CHAPTER-IV

Availability and Profit Analysis of a Clarifier System in Sugar Industry

4.1 Introduction

Impurities are inherent in any user defined/targeted product and therefore elimination of impurities is one of the most important processes to achieve the appropriate product. In the same way, in the sugar industry there are several impurities present (like small pieces of bagasse, mud etc.) in the sugarcane juice obtained from Feeding and Juice weighment systems, as discussed in previous chapters. To remove these impurities we have to go through different kind of processes according to industrial needs of different types of products targeted. Likewise, in sugar industry we put the cane juice into a process known as Clarification to get rid of different impurities.

In this chapter, the clarifier system in the sugar industry is analyzed that having three subsystems – Juice sulphitor, Raw juice heater and Sulphited juice heater. These subsystems are arranged in series. Both the subsystems ‘Juice sulphitor’ and ‘Raw juice heater’ have standby unit whereas subsystem ‘Juice sulphitor’ has single unit. The time to failure of the subsystems follows negative exponential distribution while repair time distribution is taken as arbitrary. Supplementary variable technique is adopted to derive the expression for availability and profit of the system. A particular case is also considered to show the behavior of availability and expected profit of the clarifier system in the sugar industry through tables.

Fig.4.1 Configuration diagram
4.2 System description

In the clarifier system in sugar industry, the juice obtained from juice weighment system contains impurities like small pieces of bagasse, mud etc. and hence it is passed through a number of filters in series for clarification. The juice is heated in raw juice heater (subsystem A) to attain a definite temperature (70°C). The juice remains in the heater for a certain period to achieve a given pH value. Heated juice with required pH value is sent to the sulphlonation unit (subsystem B), where Milk of lime is added and sulphur dioxide is passed through it to remove the mud. Then the juice is sent to sulphited juice heater (subsystem C) allowed to flash to its saturation temperature (120°C). The flashed juice is then transferred to a clarification which allows the suspended solids to settle. The supernatant, known as clear juice is drawn off of the clear juice and sent to the evaporators.

The clarifier system consists of three subsystems and detail descriptions of the subsystems are as follows:

1. **Subsystem A** (Raw Juice Heater)
   Subsystem A consists of three raw juice heaters out which two are working in parallel and one raw juice heater (A₁) in standby. Standby unit is in operation only when both the parallel units are failed. In this subsystem the raw juice is heated to attain a definite temperature (70°C) as well as a definite pH value. The failure of any one of the juice heaters reduces the capacity of the system. Complete failure occurs when all the juice heaters are failed. It connected with subsystems B and C in series.

2. **Subsystem B** (Juice Sulphitor)
   This subsystem consists of single juice sulphitor unit. In this subsystem Milk of lime is added and So₂ gas is passes in the cane juice, obtained from subsystem A, to attain its pH to 7.1 (raw cane juice has pH 4 to 4.5). This subsystem is connected in series with subsystems A, C. The failure of this subsystem causes complete failure of the system.

3. **Subsystem C** (Sulphited Juice Heater)
   This subsystem consists of three Sulphited juice heaters out which two are working in parallel and one Sulphited juice heater (C₁) in standby. Standby unit is in operation only when both the parallel units are failed. In this
subsystem, the juice is heated to attain a definite temperature 120° C as well as precipitates the juice. It is working in series with subsystems A and B. The failure of any one of the Sulphited juice heaters reduces the capacity of the system. Complete failure occurs when all Sulphited juice heaters are failed. The failure of subsystem C causes the complete failure of the system.

4.3 Assumptions
1. Repairmen are always available with the system.
2. Unit works as new after repair.
3. The distribution of failure times of the subsystems is taken as exponential while the distribution of repair times is arbitrary.
4. Each subsystem has a separate repair facility and there is no waiting time for repair.

4.4 Notations
The following symbols are associated with the system.

- **A, B, C**: Units working with full capacity.
- **A_1, C_1**: Standby units
- **a, c**: Units working with reduce capacity.
- **\( \bar{A}, \bar{A}, \bar{B}, \bar{C}, \bar{C}_1 \)**: Failed units.
- **\( \alpha_i \) (1 ≤ i ≤ 3), \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \)**: Failure rates of subsystems A, B, C, a, c, A_1 and C_1 respectively.
- **\( \beta_i(x), (1 ≤ i ≤ 7) \)**: Repair rates of subsystems A, B, C, a, c,(for i=1 to 5) and standby units A_1, C_1 (for i=6,7) respectively.
- **\( p_0(t) \)**: Probability that at time t the system is in good state.
- **\( p_i(x,t), (i=1,...,23) \)**: Probability that the system is in \( i^{th} \) state at time t has an elapsed repair time x.
- **\( S_i(i=0, 1,..., 23) \)**: States of the system.
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Fig. 4.2 State changeover diagram of juice clarifier system in the sugar industry
4.5 Mathematical analysis

In this section, the following set of differential-difference equations associated with the model (Fig. 4.2) developed by using supplementary variable technique.

\[ p_0(t+\Delta t) = [1-\alpha_1 - \alpha_2 + \alpha_3] p_0(t) + \int_0^{\infty} \beta_0(x)p_0(x,t)dx + \int_0^{\infty} \beta_1(x)p_1(x,t)dx + \int_0^{\infty} \beta_2(x)p_2(x,t)dx \]

Dividing both sides by \( \Delta t \)

\[ \frac{p_0(t+\Delta t) - p_0(t)}{\Delta t} = [-\alpha_1 - \alpha_2 + \alpha_3] p_0(t) + \int_0^{\infty} \beta_0(x)p_0(x,t)dx + \int_0^{\infty} \beta_1(x)p_1(x,t)dx + \int_0^{\infty} \beta_2(x)p_2(x,t)dx \]

As \( \Delta t \to 0 \)

\[ \left[ \frac{d}{dt} + \left( \int_0^{\infty} \beta_0(x)p_0(x,t)dx + \int_0^{\infty} \beta_1(x)p_1(x,t)dx + \int_0^{\infty} \beta_2(x)p_2(x,t)dx \right) \right] p_0(t) = \int_0^{\infty} \beta_1(x)p_1(x,t)dx + \int_0^{\infty} \beta_2(x)p_2(x,t)dx + \int_0^{\infty} \beta_3(x)p_3(x,t)dx \] ..(4.1)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_1 + \alpha_1 + \alpha_2 + \beta_1(x) \right] p_1(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \alpha_1 p_0(t) \] ..(4.2)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_2 + \alpha_2 + \beta_2(x) \right] p_2(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \alpha_2 p_0(t) \] ..(4.3)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_3 + \alpha_3 + \beta_3(x) \right] p_3(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \lambda_3 p_1(x,t) \] ..(4.4)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_4 + \alpha_4 + \beta_4(x) \right] p_4(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \lambda_4 p_2(x,t) \] ..(4.5)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_5 + \alpha_5 + \beta_5(x) \right] p_5(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \alpha_5 p_1(x,t) + \alpha_1 p_1(x,t) \] ..(4.6)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_6 + \alpha_6 + \beta_6(x) \right] p_6(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \alpha_6 p_1(x,t) + \alpha_1 p_1(x,t) \] ..(4.7)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_7 + \alpha_7 + \beta_7(x) \right] p_7(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \lambda_7 p_1(x,t) \] ..(4.8)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \lambda_8 + \alpha_8 + \beta_8(x) + \beta_9(x) \right] p_8(x,t) = \beta_4(x)p_4(x,t) + \beta_5(x)p_5(x,t) + \beta_6(x)p_{10}(x,t) + \alpha_8 p_1(x,t) + \alpha_1 p_1(x,t) \] ..(4.9)

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \beta_2(x) \right] p_0(x,t) = \alpha_2 p_0(t) \] ..(4.10)
\[
\begin{align*}
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{10}(x,t) &= \alpha_2 p_1(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_6(x) \right] p_{14}(x,t) &= \lambda_4 p_4(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{15}(x,t) &= \alpha_2 p_4(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_6(x) \right] p_{16}(x,t) &= \lambda_4 p_6(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{17}(x,t) &= \alpha_2 p_6(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_6(x) \right] p_{18}(x,t) &= \lambda_3 p_7(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{19}(x,t) &= \alpha_2 p_7(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_6(x) \right] p_{20}(x,t) &= \lambda_3 p_8(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{21}(x,t) &= \alpha_2 p_8(x,t) \\
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_6(x) \right] p_{22}(x,t) &= \lambda_3 p_8(x,t)
\end{align*}
\]
\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_1(x) \right] p_{22}(x,t) = \lambda_4 p_8(x,t) \]  
\text{..(4.23)}

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{23}(x,t) = \alpha_2 p_5(x,t) \]  
\text{..(4.24)}

Boundary conditions:
\[
p_1(0,t) = \alpha_1 p_0(t) \quad p_2(0,t) = \alpha_4 p_0(t) \quad p_3(0,t) = \lambda_1 p_1(t) \]
\[
p_4(0,t) = \lambda_2 p_2(t) \quad p_5(0,t) = \alpha_4 p_2(t) + \alpha_5 p_1(t) \quad p_6(0,t) = \alpha_4 p_4(t) + \lambda_2 p_5(t) \]
\[
p_7(0,t) = \alpha_5 p_3(t) + \lambda_2 p_5(t) \quad p_8(0,t) = \lambda_4 p_6(t) + \lambda_2 p_7(t) \quad p_9(0,t) = \alpha_2 p_0(t) \]
\[
p_{10}(0,t) = \alpha_2 p_1(t) \quad p_{11}(0,t) = \alpha_2 p_2(t) \quad p_{12}(0,t) = \lambda_2 p_3(t) \]
\[
p_{13}(0,t) = \alpha_2 p_3(t) \quad p_{14}(0,t) = \lambda_4 p_4(t) \quad p_{15}(0,t) = \alpha_2 p_4(t) \]
\[
p_{16}(0,t) = \lambda_4 p_6(t) \quad p_{17}(0,t) = \alpha_2 p_6(t) \quad p_{18}(0,t) = \alpha_2 p_7(t) \]
\[
p_{19}(0,t) = \lambda_5 p_7(t) \quad p_{20}(0,t) = \alpha_2 p_8(t) \quad p_{21}(0,t) = \lambda_5 p_8(t) \]
\[
p_{22}(0,t) = \lambda_4 p_8(t) \quad p_{23}(0,t) = \alpha_2 p_3(t) \]  
\text{..(4.25)}

Initial conditions:
\[
p_0(0) = 1 \quad p_i(0) = 0, \ i = 1 \ to \ 23 \]  
\text{..(4.26)}

The system of differential equations (4.1) – (4.24) together with the boundary conditions (4.25) and initial conditions (4.26) is called Chapman- Kolmogorov differential difference equation. In order to find the reliability of the system, the governing equations (4.1)-(4.24) along with the boundary conditions (4.26) have been solved to get probabilities \( p_i(t) \) \( i= 0 \ to \ 4 \)

Taking Laplace transforms of equation (4.1)-(4.24) and using initial conditions, we obtain
\[
[s + \alpha_1 + \alpha_2 + \alpha_3] p_0(t) - 1 = \int_0^\infty \beta_1(x)p_1(x,s)dx + \int_0^\infty \beta_2(x)p_0(x,s)dx + \int_0^\infty \beta_3(x)p_2(x,s)dx \]  
\text{..(4.27)}
\[
\frac{d}{dx} + s + \lambda_1 + \alpha_1 + \alpha_2 + \beta(x) \right] p_1(x, s) = \beta(x)p_1(x, s) + \beta_1(x)p_2(x, s) + \beta_2(x)p_3(x, s) + \alpha p_0(s) \\
\text{..(4.28)}
\]

\[
\frac{d}{dx} + s + \lambda_2 + \alpha_1 + \alpha_2 + \beta_1(x) \right] p_2(x, s) = \beta_1(x)p_2(x, s) + \beta_1(x)p_3(x, s) + \beta_2(x)p_4(x, s) + \alpha p_0(s) \\
\text{..(4.29)}
\]

\[
\frac{d}{dx} + s + \alpha_1 + \lambda_2 + \alpha_2 + \beta_1(x) \right] p_3(x, s) = \beta(x)p_3(x, s) + \beta_1(x)p_4(x, s) + \beta_2(x)p_5(x, s) + \alpha p_1(s) \\
\text{..(4.30)}
\]

\[
\frac{d}{dx} + s + \alpha_1 + \lambda_2 + \alpha_2 + \beta_1(x) \right] p_4(x, s) = \beta(x)p_4(x, s) + \beta_1(x)p_5(x, s) + \beta_2(x)p_6(x, s) + \alpha p_1(s) \\
\text{..(4.31)}
\]

\[
\frac{d}{dx} + s + \alpha_1 + \lambda + \lambda_2 + \beta(x) + \beta_1(x) \right] p_6(x, s) = \beta(x)p_6(x, s) + \beta_1(x)p_7(x, s) + \beta_2(x)p_8(x, s) + \alpha p_4(s) \\
\text{..(4.32)}
\]

\[
\frac{d}{dx} + s + \alpha_1 + \lambda_2 + \beta(x) + \beta_1(x) \right] p_7(x, s) = \beta(x)p_7(x, s) + \beta_1(x)p_8(x, s) + \beta_2(x)p_9(x, s) + \alpha p_5(s) \\
\text{..(4.33)}
\]

\[
\frac{d}{dx} + s + \alpha_1 + \lambda_2 + \beta(x) + \beta_1(x) \right] p_8(x, s) = \beta(x)p_8(x, s) + \beta_1(x)p_9(x, s) + \beta_2(x)p_10(x, s) + \alpha p_6(s) \\
\text{..(4.34)}
\]

\[
\frac{d}{dx} + s + \alpha_1 + \lambda_2 + \beta(x) + \beta_1(x) \right] p_9(x, s) = \beta(x)p_9(x, s) + \beta_1(x)p_{10}(x, s) + \beta_2(x)p_{11}(x, s) + \alpha p_7(s) \\
\text{..(4.35)}
\]

\[
\frac{d}{dx} + s + \beta_2(x) \right] p_2(x, s) = \alpha_2 p_0(s) \\
\text{..(4.36)}
\]

\[
\frac{d}{dx} + s + \beta_2(x) \right] p_{10}(x, s) = \alpha_2 p_1(x, s) \\
\text{..(4.37)}
\]

\[
\frac{d}{dx} + s + \beta_2(x) \right] p_{11}(x, s) = \alpha_2 p_2(x, s) \\
\text{..(4.38)}
\]

\[
\frac{d}{dx} + s + \beta_6(x) \right] p_{12}(x, s) = \lambda_3 p_3(x, s) \\
\text{..(4.39)}
\]

\[
\frac{d}{dx} + s + \beta_2(x) \right] p_{13}(x, s) = \alpha_2 p_3(x, s) \\
\text{..(4.40)}
\]

\[
\frac{d}{dx} + s + \beta_7(x) \right] p_{14}(x, s) = \lambda_4 p_4(x, s) \\
\text{..(4.41)}
\]

\[
\frac{d}{dx} + s + \beta_2(x) \right] p_{15}(x, s) = \alpha_2 p_4(x, s) \\
\text{..(4.42)}
\]
\[
\frac{d}{dx} + s + \beta_7(x) \quad p_{16}(x, s) = \lambda_4 p_6(x, s)
\]
\[(4.43)\]
\[
\frac{d}{dx} + s + \beta_2(x) \quad p_{17}(x, s) = \alpha_2 p_6(x, s)
\]
\[(4.44)\]
\[
\frac{d}{dx} + s + \beta_2(x) \quad p_{18}(x, s) = \alpha_2 p_7(x, s)
\]
\[(4.45)\]
\[
\frac{d}{dx} + s + \beta_6(x) \quad p_{19}(x, s) = \lambda_3 p_7(x, s)
\]
\[(4.46)\]
\[
\frac{d}{dx} + s + \beta_2(x) \quad p_{20}(x, s) = \alpha_2 p_8(x, s)
\]
\[(4.47)\]
\[
\frac{d}{dx} + s + \beta_6(x) \quad p_{21}(x, s) = \lambda_3 p_8(x, s)
\]
\[(4.48)\]
\[
\frac{d}{dx} + s + \beta_6(x) \quad p_{22}(x, s) = \alpha_2 p_9(x, s)
\]
\[(4.49)\]
\[
\frac{d}{dx} + s + \beta_2(x) \quad p_{23}(x, s) = \alpha_2 p_5(x, s)
\]
\[(4.50)\]

Laplace transformations of boundary conditions:

\[
p_1(0, s) = \alpha_1 p_0(s) \quad p_2(0, s) = \alpha_3 p_3(s) \quad p_3(0, s) = \lambda_4 p_1(s)
\]
\[
p_4(0, s) = \lambda_2 p_2(s) \quad p_5(0, s) = \alpha_2 p_2(s) + \alpha_3 p_1(s)
\]
\[
p_6(0, s) = \alpha_4 p_4(s) + \lambda_2 p_5(s) \quad p_7(0, s) = \alpha_3 p_3(s) + \lambda_4 p_5(s) \quad p_8(0, s) = \lambda_4 p_6(s) + \lambda_2 p_7(s)
\]
\[
p_9(0, s) = \alpha_2 p_0(s) \quad p_{10}(0, s) = \alpha_2 p_1(s) \quad p_{11}(0, s) = \lambda_4 p_2(s)
\]
\[
p_{12}(0, s) = \lambda_2 p_3(s) \quad p_{13}(0, s) = \alpha_2 p_3(s) \quad p_{14}(0, s) = \lambda_4 p_4(s)
\]
\[
p_{16}(0, s) = \lambda_4 p_6(s) \quad p_{17}(0, s) = \alpha_2 p_6(s) \quad p_{18}(0, s) = \alpha_2 p_7(s)
\]
\[
p_{19}(0, s) = \lambda_3 p_7(s) \quad p_{20}(0, s) = \alpha_2 p_8(s) \quad p_{21}(0, s) = \lambda_3 p_8(s)
\]
\[
p_{22}(0, s) = \lambda_4 p_8(s) \quad p_{23}(0, s) = \alpha_2 p_5(s)
\]
\[(4.51)\]

The equations (4.27)-(4.50) have been solved by using boundary conditions to get the transition state probabilities \( p_i(s), \ (i = 0 \text{ to } 23) \) of clarifier system in the sugar industry as
\[ p_{23}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \alpha_2 p_5(s) + \int_0^\infty \alpha_2 p_5(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.52)

\[ p_{22}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \lambda_4 p_8(s) + \int_0^\infty \lambda_4 p_8(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.53)

\[ p_{21}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \lambda_3 p_7(s) + \int_0^\infty \lambda_3 p_7(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.54)

\[ p_{20}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \alpha_2 p_8(s) + \int_0^\infty \alpha_2 p_8(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.55)

\[ p_{19}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \lambda_3 p_7(s) + \int_0^\infty \lambda_3 p_7(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.56)

\[ p_{18}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \alpha_2 p_7(s) + \int_0^\infty \alpha_2 p_7(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.57)

\[ p_{17}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \alpha_2 p_6(s) + \int_0^\infty \alpha_2 p_6(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.58)

\[ p_{16}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \lambda_4 p_6(s) + \int_0^\infty \lambda_4 p_6(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.59)

\[ p_{15}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \alpha_2 p_4(s) + \int_0^\infty \alpha_2 p_4(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.60)

\[ p_{14}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \lambda_4 p_4(s) + \int_0^\infty \lambda_4 p_4(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.61)

\[ p_{13}(x,s) = e^{-\int_0^x (s+\beta_s(x))dx} \left\{ \alpha_2 p_3(s) + \int_0^\infty \alpha_2 p_3(x,s) e^{\int_0^x (s+\beta_s(x))dx} \right\} \] ..(4.62)
\[ p_{12}(x,s) = e^{-\int_0^s (x+x_0) \, dx} \left\{ \lambda_1 p_1(s) + \int_0^\infty \lambda_1 p_1(x,s) e^x \, dx \right\} \]  

(4.63)

\[ p_{11}(x,s) = e^{-\int_0^s (x+x_0) \, dx} \left\{ \alpha_2 p_2(s) + \int_0^\infty \alpha_2 p_2(x,s) e^x \, dx \right\} \]  

(4.64)

\[ p_{10}(x,s) = e^{-\int_0^s (x+x_0) \, dx} \left\{ \alpha_2 p_1(s) + \int_0^\infty \alpha_2 p_1(x,s) e^x \, dx \right\} \]  

(4.65)

\[ p_9(x,s) = e^{-\int_0^s (x+x_0) \, dx} \left\{ \alpha_2 p_0(s) + \int_0^\infty \alpha_2 p_0(x,s) e^x \, dx \right\} \]  

(4.66)

\[ p_8(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1) \, dx} \left\{ \lambda_4 p_6(s) + \lambda_5 p_7(s) + \int_0^\infty \lambda_4 p_6(x,s) e^{(x+x_0+\alpha_2 \lambda_1)} \, dx \right\} \]  

(4.67)

\[ p_7(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1) \, dx} \left\{ \lambda_4 p_3(s) + \lambda_5 p_4(s) + \int_0^\infty \lambda_4 p_3(x,s) e^{(x+x_0+\alpha_2 \lambda_1)} \, dx \right\} \]  

(4.68)

\[ p_6(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2) \, dx} \left\{ \alpha_4 p_4(s) + \lambda_5 p_5(s) + \int_0^\infty \alpha_4 p_4(x,s) e^{(x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2)} \, dx \right\} \]  

(4.69)

\[ p_5(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2) \, dx} \left\{ \alpha_4 p_2(s) + \lambda_5 p_3(s) + \int_0^\infty \alpha_4 p_2(x,s) e^{(x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2)} \, dx \right\} \]  

(4.70)

\[ p_4(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2) \, dx} \left\{ \lambda_4 p_2(s) + \int_0^\infty \lambda_4 p_2(x,s) e^{(x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2)} \, dx \right\} \]  

(4.71)

\[ p_3(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2) \, dx} \left\{ \lambda_4 p_1(s) + \int_0^\infty \lambda_4 p_1(x,s) e^{(x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2)} \, dx \right\} \]  

(4.72)

\[ p_2(x,s) = e^{-\int_0^s (x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2) \, dx} \left\{ \alpha_2 p_0(s) + \int_0^\infty \alpha_2 p_0(x,s) e^{(x+x_0+\alpha_2 \lambda_1+\alpha_2 \lambda_2)} \, dx \right\} \]  

(4.73)
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\[
p_1(x,s) = e^{-\int_{0}^{\infty} (\alpha_1 + \lambda_1 + \beta_1(x)) dx} \left\{ \alpha_1 p_0(s) + \int_{0}^{\infty} \beta_1(x) p_1(x,s) dx \right\} \quad ..(4.74)
\]

\[
p_0(s) = \frac{1}{s + \alpha_1 + \alpha_2 + \alpha_3} \left\{ 1 + \int_{0}^{\infty} \beta_1(x) p_1(x,s) dx + \int_{0}^{\infty} \beta_2(x) p_0(x,s) dx + \int_{0}^{\infty} \beta_3(x) p_2(x,s) dx \right\} \quad ..(4.75)
\]

where

\[
y_1 = \beta_1(x) p_{22}(x,s) + \beta_0(x) p_{21}(x,s) + \beta_2(x) p_{20}(x,s) + \lambda_1 p_0(x,s) + \lambda_2 p_1(x,s)
\]

\[
y_2 = \beta_2(x) p_{18}(x,s) + \beta_0(x) p_{19}(x,s) + \beta_3(x) p_8(x,s) + \alpha_3 p_3(x,s) + \lambda_4 p_5(x,s)
\]

\[
y_3 = \beta_1(x) p_{66}(x,s) + \beta_2(x) p_{17}(x,s) + \alpha_4 p_4(x,s) + \lambda_5 p_5(x,s) + \beta_4(x) p_8(x,s)
\]

\[
y_4 = \beta_4(x) p_{77}(x,s) + \beta_5(x) p_{66}(x,s) + \beta_2(x) p_{23}(x,s) + \alpha_5 p_2(x,s) + \alpha_5 p_1(x,s)
\]

\[
y_5 = \beta_1(x) p_6(x,s) + \beta_1(x) p_{14}(x,s) + \beta_2(x) p_{15}(x,s) + \lambda_2 p_2(x,s)
\]

\[
y_6 = \beta_3(x) p_7(x,s) + \beta_6(x) p_{12}(x,s) + \beta_2(x) p_{13}(x,s) + \lambda_4 p_1(x,s)
\]

\[
y_7 = \beta_3(x) p_4(x,s) + \beta_5(x) p_3(x,s) + \beta_2(x) p_{11}(x,s) + \alpha_3 p_0(x,s)
\]

\[
y_8 = \beta_4(x) p_3(x,s) + \beta_3(x) p_5(x,s) + \beta_2(x) p_{10}(x,s) + \alpha_4 p_0(x,s)
\]

Reliability of the system in terms of Laplace transform is given as

\[
R(s) = p_0(s) + \int_{0}^{\infty} \sum_{i=1}^{8} p_i(x,s) dx \quad ..(4.76)
\]

Thus, the time dependent Reliability \( R(t) \) of the system model is computed by taking the inverse Laplace transformation of equation (4.76) as

\[
R(t) = p_0(t) + \int_{0}^{\infty} \sum_{i=1}^{8} p_i(x,t) dx
\]

4.6 Particular case

To show the importance of results and characterize the behavior of availability and profit of the manure system, here we assume the repair times are exponentially distributed. For this particular case system of equations (4.1)-(4.24) reduces as follows:
\[
\frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 \quad p_0 (t) = \beta_1 p_1 (t) + \beta_2 p_9 (t) + \beta_3 p_2 (t) \tag{4.77}
\]
\[
\frac{d}{dt} + \lambda_1 + \alpha_1 + \alpha_2 + \beta_1 \quad p_1 (t) = \beta_4 p_4 (t) + \beta_5 p_5 (t) + \beta_2 p_{10} (t) + \alpha_1 p_0 (t) \tag{4.78}
\]
\[
\frac{d}{dt} + \lambda_2 + \alpha_1 + \alpha_2 + \beta_3 \quad p_2 (t) = \beta_3 p_4 (t) + \beta_5 p_5 (t) + \beta_2 p_{11} (t) + \alpha_3 p_0 (t) \tag{4.79}
\]
\[
\frac{d}{dt} + \alpha_5 + \lambda_3 + \alpha_2 + \beta_4 \quad p_3 (t) = \beta_3 p_7 (t) + \beta_6 p_{12} (t) + \beta_2 p_{13} (t) + \lambda_4 p_1 (t) \tag{4.80}
\]
\[
\frac{d}{dt} + \alpha_4 + \lambda_4 + \alpha_2 + \beta_3 \quad p_4 (t) = \beta_5 p_9 (t) + \beta_7 p_{14} (t) + \beta_2 p_{15} (t) + \lambda_2 p_2 (t) \tag{4.81}
\]
\[
\frac{d}{dt} + \alpha_2 + \lambda_2 + \beta_1 + \beta_3 \quad p_5 (t) = \beta_5 p_7 (t) + \beta_8 p_8 (t) + \beta_2 p_{23} (t) + \alpha_1 p_0 (t) + \alpha_3 p_1 (t) \tag{4.82}
\]
\[
\frac{d}{dt} + \alpha_2 + \lambda_4 + \beta_1 + \beta_3 \quad p_6 (t) = \beta_5 p_{16} (t) + \beta_8 p_{17} (t) + \alpha_4 p_4 (t) + \lambda_2 p_4 (t) + \beta_4 p_6 (t) \tag{4.83}
\]
\[
\frac{d}{dt} + \alpha_2 + \lambda_2 + \beta_3 + \beta_4 \quad p_7 (t) = \beta_6 p_{18} (t) + \beta_8 p_{19} (t) + \beta_2 p_9 (t) + \alpha_3 p_3 (t) + \lambda_4 p_3 (t) \tag{4.84}
\]
\[
\frac{d}{dt} + \alpha_4 + \lambda_3 + \beta_4 + \beta_5 \quad p_8 (t) = \beta_7 p_{22} (t) + \beta_8 p_{21} (t) + \beta_2 p_{20} (t) + \lambda_2 p_6 (t) + \lambda_2 p_4 (t) \tag{4.85}
\]
\[
\frac{d}{dt} + \beta_2 \quad p_9 (t) = \alpha_2 p_0 (t) \tag{4.86}
\]
\[
\frac{d}{dt} + \beta_2 \quad p_{10} (t) = \alpha_2 p_1 (t) \tag{4.87}
\]
\[
\frac{d}{dt} + \beta_2 \quad p_{11} (t) = \alpha_2 p_2 (t) \tag{4.88}
\]
\[
\frac{d}{dt} + \beta_6 \quad p_{12} (t) = \lambda_4 p_3 (t) \tag{4.89}
\]
\[
\frac{d}{dt} + \beta_2 \quad p_{13} (t) = \alpha_2 p_3 (t) \tag{4.90}
\]
\[
\frac{d}{dt} + \beta_7 \quad p_{14} (t) = \lambda_4 p_4 (t) \tag{4.91}
\]
\[
\frac{d}{dt} + \beta_2 \quad p_{15} (t) = \alpha_2 p_4 (t) \tag{4.92}
\]
\[
\begin{align*}
\left[ \frac{d}{dt} + \beta_2 \right] p_{16}(t) &= \lambda_4 p_6(t) \\
\left[ \frac{d}{dt} + \beta_2 \right] p_{17}(t) &= \alpha_2 p_6(t) \\
\left[ \frac{d}{dt} + \beta_2 \right] p_{18}(t) &= \alpha_2 p_7(t) \\
\left[ \frac{d}{dt} + \beta_6 \right] p_{19}(t) &= \lambda_3 p_7(t) \\
\left[ \frac{d}{dt} + \beta_2 \right] p_{20}(t) &= \alpha_2 p_8(t) \\
\left[ \frac{d}{dt} + \beta_6 \right] p_{21}(t) &= \lambda_3 p_8(t) \\
\left[ \frac{d}{dt} + \beta_2 \right] p_{22}(t) &= \lambda_4 p_8(t) \\
\left[ \frac{d}{dt} + \beta_2 \right] p_{23}(t) &= \alpha_2 p_5(t)
\end{align*}
\]

..(4.93)  

..(4.94)  

..(4.95)  

..(4.96)  

..(4.97)  

..(4.98)  

..(4.99)  

..(4.100)

In process industry, systems are required to run for long time, the long run or steady-state availability is defined as the proportion of the time during which an equipment is available for use (Balagurusamy (1984)). So the long-run availability of the system is calculated by taking \( \frac{d}{dt} = 0 \) as \( t \rightarrow \infty \), \( p_i(t) = p_i \) in each equation (4.77) to (4.100); we have steady-state probabilities as follows

\[
\begin{align*}
p_8 &= \frac{x_{47}}{\beta_4 + \beta_5} p_0 \\
p_7 &= \frac{x_{46}}{x_3} p_0 \\
p_6 &= \frac{x_{45}}{x_2} p_0 \\
p_5 &= \frac{x_{44}}{x_0} p_0 \\
p_4 &= \frac{x_{43}}{x_{28}} p_0 \\
p_3 &= \frac{x_{42}}{x_{24}} p_0 \\
p_2 &= \frac{x_{41}}{x_{39}} p_0 \\
p_1 &= \frac{x_{40}}{x_5} p_0
\end{align*}
\]

Using normalizing condition \( \sum p_i = 1 \) we have

\[
p_0 = \left[ \left( 1 + \frac{\alpha_2}{\beta_2} \right) \left( 1 + \frac{x_{41}}{x_5} + \frac{x_{40}}{x_{39}} + \frac{x_{44}}{x_0} \right) + \left( 1 + \frac{\alpha_2}{\beta_2} + \frac{\lambda_3}{\beta_6} \right) \frac{x_{42}}{x_{24}} + \frac{x_{46}}{x_3} \right]^{-1} \\
+ \left[ \left( 1 + \frac{\alpha_2}{\beta_2} + \frac{\lambda_4}{\beta_7} \right) \frac{x_{43}}{x_{28}} + \frac{x_{45}}{x_2} \right] + \left[ 1 + \frac{\alpha_2}{\beta_2} + \frac{\lambda_4}{\beta_6} + \frac{\lambda_4}{\beta_7} \right] \frac{x_{47}}{\beta_4 + \beta_5}
\]
where

\[
\begin{align*}
    x_1 &= \lambda_2 + \beta_4 + \beta_3 - \frac{\lambda_2 \beta_5}{\beta_4 + \beta_5} \\
    x_3 &= x_1 - \frac{\lambda_1 \lambda_2 \beta_4 \beta_5}{x_2 (\beta_4 + \beta_5)} \\
    x_5 &= \lambda_3 + \frac{\lambda \lambda_2 \beta_5}{x_2 (\beta_4 + \beta_5)} \\
    x_7 &= \lambda_3 + \frac{x_5 \lambda_2 \beta_4}{x_3 (\beta_4 + \beta_5)} \\
    x_9 &= \lambda_4 + \lambda_3 + \beta_1 + \beta_3 - \frac{x_4 \beta_4}{x_5} - \frac{x_7 \beta_5}{x_2} \\
    x_{11} &= \frac{\alpha \lambda_4 \beta_4}{x_3} + \frac{x_5 \beta_3}{x_2} \\
    x_{13} &= x_8 + \frac{x_{11} \beta_1}{x_9} \\
    x_{15} &= \frac{x_2 \lambda_4}{x_9} \\
    x_{17} &= \frac{x_1 \lambda_1 \beta_4}{x_9} \\
    x_{18} &= \frac{x_4 \alpha_4}{x_9} \\
    x_{19} &= \frac{x_4 \alpha_4}{x_9} \\
    x_{20} &= \alpha_4 + \beta_5 - \frac{x_5 \beta_1}{x_2} \\
    x_{22} &= \alpha_4 + \beta_5 - \frac{x_5 \beta_1}{x_2} \\
    x_{24} &= \alpha_5 + \beta_4 - \frac{x_5 \beta_1}{x_3} \\
    x_{26} &= \frac{x_6 \beta_3}{x_3} \\
    x_{28} &= x_{20} - \frac{x_21 \beta_3}{x_3} \\
    x_{30} &= x_{23} + \frac{x_21 \beta_3}{x_3} \\
    x_{32} &= x_{27} + \frac{x_25 \beta_3}{x_28} \\
    x_{34} &= \alpha_3 + \frac{x_10 \beta_3}{x_28} + \frac{x_{11} \beta_3}{x_24} \\
    x_{36} &= \alpha_3 + \beta_5 - \frac{x_2 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{40} &= \alpha_3 + \beta_5 - \frac{x_3 \beta_4}{x_3} - \frac{x_5 \beta_4}{x_9} \\
    x_{42} &= \alpha_3 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{44} &= \alpha_4 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{46} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{48} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{50} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{52} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{54} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{56} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{58} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{60} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{62} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{64} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{66} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{68} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{70} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{72} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{74} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{76} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{78} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{80} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{82} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{84} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{86} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{88} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{90} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{92} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{94} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{96} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{98} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{100} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{102} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{104} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{106} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{108} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \\
    x_{110} &= \alpha_5 + \beta_5 - \frac{x_4 \beta_4}{x_4} - \frac{x_5 \beta_4}{x_9} \quad (\text{as an example of the pattern})
\end{align*}
\]
The long-run availability of the system ($A_v$) is given as

\[ A_v = p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 \]

\[
A_v = \left[ 1 + \frac{x_{41}}{x_5} + \frac{x_{40}}{x_{39}} + \frac{x_{42}}{x_{24}} + \frac{x_{43}}{x_{28}} + \frac{x_{44}}{x_9} + \frac{x_{45}}{x_2} + \frac{x_{46}}{x_3} + \frac{x_{47}}{\beta_4 + \beta_5} \right] \\
\left[ 1 + \frac{\alpha_2}{\beta_2} \right] \left[ 1 + \frac{x_{41}}{x_5} + \frac{x_{40}}{x_{39}} + \frac{x_{42}}{x_{24}} + \frac{x_{43}}{x_{28}} + \frac{x_{44}}{x_9} + \frac{x_{45}}{x_2} + \frac{x_{46}}{x_3} + \frac{x_{47}}{\beta_4 + \beta_5} \right] \\
\left[ 1 + \frac{\alpha_2 + \lambda_4}{\beta_2 + \beta_7} \right] \left[ 1 + \frac{x_{41}}{x_5} + \frac{x_{40}}{x_{39}} + \frac{x_{42}}{x_{24}} + \frac{x_{43}}{x_{28}} + \frac{x_{44}}{x_9} + \frac{x_{45}}{x_2} + \frac{x_{46}}{x_3} + \frac{x_{47}}{\beta_4 + \beta_5} \right] \cdots (4.101)
\[ \begin{aligned}
&K_1 \left[ \frac{1 + \frac{x_{41}}{x_4} + \frac{x_{40}}{x_{39}} + \frac{x_{42}}{x_{24}} + \frac{x_{43}}{x_{28}} + \frac{x_{44}}{x_9} + \frac{x_{45}}{x_2} + \frac{x_{46}}{x_3} + \frac{x_{47}}{\beta_4 + \beta_5}}{1 + \frac{\alpha_2}{\beta_2} \left( 1 + \frac{x_{41}}{x_5} + \frac{x_{40}}{x_{39}} + \frac{x_{44}}{x_9} \right)} + \frac{\alpha_2 \lambda_2}{\beta_2 \beta_7} \left( \frac{x_{43}}{x_{28}} + \frac{x_{45}}{x_2} \right) + \frac{\alpha_2 \lambda_3 \lambda_4}{\beta_2 \beta_6 \beta_7} \left( \frac{x_{47}}{\beta_4 + \beta_5} \right) \right] - K_2 \\
\end{aligned} \]

where

\[ A_v = \text{Steady state availability of the system.} \]

### 4.8 Numerical analysis

The effect of failure rates and repair rates on Availability and Profit of clarifier system in sugar industry is given in tables 4.1, 4.2, 4.3 and 4.4 as follows:
**Effect of failure rates of subsystems on availability of the clarifier system**

**Table 4.1**

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
<th>$\alpha_2=0.02, \alpha_3=0.01, \lambda_1=0.035, \lambda_3=0.04, \lambda_4=0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1=0.01, \beta_2=0.2, \beta_3=0.1, \beta_4=0.02, \beta_5=0.1, \beta_6=0.15, \beta_7=0.15$</td>
<td>0.878654</td>
<td>0.841676</td>
<td>0.876876</td>
<td>0.873344</td>
<td>0.877781</td>
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<td>0.82538</td>
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Effect of failure rates of subsystems on profit of the clarifier system

Table 4.2

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<td>3816.118 3637.537 3815.864 3777.496 3814.148 3769.309 3809.011</td>
</tr>
<tr>
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<td>3761.877 3587.666 3766.379 3717.057 3761.241 3703.017 3755.101</td>
</tr>
<tr>
<td>0.04</td>
<td>3721.66 3550.655 3728.962 3673.408 3721.823 3654.122 3715.125</td>
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<tr>
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Effect of repair rates of subsystems on availability of the clarifier system

Table 4.3

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</table>

| $\alpha_1=0.01$, $\alpha_2=0.02$, $\alpha_3=0.01$, $\lambda_1=0.035$, $\lambda_2=0.035$, $\lambda_3=0.04$, $\lambda_3=0.04$ |

| 0.1 | 0.878654 | 0.905164 | 0.879694 | 0.88466 | 0.881945 | 0.888088 | 0.881093 |
| 0.11 | 0.880303 | 0.906915 | 0.881498 | 0.885955 | 0.883594 | 0.889091 | 0.88276 |
| 0.12 | 0.881738 | 0.908438 | 0.883066 | 0.887074 | 0.885029 | 0.889962 | 0.884209 |
| 0.13 | 0.882998 | 0.909775 | 0.884442 | 0.88805 | 0.886288 | 0.890726 | 0.885482 |
| 0.14 | 0.884112 | 0.910959 | 0.885658 | 0.888908 | 0.887403 | 0.8914 | 0.886608 |
| 0.15 | 0.885105 | 0.912013 | 0.886742 | 0.88967 | 0.888396 | 0.892001 | 0.887611 |
| 0.16 | 0.885996 | 0.912959 | 0.887714 | 0.890349 | 0.889286 | 0.89254 | 0.888511 |
| 0.17 | 0.886799 | 0.913811 | 0.888589 | 0.89096 | 0.89009 | 0.893025 | 0.889322 |
| 0.18 | 0.887527 | 0.914584 | 0.889383 | 0.891511 | 0.890818 | 0.893464 | 0.890057 |
| 0.19 | 0.88819 | 0.915288 | 0.890105 | 0.892011 | 0.891481 | 0.893864 | 0.890727 |
Effect of repair rates of subsystems on profit of the clarifier system

Table 4.4

| $\beta_1$ | $\beta_2=0.3$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ | $\beta_2=0.2$, $\beta_3=0.1$, $\beta_4=0.02$, $\beta_5=0.1$, $\beta_6=0.15$, $\beta_7=0.15$ |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0.1 | 3893.268 | 4025.822 | 3898.469 | 3923.299 | 3909.726 | 3940.442 | 3905.465 |
| 0.11 | 3901.516 | 4034.576 | 3907.489 | 3929.777 | 3917.971 | 3945.455 | 3913.798 |
| 0.12 | 3908.691 | 4042.192 | 3915.329 | 3935.37 | 3925.143 | 3949.81 | 3921.046 |
| 0.13 | 3914.989 | 4048.877 | 3922.208 | 3940.248 | 3931.440 | 3953.628 | 3927.408 |
| 0.14 | 3920.561 | 4054.793 | 3928.292 | 3944.541 | 3937.013 | 3957.002 | 3933.038 |
| 0.15 | 3925.527 | 4060.065 | 3933.71 | 3948.348 | 3941.979 | 3960.006 | 3938.055 |
| 0.16 | 3929.98 | 4064.793 | 3938.568 | 3951.746 | 3946.432 | 3962.698 | 3942.553 |
| 0.17 | 3933.995 | 4069.056 | 3942.947 | 3954.798 | 3950.449 | 3965.123 | 3946.610 |
| 0.18 | 3937.635 | 4072.921 | 3946.914 | 3957.555 | 3954.089 | 3967.320 | 3950.287 |
| 0.19 | 3940.948 | 4076.44 | 3950.526 | 3960.057 | 3957.405 | 3969.319 | 3953.636 |
4.9 Conclusion

The effect of failure rates of subsystems on availability and profit of the clarifier system has been studied for arbitrary values of various parameters and cost in tables 4.1 and 4.2 respectively. From tables 4.1 and 4.2 it is observed that availability and profit of the clarifier system go on decreasing with the increase of failure rates of the subsystems A, B and C. However, the effect of failure rates of subsystems A and B is much more as compared to the failure rates of subsystem C. Therefore, there is need to control the failure rates of subsystems A and B in order to make the clarifier system of sugar industry more profitable.

Behavior of availability and profit of the clarifier system keep on moving up with the increase of repair rates of all the subsystems as shown in tables 4.3 and 4.4 respectively. The effect of repair rates of subsystems A and B on availability and profit is more as compared to subsystem C. Thus the study reveals that clarifier system of a sugar industry can play vital role in improving availability and profit of whole sugar industry provided that subsystems A and B have to be given more attention.