \tag{4.29}
\]

3. **Pred R-squared**: It is the measure of the amount of variation in new data explained by the model.

\[
Pred \ R^2 = 1 - \frac{PRESS}{SS \ total - SS \ block} \tag{4.30}
\]

In the present work this analysis was carried out for a significance level of \(\alpha=0.05\), i.e. for a confidence level of 95%. ANOVA tests were carried out for response surface quadratic models for wear loss under two-body and three-body abrasion conditions, and are summarized in Table 4.6 and Table 4.7 respectively. The tables show the value of “Prob.>F” for models to be 0.0001 which is less than 0.05, that indicates the models are significant. For two-body abrasion, the value of “Prob.>F” for main effect of load, number of revolutions, grit size and two-level interaction of load and number of revolutions, load and grit size, grit size and number of revolutions, and second-order effect of load and number of revolutions was 0.05, thereby, indicating that these terms were significant model terms. The value of “Prob.>F” for lack-of-fit was 0.0671 which was greater than 0.05, indicating that lack of fit was insignificant, which is a desirable condition to fit the model. Similarly, for three-body abrasion, “Prob.>F” value for main effects [load (A), speed (B), no. of revolutions (C) and abrasives flow rate (D)], interaction effects (AB, AC, AD and CD) and second order effects (A², B², C² and D²) were found to be less than 0.05, thereby, indicating that these terms were significant model terms. The value of “Prob.>F” for lack of fit was found to be 0.0560
which is greater than 0.05, indicating that lack of fit was insignificant and thus the desirable condition to fit the model.

The total variation in the model can be explained by the $R^2$ value which is 0.996 and 0.987 or close to 1 for two-body and three-body abrasion conditions respectively. The adjusted $R^2$ value is useful when comparing models with different number of terms. For two-body and three-body abrasion, the adjusted $R^2$ was found to be 0.994 and 0.977 respectively, and the difference between adjusted $R^2$ value and ordinary $R^2$ value was found to be within the limit of 20%. Further the adequate precision value was used to compare the range of the predicted values at the design points to the average prediction error.

**Table 4.6: Resulting ANOVA table for response surface quadratic model for two-body abrasion test**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean square</th>
<th>F-value</th>
<th>P-value Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>17410.877</td>
<td>8</td>
<td>2176.360</td>
<td>481.631</td>
<td>0.0001 S</td>
</tr>
<tr>
<td>A-Load (N)</td>
<td>980.264</td>
<td>1</td>
<td>980.264</td>
<td>216.934</td>
<td>0.0001</td>
</tr>
<tr>
<td>B-Revolutions (Rev.)</td>
<td>3910.519</td>
<td>1</td>
<td>3910.519</td>
<td>865.402</td>
<td>0.0001</td>
</tr>
<tr>
<td>C-Grit size</td>
<td>11000.425</td>
<td>1</td>
<td>11000.425</td>
<td>2434.407</td>
<td>0.0001</td>
</tr>
<tr>
<td>AB</td>
<td>41.861</td>
<td>1</td>
<td>41.861</td>
<td>9.264</td>
<td>0.0073</td>
</tr>
<tr>
<td>AC</td>
<td>334.531</td>
<td>1</td>
<td>334.531</td>
<td>74.032</td>
<td>0.0001</td>
</tr>
<tr>
<td>BC</td>
<td>979.466</td>
<td>1</td>
<td>979.466</td>
<td>216.757</td>
<td>0.0001</td>
</tr>
<tr>
<td>$A^2$</td>
<td>42.761</td>
<td>1</td>
<td>42.761</td>
<td>9.463</td>
<td>0.0068</td>
</tr>
<tr>
<td>$B^2$</td>
<td>138.326</td>
<td>1</td>
<td>138.326</td>
<td>30.612</td>
<td>0.0001</td>
</tr>
<tr>
<td>Residual</td>
<td>76.818</td>
<td>17</td>
<td>4.519</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of fit</td>
<td>59.358</td>
<td>9</td>
<td>6.595</td>
<td>3.022</td>
<td>0.0671 NS</td>
</tr>
<tr>
<td>Pure error</td>
<td>17.460</td>
<td>8</td>
<td>2.183</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor. total</td>
<td>17487.695</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Std. dev. 2.126 R-squared 0.996
Mean 45.331 Adj. R-squared 0.994
C.V. % 4.689 Pred. R-squared 0.987
PRESS 229.041 Adeq. precision 73.343
Table 4.7: Resulting ANOVA table for response surface quadratic model for three-body abrasion test

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of squares</th>
<th>Degree of freedom</th>
<th>Mean square</th>
<th>F-value</th>
<th>P-value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>6.101</td>
<td>12</td>
<td>0.508</td>
<td>103.793</td>
<td>0.0001</td>
<td>S</td>
</tr>
<tr>
<td>A-Load (N)</td>
<td>2.792</td>
<td>1</td>
<td>2.792</td>
<td>570.039</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>B-Speed (m/s)</td>
<td>0.258</td>
<td>1</td>
<td>0.258</td>
<td>52.658</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>C-No. of revolutions (Rev.)</td>
<td>2.370</td>
<td>1</td>
<td>2.370</td>
<td>483.876</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>D-Abrasive flow rate (g/min)</td>
<td>0.316</td>
<td>1</td>
<td>0.316</td>
<td>64.613</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>0.061</td>
<td>1</td>
<td>0.061</td>
<td>12.355</td>
<td>0.0027</td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td>0.137</td>
<td>1</td>
<td>0.137</td>
<td>27.950</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>0.029</td>
<td>1</td>
<td>0.029</td>
<td>5.831</td>
<td>0.0273</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>0.024</td>
<td>1</td>
<td>0.024</td>
<td>5.000</td>
<td>0.0390</td>
<td></td>
</tr>
<tr>
<td>A²</td>
<td>0.048</td>
<td>1</td>
<td>0.048</td>
<td>9.898</td>
<td>0.0059</td>
<td></td>
</tr>
<tr>
<td>B²</td>
<td>0.020</td>
<td>1</td>
<td>0.020</td>
<td>4.020</td>
<td>0.0612</td>
<td></td>
</tr>
<tr>
<td>C²</td>
<td>0.042</td>
<td>1</td>
<td>0.042</td>
<td>8.481</td>
<td>0.0097</td>
<td></td>
</tr>
<tr>
<td>D²</td>
<td>0.050</td>
<td>1</td>
<td>0.050</td>
<td>10.135</td>
<td>0.0054</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>0.083</td>
<td>17</td>
<td>0.005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of fit</td>
<td>0.076</td>
<td>12</td>
<td>0.006</td>
<td>4.420</td>
<td>0.0560</td>
<td>NS</td>
</tr>
<tr>
<td>Pure error</td>
<td>0.007</td>
<td>5</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor. total</td>
<td>6.184</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Std. dev.                          | 0.070          | R-squared         | 0.987       |
| Mean                               | 1.129          | Adj. R-squared    | 0.977       |
| C.V. %                             | 6.198          | Pred. R-squared   | 0.943       |
| PRESS                              | 0.351          | Adeq. precision   | 39.271      |

4.4.8 Analysis of results

This section discusses the viability of assumptions of ANOVA for residuals in the form of graphs namely; plot of residuals versus fitted values, normal probability plots and actual versus predicted plots followed by the development of mathematical models for two-body and three-body abrasion wear phenomena.

4.4.8.1 Plot of residual vs. fitted values

If the model is correct & if the assumptions are satisfied, the residual should be structure less, in particular, they should be unrelated to any other variable, including the response. A defect that occasionally shows up on this plot is non-constant variance.
Sometimes, the variance of the observations increases as the magnitude of the observation increases. This would be the case if the error was a constant percentage of the size of the observations (this mainly happens with many measuring instruments-error in a percent of the scale reading). If this were the case, the residual get larger as the response get larger and the plot of the residual v/s response will take the shape upward opening like funnel. Non-constant variance also rises in case when the data follows a non normal, skewed distribution. Erratic response to the treatment can also cause inequality of the variance.

To test the assumption of constant variance, graphs were plotted between residuals and predicted response for wear loss during two-body and three-body abrasion shown in Figure 4.7. The random scatter in the figures signifies that the assumption has not been violated.

![Figure 4.7: Plot of residuals vs. predicted response for different wear testing conditions of base metal: (a) two-body abrasion; (b) three-body abrasion](image)

**4.4.8.2 Normal plot of residuals**

It is a graph between the normal probability and residuals. The normal probability plot indicates that whether the residuals follow a normal distribution, in which case, the points follow the straight line. It shows some scatter even with the normal data. It can be clearly inferred from the normal probability plots drawn for two-body and three-
body abrasion shown in Figure 4.8 that the residuals fall on a straight line implying that the errors are distributed normally.

![Normal Plot of Residuals](image1)

**Figure 4.8:** Normal probability plot of residuals for different wear testing conditions of base metal: (a) two-body abrasion; (b) three-body abrasion

### 4.4.8.3 Actual vs. predicted plot

It is a graph of actual response values vs. predicted response values and shown in Figure 4.9. Actual values are achieved through experiments & predicted are those, which are predicted from the model. It helps to detect a value, or group of values, that are not easily predicted by the model.

![Predicted vs Actual](image2)

**Figure 4.9:** Plot of predicted vs. actual response for different wear testing conditions of base metal: (a) two-body abrasion; (b) three-body abrasion

Figure 4.9 clearly shows that the data points are evenly split along the 45° line, confirming the close proximity between the actual and predicted values of the weight loss for two-body and three-body abrasion.
4.4.8.4 Mathematical models

The mathematical models developed for two-body and three-body abrasion wear conditions both in coded and actual forms are presented in the following sub-section.

4.4.8.4.1 Mathematical models for two-body abrasion test

The final regression model for wear loss in terms of coded factors is represented in Equation (4.31).

\[
\text{Wear loss} = 48.35 + 7.83A + 15.63B - 20.57C + 2.29AB - 4.57AC - 7.82BC - 1.75A^2 \\
- 3.15B^2
\]  

(4.31)

Although, the two-way interaction terms of AC and BC were found significant, however, owing to very low values of their coefficients, these terms are neglected in the final empirical models in terms of actual factors, and are given in Equation (4.32) and Equation (4.33) for grit size of 80 and 180 respectively.

\[
\text{Wear loss (80 grit)} = -29.86 + 3.67\text{Load} + 0.29\text{Revolutions} + 0.005\text{Load} \times \text{Revolutions} \\
- 0.07\text{Load}^2 - 0.003\text{Revolutions}^2
\]  

(4.32)

\[
\text{Wear loss (180 grit)} = -12.27 + 1.84\text{Load} + 0.14\text{Revolutions} + 0.005\text{Load} \times \text{Revolutions} \\
- 0.07\text{Load}^2 - 0.003\text{Revolutions}^2
\]  

(4.33)

4.4.8.4.2 Mathematical models for three-body abrasion test

The final regression models for wear loss in terms of coded and actual factors are given in Equation (4.34) and Equation (4.35) respectively.

\[
\text{Wear} = 1.25 + 0.34A - 0.10B + 0.31C + 0.11D - 0.061AB + 0.093AC + 0.042AD \\
+ 0.039CD - 0.042A^2 - 0.039C^2 - 0.043D^2
\]  

(4.34)

\[
\text{Wear} = -1.85 + 0.02 \times \text{Load} + 0.30 \times \text{Speed} - 3.57 \text{E} - 05 \times \text{No. of revolutions} + \\
5.07 \text{E} - 003 \times \text{Abrasive flow rate} - 3.075 \text{E} - 03 \times \text{Load} \times \text{Speed} + 9.25 \text{E} - \\
06 \times \text{Load} \times \text{No. of revolutions} + 4.225 \text{E} - 05 \times \text{Load} \times \text{Abrasive flow rate} + \\
1.56 \text{E} - 06 \times \text{No. of revolutions} \times \text{Abrasive flow rate} - 1.05 \text{E} - 04 \times \text{Load}^2 - \\
1.56 \text{E} - 07 \times \text{No. of revolutions}^2 - 1.70 \text{E} - 05 \times \text{Abrasive flow rate}^2
\]  

(4.35)
4.4.9 Optimization

The optimization searches for a combination of factor levels that simultaneously satisfy the requirements placed on each of the responses and factors. Before optimization, each response was established with the appropriate model. Optimization of one response or the simultaneous optimization of multiple responses can be performed graphically or numerically. In the present work, the aim was to find the optimal values of input variables in order to ascertain the minimum and maximum wear loss. The optimal solutions for the two-body and three-body abrasion wear conditions are reported in Table 4.8 and Table 4.9 respectively.

Table 4.8: Optimization results for two-body abrasion wear test

<table>
<thead>
<tr>
<th>Wear condition</th>
<th>Solution no.</th>
<th>Load (N)</th>
<th>Revolutions (Rev.)</th>
<th>Grit size (mesh)</th>
<th>Wear (mg)</th>
<th>Desirability</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>1</td>
<td>10</td>
<td>100</td>
<td>180</td>
<td>14.098</td>
<td>0.9604</td>
<td>Selected</td>
</tr>
<tr>
<td>Maximum</td>
<td>2</td>
<td>20</td>
<td>300</td>
<td>80</td>
<td>102.158</td>
<td>0.9814</td>
<td>Selected</td>
</tr>
</tbody>
</table>

Table 4.9: Optimization results for three-body abrasion wear test

<table>
<thead>
<tr>
<th>Wear condition</th>
<th>Solution no.</th>
<th>Load (N)</th>
<th>Speed (m/s)</th>
<th>No. of Revolutions (Rev)</th>
<th>Abrasive flow rate (gm/min)</th>
<th>Wear (mg)</th>
<th>Desirability</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>1</td>
<td>60</td>
<td>4</td>
<td>1000</td>
<td>200</td>
<td>0.472</td>
<td>0.936</td>
<td>Selected</td>
</tr>
<tr>
<td>Maximum</td>
<td>2</td>
<td>100</td>
<td>2</td>
<td>2000</td>
<td>300</td>
<td>2.217</td>
<td>0.967</td>
<td>Selected</td>
</tr>
</tbody>
</table>

4.4.10 Confirmation experiments

Statistically developed mathematical models for wear loss were found to be significant and were validated through F-tests and lack-of-fit tests. The higher values of coefficient of variation ($R^2$) for models indicate the effectiveness of making predictions. This conclusion is further supported through the confirmation runs. A set of two confirmation runs under each two-body and three-body abrasive wear conditions were performed to verify the prediction ability of the developed wear model. The details of the confirmation runs and their comparative evaluation with predicted values are given
in Table 4.10 and Table 4.11. The percentage error between the experimental and the predicted values for both the wear test conditions were found to be less than 5%, which show that all the experimental values were within the 95% prediction interval, which clearly demonstrates the accuracy of the models developed in this study.

**Table 4.10: Plan of confirmation experiments and results for two-body abrasion test**

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Load (N)</th>
<th>Revolutions (Rev.)</th>
<th>Grit size (mesh)</th>
<th>Wear loss (mg) Predicted</th>
<th>Wear loss (mg) Experimental</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>100</td>
<td>180</td>
<td>14.098</td>
<td>14.809</td>
<td>4.8</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>300</td>
<td>80</td>
<td>102.158</td>
<td>105.863</td>
<td>3.5</td>
</tr>
</tbody>
</table>

**Table 4.11: Plan of confirmation experiments and results for three-body abrasion test**

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Load (N)</th>
<th>Speed (m/s)</th>
<th>Revolutions (Rev)</th>
<th>Abrasive flow rate (gm/min)</th>
<th>Wear loss (mg) Predicted</th>
<th>Wear loss (mg) Experimental</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>4</td>
<td>1000</td>
<td>200</td>
<td>0.472</td>
<td>0.496</td>
<td>4.8</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>2</td>
<td>2000</td>
<td>300</td>
<td>2.217</td>
<td>2.312</td>
<td>4.1</td>
</tr>
</tbody>
</table>

### 4.5 Abrasion wear of weldments

Due to the exposure of these steels to a wide variety of wear conditions, series of experiments were carried out to examine the abrasion wear behavior of the different regimes of welded joints fabricated using different filler wire compositions under two-body and three-body abrasive wear conditions. Wear tests were performed on two types of specimen configurations: first, on fusion zone of the welded joints exhibiting a specific microstructure, and second, the different regions of heat affected zones comprising of mixed microstructures. The above study in collaboration with microstructural and microhardness studies of the fabricated joints helped in establishing a correlation among wear characteristics of the various joints and established a pathway to determine the relative wear rate of the different zones with respect to the base metal.
Further, the above established wear performance criterion were taken as the basis for comparative evaluation of the wear behavior of the welded joints.

4.5.1 Experimental procedure

The optimum set of parameters obtained from Section 4.4.9 for two-body and three-body abrasion wear phenomenon have been used to investigate the abrasion wear performance of the welded joints fabricated using varied filler material compositions. This section gives a brief outline of the fabrication and abrasion testing procedures employed for the welded joints.

4.5.1.1 Fabrication of welded joints

Two-body abrasion wear test specimens in the form of cylindrical pins of diameter 6 mm and length 15 mm were extracted from the welded plates fabricated using different filler material compositions as detailed in Chapter 3. In order to reduce the experimental and measurement error, set of three pins each from fusion zone and heat affected zone were extracted using wire WEDM process from all the welded plates (viz. WJM, WJA, WJMA, WJG and WJX). For three-body abrasion wear test, rectangular specimens of size \((76.2 \times 25.4 \times 12.7)\) mm comprising of fusion zone, HAZ and unaffected base metal were extracted from the welded plates fabricated using different filler material compositions.

4.5.1.2 Abrasion wear test

All the pins and rectangular specimens extracted from the welded joints were subjected to abrasion wear testing on pin-on-disc and dry sand rubber wheel apparatus respectively, at room temperature under the maximum wear rate conditions as shown in Table 4.10 and Table 4.11. The same experimental procedures were followed for the preparation and experimentation of the specimens as already discussed in Chapter 3.
4.6 Summary

This chapter has presented the important aspects related to wear mainly a detailed discussion on abrasion wear phenomenon. This was followed by the implementation of design of experiments approach using response surface methodology for identifying and establishing empirical relationships between different wear variables and wear loss under two-body and three-body abrasive wear conditions for base metal. The results obtained from the detailed experimentation and the abrasion wear studies of the base metal under two-body and three-body abrasive wear conditions have been presented and discussed in the next chapter.