CHAPTER-II

Availability and Profit Analysis of a Feeding System in Sugar Industry

2.1 Introduction

It is fact that every product is the result of a process; such is the case with sugar in sugar manufacturing process. After the arrival of the sugarcane in the sugar industry from the fields or farms, it passes through a process. This process of sugar manufacturing has a chain of systems such as; Feeding system, Juice weighment system, Clarifier system, Manure system, Evaporation system and Crystallization system. The feeding system in the sugar industry is main functioning part of the sugar making process and failure of this system has great impact on the whole working of the remaining process of the sugar industry. Kumar et al. (1988) discussed availability of the feeding system in Sugar industry without considering the main functioning part of the feeding system like cane unloaders and turbines. As Turbine is the main subsystem of feeding system that plays an important role in generation of electricity in sugar industry. So, the study of feeding system along with unloaders and turbines will provide new dimensions to increase profit of the sugar industry. In the feeding system, the sugar cane (transported from fields) unloaded by unloaders and fed to cutters by a chain conveyor. Then chopped pieces are sent to crushers to squeeze the sugar cane for maximum possible extraction of juice. The squeezed sugar cane (bagasse) is used up in the plant boiler as a fuel and also used in paper plant as the raw material. Turbines take the steam from boiler and convert it into electricity which is then used in the industry itself and also excessive electricity is transported outside for some other use.

The industry personals provide us all the necessary information about the working of the feeding system in the sugar industry which consists of six subsystems - Unloaders, Cane carrier and cutter unit, Crushing unit, Boiler unit, Bagasse carrier unit and Turbines. Cane carrier and cutter unit, Crushing unit, Bagasse carrier unit and Boiler unit are working in series. In subsystem Unloaders one unloader is operative and another is kept as cold standby whereas, in subsystem Turbines both the turbines work in parallel. Keeping in view, in this chapter, the reliability model of the feeding system is developed. The availability and profit of the feeding system are analyzed by using supplementary variable technique. The distribution of failure times of subsystems of feeding system is taken as exponential while the
distribution of repair times of subsystems is considered as arbitrary. The performance analysis of availability and profit of the feeding system have been studied numerically through tables for a particular case.

2.2 System Description

In a feeding system (Fig. 2.1), the sugar cane (transported from field) unloaded by unloaders and fed to cutters by a chain conveyor. Then chopped pieces are sent to crushers to squeeze the sugar cane for maximum possible extraction of juice. The squeezed sugar cane (bagasse) is used up in the plant boiler as a fuel and also used in paper plant as a raw material. Turbines take the steam from boiler and convert it into electricity. As described above, feeding system consists of six subsystems and the detail descriptions of the subsystems are as follows:

1. **Subsystem A** (Unloaders)

   This subsystem consists of two unloaders; one is operative and another is kept as cold standby. Unloaders are used to unload the cane from storage. Complete failure of the system occurs only when all the unloaders fail.
2. **Subsystem B** (Cane carrier and cutter unit)
   This subsystem consists of cane carrier and cane cutter. They work in series. Failure of any one causes the complete failure of the system. Cane carrier takes cane from unloaders, after that cane cutter starts cutting of cane in fine small pieces.

3. **Subsystem C** (Crushing unit)
   In subsystem C, the small pieces of cane are crushed and raw juice is obtained. It works in series with subsystems B, D and E. The failure of subsystem C causes the complete failure of the system.

4. **Subsystem D** (Bagasse carrier unit)
   It works in series with subsystems B, C and E. Failure of this unit causes the complete failure of the system.

5. **Subsystem E** (Boiler unit)
   Subsystem E is working in series with subsystems B, C and D. The failure of this unit causes the complete failure of the system.

6. **Subsystem T** (Turbines)
   This subsystem consists of two non-identical turbines and both turbines work in parallel. The complete failure of the system occurred when both the turbines fail.

### 2.3 Assumptions

1. Repairmen always remain with the system.
2. Unit works as new after repair.
3. The distribution of failure times of subsystems of feeding system is taken as exponential while the distribution of repair times is arbitrary.
4. Each subsystem has a separate repair facility and there is no waiting time for repair in subsystems.

### 2.4 Notations

The following symbols are associated with the system

\[ A_1, A, B, C, D, E, T_1, T \]

: Working units with full capacity
Failed units

$\alpha_i \quad (1 \leq i \leq 7)$ : Failure rates of $A_1$, $A$, $B$, $C$, $D$, $E$, $T_1$ and $T$ units respectively

$\beta_i (x), \quad (1 \leq i \leq 7)$ : Repair rates of $A_1$, $A$, $B$, $C$, $D$, $E$, $T_1$ and $T$ units respectively

$p_0 (t)$ : The probability that the system is in full capacity

$p_i (x, t) \Delta, \quad (i= 1, \ldots, 34)$ : The probability that the system is in $i^{th}$ state at time $t$ has an elapsed repair time $x$.

$S_i \quad (i=0, 1, \ldots, 34)$ : States of the system.

### 2.5 Mathematical modeling of the system transient states

In this section, the following set of differential- difference equations associated with the model (Fig 2.2) can be obtained by using supplementary variable technique.

\[
\begin{align*}
\frac{p_0(t + \Delta t) - p_0(t)}{\Delta t} &= [-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5 - \alpha_6 - \alpha_7] p_0(t) + \int_{0}^{\infty} \beta_1(x) p_1(x, t) \, dx + \int_{0}^{\infty} \beta_2(x) p_2(x, t) \, dx \\
&\quad + \int_{0}^{\infty} \beta_3(x) p_3(x, t) \, dx + \int_{0}^{\infty} \beta_4(x) p_4(x, t) \, dx + \int_{0}^{\infty} \beta_5(x) p_5(x, t) \, dx + \int_{0}^{\infty} \beta_6(x) p_6(x, t) \, dx + \int_{0}^{\infty} \beta_7(x) p_7(x, t) \, dx
\end{align*}
\]

Dividing both sides by $\Delta t$, we get

\[
\begin{align*}
\frac{p_0(t + \Delta t) - p_0(t)}{\Delta t} &= [-\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5 - \alpha_6 - \alpha_7] p_0(t) + \int_{0}^{\infty} \beta_1(x) p_1(x, t) \, dx + \int_{0}^{\infty} \beta_2(x) p_2(x, t) \, dx \\
&\quad + \int_{0}^{\infty} \beta_3(x) p_3(x, t) \, dx + \int_{0}^{\infty} \beta_4(x) p_4(x, t) \, dx + \int_{0}^{\infty} \beta_5(x) p_5(x, t) \, dx + \int_{0}^{\infty} \beta_6(x) p_6(x, t) \, dx + \int_{0}^{\infty} \beta_7(x) p_7(x, t) \, dx
\end{align*}
\]

As $\Delta t \to 0$

\[
\begin{align*}
\left[ \frac{dp_0(t)}{dt} \right] + [\alpha + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7] p_0(t) &= \int_{0}^{\infty} \beta_1(x) p_1(x, t) \, dx + \int_{0}^{\infty} \beta_2(x) p_2(x, t) \, dx \\
&\quad + \int_{0}^{\infty} \beta_3(x) p_3(x, t) \, dx + \int_{0}^{\infty} \beta_4(x) p_4(x, t) \, dx + \int_{0}^{\infty} \beta_5(x) p_5(x, t) \, dx + \int_{0}^{\infty} \beta_6(x) p_6(x, t) \, dx + \int_{0}^{\infty} \beta_7(x) p_7(x, t) \, dx
\end{align*}
\]

\[
\begin{align*}
\left[ \frac{d}{dt} + u_0 \right] p_0(t) &= f_0 \quad ..(2.1)
\end{align*}
\]

Similarly,
\[
\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + u_i(x) \right] p_i(x,t) = f_i(x,t) \quad i = 1, 2, 3, 4, 5
\]
\[[2.2]\]
Feeding System

Fig. 2.2 State changeover diagram of a feeding system in sugar industry

Operative in full capacity
Operative in reduced capacity
failed state
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_6(x) + \beta_7(x) \left[ p_6(x,t) = \alpha_6 p_2(x,t) + \alpha_7 p_3(x,t) \right] \quad \text{.(2.3)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_8(x) + \beta_9(x) \left[ p_7(x,t) = \alpha_8 p_4(x,t) + \alpha_9 p_5(x,t) \right] \quad \text{.(2.4)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \left[ p_8(x,t) = \alpha_2 p_0(t) \right] \quad \text{.(2.5)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \left[ p_9(x,t) = \alpha_3 p_0(t) \right] \quad \text{.(2.6)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \left[ p_{10}(x,t) = \alpha_4 p_0(t) \right] \quad \text{.(2.7)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \left[ p_{11}(x,t) = \alpha_5 p_0(t) \right] \quad \text{.(2.8)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \left[ p_{12}(x,t) = \alpha_2 p_1(x,t) \right] \quad \text{.(2.9)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \left[ p_{13}(x,t) = \alpha_3 p_1(x,t) \right] \quad \text{.(2.10)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \left[ p_{14}(x,t) = \alpha_4 p_1(x,t) \right] \quad \text{.(2.11)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \left[ p_{15}(x,t) = \alpha_5 p_1(x,t) \right] \quad \text{.(2.12)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \left[ p_{16}(x,t) = \alpha_2 p_2(x,t) \right] \quad \text{.(2.13)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \left[ p_{17}(x,t) = \alpha_3 p_2(x,t) \right] \quad \text{.(2.14)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \left[ p_{18}(x,t) = \alpha_4 p_2(x,t) \right] \quad \text{.(2.15)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \left[ p_{19}(x,t) = \alpha_5 p_2(x,t) \right] \quad \text{.(2.16)}
\]
\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \left[ p_{20}(x,t) = \alpha_2 p_3(x,t) \right] \quad \text{.(2.17)}
\]
\[
\begin{align*}
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{21}(x,t) &= \alpha_2 p_1(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \right] p_{22}(x,t) &= \alpha_3 p_1(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \right] p_{23}(x,t) &= \alpha_4 p_1(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \right] p_{24}(x,t) &= \alpha_5 p_1(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x) \right] p_{25}(x,t) &= \alpha_2 p_2(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \right] p_{26}(x,t) &= \alpha_3 p_2(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \right] p_{27}(x,t) &= \alpha_4 p_2(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \right] p_{28}(x,t) &= \alpha_5 p_2(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \right] p_{29}(x,t) &= \alpha_3 p_3(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \right] p_{30}(x,t) &= \alpha_4 p_3(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \right] p_{31}(x,t) &= \alpha_5 p_3(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x) \right] p_{32}(x,t) &= \alpha_3 p_4(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_4(x) \right] p_{33}(x,t) &= \alpha_4 p_4(x,t) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_5(x) \right] p_{34}(x,t) &= \alpha_5 p_4(x,t)
\end{align*}
\]

where

\[ u_0 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7, \]
\[ u_1(x) = u_0 + \beta_1(x) \quad u_2(x) = u_0 - \alpha_7 + \beta_7(x) \quad u_3(x) = u_0 - \alpha_6 + \beta_6(x) \]
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\[ u_4(x) = u_0 - \alpha_1(x) + \beta_1(x) + \beta_2(x) \quad u_5(x) = u_0 - \alpha_6(x) + \beta_1(x) + \beta_6(x) \]

\[ f_0 = \int_0^\infty \beta_1(x)p_1(x,t)dx + \int_0^\infty \beta_2(x)p_2(x,t)dx + \int_0^\infty \beta_3(x)p_3(x,t)dx + \int_0^\infty \beta_4(x)p_4(x,t)dx + \int_0^\infty \beta_5(x)p_5(x,t)dx \]

\[ f_1(x,t) = \beta_1(x)p_{11}(x,t) + \beta_2(x)p_{12}(x,t) + \beta_3(x)p_{13}(x,t) + \beta_4(x)p_{14}(x,t) + \beta_5(x)p_{15}(x,t) + \beta_6(x)p_{16}(x,t) + \alpha_6 p_0(t) \]

\[ f_2(x,t) = \beta_1(x)p_{21}(x,t) + \beta_2(x)p_{22}(x,t) + \beta_3(x)p_{23}(x,t) + \beta_4(x)p_{24}(x,t) + \beta_5(x)p_{25}(x,t) + \beta_6(x)p_{26}(x,t) + \alpha_6 p_0(t) \]

\[ f_3(x,t) = \beta_1(x)p_{31}(x,t) + \beta_2(x)p_{32}(x,t) + \beta_3(x)p_{33}(x,t) + \beta_4(x)p_{34}(x,t) + \beta_5(x)p_{35}(x,t) + \beta_6(x)p_{36}(x,t) + \alpha_6 p_0(t) \]

\[ f_4(x,t) = \beta_1(x)p_{41}(x,t) + \beta_2(x)p_{42}(x,t) + \beta_3(x)p_{43}(x,t) + \beta_4(x)p_{44}(x,t) + \beta_5(x)p_{45}(x,t) + \beta_6(x)p_{46}(x,t) + \alpha_6 p_0(t) \]

\[ f_5(x,t) = \beta_1(x)p_{51}(x,t) + \beta_2(x)p_{52}(x,t) + \beta_3(x)p_{53}(x,t) + \beta_4(x)p_{54}(x,t) + \beta_5(x)p_{55}(x,t) + \beta_6(x)p_{56}(x,t) + \alpha_6 p_0(t) \]

Boundary conditions:

\[ p_1(0,t) = \alpha_1 p_0(t) \quad p_2(0,t) = \alpha_2 p_0(t) \quad p_3(0,t) = \alpha_3 p_0(t) \]

\[ p_4(0,t) = \alpha_4 p_1(t) + \alpha_5 p_2(t) \quad p_5(0,t) = \alpha_6 p_1(t) + \alpha_6 p_2(t) \quad p_6(0,t) = \alpha_6 p_2(t) + \alpha_6 p_3(t) \]

\[ p_7(0,t) = \alpha_6 p_4(t) + \alpha_6 p_5(t) \quad p_8(0,t) = \alpha_2 p_0(t) \quad p_9(0,t) = \alpha_3 p_0(t) \]

\[ p_{10}(0,t) = \alpha_4 p_0(t) \quad p_{11}(0,t) = \alpha_5 p_0(t) \quad p_{12}(0,t) = \alpha_2 p_1(t) \]

\[ p_{13}(0,t) = \alpha_3 p_1(t) \quad p_{14}(0,t) = \alpha_4 p_1(t) \quad p_{15}(0,t) = \alpha_1 p_1(t) \]

\[ p_{16}(0,t) = \alpha_5 p_1(t) \quad p_{17}(0,t) = \alpha_2 p_2(t) \quad p_{18}(0,t) = \alpha_3 p_2(t) \]

\[ p_{19}(0,t) = \alpha_4 p_2(t) \quad p_{20}(0,t) = \alpha_5 p_2(t) \quad p_{21}(0,t) = \alpha_2 p_3(t) \]

\[ p_{22}(0,t) = \alpha_3 p_3(t) \quad p_{23}(0,t) = \alpha_4 p_3(t) \quad p_{24}(0,t) = \alpha_3 p_3(t) \]

\[ p_{25}(0,t) = \alpha_2 p_5(t) \quad p_{26}(0,t) = \alpha_3 p_5(t) \quad p_{27}(0,t) = \alpha_4 p_5(t) \]

\[ p_{28}(0,t) = \alpha_5 p_3(t) \quad p_{29}(0,t) = \alpha_4 p_3(t) \quad p_{30}(0,t) = \alpha_2 p_4(t) \]

\[ p_{31}(0,t) = \alpha_5 p_4(t) \quad p_{32}(0,t) = \alpha_4 p_4(t) \quad p_{33}(0,t) = \alpha_5 p_4(t) \]

\[ p_{34}(0,t) = \alpha_4 p_4(t) \]

..(2.32)
Initial conditions:
\[ p_0(0) = 1 \]
\[ p_i(0) = 0, i = 1 \text{ to } 34 \] ..(2.33)

The system of differential equations (2.1) – (2.31) together with the boundary conditions (2.32) and initial conditions (2.33) is called Chapman- Kolmogorov differential difference equation. Equation (2.1) is linear differential equation of first order and other equations (2.2)-(2.31) are linear partial differential equations. In order to find the reliability of the system, the governing equations (2.1)-(2.31) along with the boundary conditions (2.32) have been solved to get probabilities \( p_i(t) \) (i= 0 to 34) by using Lagrange’s Method as

\[
p_{34}(x,t) = \varphi_{34}(t-x) + \alpha_4 p_4(t)e^{-\int \beta_4(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_4(x)dx} \right\} \] ..(2.34)

\[
p_{33}(x,t) = \varphi_{33}(t-x) + \alpha_3 p_4(t)e^{-\int \beta_3(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_3(x)dx} \right\} \] ..(2.35)

\[
p_{32}(x,t) = \varphi_{32}(t-x) + \alpha_4 p_4(t)e^{-\int \beta_2(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_2(x)dx} \right\} \] ..(2.36)

\[
p_{31}(x,t) = \varphi_{31}(t-x) + \alpha_3 p_4(t)e^{-\int \beta_1(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_1(x)dx} \right\} \] ..(2.37)

\[
p_{30}(x,t) = \varphi_{30}(t-x) + \alpha_2 p_4(t)e^{-\int \beta_0(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_0(x)dx} \right\} \] ..(2.38)

\[
p_{29}(x,t) = \varphi_{29}(t-x) + \alpha_5 p_5(t)e^{-\int \beta_5(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_5(x)dx} \right\} \] ..(2.39)

\[
p_{28}(x,t) = \varphi_{28}(t-x) + \alpha_5 p_5(t)e^{-\int \beta_5(x)dx} \left\{ 1 + \int_0^\infty e^{\int \beta_5(x)dx} \right\} \] ..(2.40)
\[ p_{27}(x, t) = \varphi_{27}(t-x) + \alpha_4 p_5(t)e^{-\int_0^x \beta_4(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_4(x)dx} \right\} \] ..(2.41)

\[ p_{26}(x, t) = \varphi_{26}(t-x) + \alpha_3 p_5(t)e^{-\int_0^x \beta_3(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_3(x)dx} \right\} \] ..(2.42)

\[ p_{25}(x, t) = \varphi_{25}(t-x) + \alpha_2 p_5(t)e^{-\int_0^x \beta_2(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_2(x)dx} \right\} \] ..(2.43)

\[ p_{24}(x, t) = \varphi_{24}(t-x) + \alpha_5 p_3(t)e^{-\int_0^x \beta_5(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_5(x)dx} \right\} \] ..(2.44)

\[ p_{23}(x, t) = \varphi_{23}(t-x) + \alpha_4 p_3(t)e^{-\int_0^x \beta_4(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_4(x)dx} \right\} \] ..(2.45)

\[ p_{22}(x, t) = \varphi_{22}(t-x) + \alpha_3 p_3(t)e^{-\int_0^x \beta_3(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_3(x)dx} \right\} \] ..(2.46)

\[ p_{21}(x, t) = \varphi_{21}(t-x) + \alpha_2 p_3(t)e^{-\int_0^x \beta_2(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_2(x)dx} \right\} \] ..(2.47)

\[ p_{20}(x, t) = \varphi_{20}(t-x) + \alpha_5 p_2(t)e^{-\int_0^x \beta_5(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_5(x)dx} \right\} \] ..(2.48)

\[ p_{19}(x, t) = \varphi_{19}(t-x) + \alpha_4 p_2(t)e^{-\int_0^x \beta_4(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_4(x)dx} \right\} \] ..(2.49)

\[ p_{18}(x, t) = \varphi_{18}(t-x) + \alpha_3 p_2(t)e^{-\int_0^x \beta_3(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_3(x)dx} \right\} \] ..(2.50)

\[ p_{17}(x, t) = \varphi_{17}(t-x) + \alpha_2 p_2(t)e^{-\int_0^x \beta_2(x)dx} \left\{ 1 + \int_0^\infty e^{\int_0^x \beta_2(x)dx} \right\} \] ..(2.51)
\[ p_{16}(x,t) = \varphi_{16}(t-x) + \alpha_5 p_1(t)e^{-\int_0^t \beta_5(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_5(\lambda)\,d\lambda \right\} \] ..(2.52)

\[ p_{15}(x,t) = \varphi_{15}(t-x) + \alpha_4 p_1(t)e^{-\int_0^t \beta_4(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_4(\lambda)\,d\lambda \right\} \] ..(2.53)

\[ p_{14}(x,t) = \varphi_{14}(t-x) + \alpha_4 p_1(t)e^{-\int_0^t \beta_4(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_4(\lambda)\,d\lambda \right\} \] ..(2.54)

\[ p_{13}(x,t) = \varphi_{13}(t-x) + \alpha_3 p_1(t)e^{-\int_0^t \beta_3(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_3(\lambda)\,d\lambda \right\} \] ..(2.55)

\[ p_{12}(x,t) = \varphi_{12}(t-x) + \alpha_2 p_1(t)e^{-\int_0^t \beta_2(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_2(\lambda)\,d\lambda \right\} \] ..(2.56)

\[ p_{11}(x,t) = \varphi_{11}(t-x) + \alpha_1 p_1(t)e^{-\int_0^t \beta_1(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_1(\lambda)\,d\lambda \right\} \] ..(2.57)

\[ p_{10}(x,t) = \varphi_{10}(t-x) + \alpha_0 p_1(t)e^{-\int_0^t \beta_0(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_0(\lambda)\,d\lambda \right\} \] ..(2.58)

\[ p_9(x,t) = \varphi_9(t-x) + \alpha_5 p_0(t)e^{-\int_0^t \beta_5(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_5(\lambda)\,d\lambda \right\} \] ..(2.59)

\[ p_8(x,t) = \varphi_8(t-x) + \alpha_4 p_0(t)e^{-\int_0^t \beta_4(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_4(\lambda)\,d\lambda \right\} \] ..(2.60)

\[ p_7(x,t) = \varphi_7(t-x) + (\alpha_5 p_1(t) + \alpha_4 p_0(t))e^{-\int_0^t \beta_5(x)\,dx} \left\{ 1 + \int_0^\infty e^\beta_5(\lambda)\,d\lambda \right\} \] ..(2.61)
\[ p_6(x,t) = \varphi_6(t-x) + \left( \alpha_6 p_2(t) + \alpha_7 p_3(t) \right) e^{\int_0^t (\beta_6(t-x) - \beta_7(t-x)) dt} \left( 1 + \int_0^\infty e^{\int_0^t (\beta_6(t-x) - \beta_7(x)) dt} dx \right) \] ..(2.62)

\[ p_5(x,t) = \varphi_5(t-x) + e^{\int_0^t \nu_s(t-x) dt} \left( \alpha_6 p_1(t) + \alpha_4 p_3(t) + \int_0^\infty f_5(x,t)e^{\int_0^t \nu_s(t-x) dt} dx \right) \] ..(2.63)

\[ p_4(x,t) = \varphi_4(t-x) + e^{\int_0^t \nu_s(t-x) dt} \left( \alpha_7 p_1(t) + \alpha_2 p_2(t) + \int_0^\infty f_4(x,t)e^{\int_0^t \nu_s(t-x) dt} dx \right) \] ..(2.64)

\[ p_3(x,t) = \varphi_3(t-x) + e^{\int_0^t \nu_s(t-x) dt} \left( \alpha_6 p_0(t) + \int_0^\infty f_3(x,t)e^{\int_0^t \nu_s(t-x) dt} dx \right) \] ..(2.65)

\[ p_2(x,t) = \varphi_2(t-x) + e^{\int_0^t \nu_s(t-x) dt} \left( \alpha_7 p_0(t) + \int_0^\infty f_2(x,t)e^{\int_0^t \nu_s(t-x) dt} dx \right) \] ..(2.66)

\[ p_1(x,t) = \varphi_1(t-x) + e^{\int_0^t \nu_s(t-x) dt} \left( \alpha_1 p_0(t) + \int_0^\infty f_1(x,t)e^{\int_0^t \nu_s(t-x) dt} dx \right) \] ..(2.67)

\[ p_0(t) = e^{\int_0^t \nu_s dt} \left( 1 + \int_0^\infty f_0 e^{\int_0^t \nu_s dt} dt \right) \] ..(2.68)

Thus, the time dependent Reliability \[ R(t) \] of the system model is computed as

\[ R(t) = p_0(t) + \int_0^\infty \sum_{i=1}^5 p_i(x,t) dx \]

### 2.6 Particular case

To show the importance of results and characterize the behavior of availability and profit of the feeding system, here we assume the repair times are exponentially distributed.

For this particular case system of equations (2.1)-(2.31) reduces as follows:

\[ \left[ \frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \right] p_i(t) = \beta_1 p_1(t) + \beta_2 p_2(t) + \beta_3 p_3(t) + \beta_4 p_4(t) + \beta_5 p_5(t) + \beta_6 p_6(t) + \beta_7 p_7(t) \]

..(2.69)
\[
\frac{d}{dt} \left[ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \beta_1 \right] p_1(t) = \beta_1 p_1(t) + \beta_2 p_2(t) + \beta_3 p_3(t) + \beta_4 p_4(t) + \beta_5 p_5(t) + \beta_6 p_6(t) + \alpha_1 p_1(t) \quad \ldots \text{(2.70)}
\]

\[
\frac{d}{dt} \left[ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \beta_2 \right] p_2(t) = \beta_2 p_2(t) + \beta_3 p_3(t) + \beta_4 p_4(t) + \beta_5 p_5(t) + \beta_6 p_6(t) + \alpha_2 p_2(t) \quad \ldots \text{(2.71)}
\]

\[
\frac{d}{dt} \left[ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \beta_3 \right] p_3(t) = \beta_3 p_3(t) + \beta_4 p_4(t) + \beta_5 p_5(t) + \beta_6 p_6(t) + \alpha_3 p_3(t) \quad \ldots \text{(2.72)}
\]

\[
\frac{d}{dt} \left[ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \beta_4 \right] p_4(t) = \beta_4 p_4(t) + \beta_5 p_5(t) + \beta_6 p_6(t) + \alpha_4 p_4(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t) + \alpha_3 p_3(t) \quad \ldots \text{(2.73)}
\]

\[
\frac{d}{dt} \left[ \beta_1 + \beta_2 \right] p_5(t) = \alpha_5 p_5(t) + \alpha_6 p_6(t) \quad \ldots \text{(2.74)}
\]

\[
\frac{d}{dt} \left[ \beta_3 + \beta_4 \right] p_6(t) = \alpha_6 p_6(t) + \alpha_2 p_2(t) \quad \ldots \text{(2.75)}
\]

\[
\frac{d}{dt} \left[ \beta_5 + \beta_6 \right] p_8(t) = \alpha_2 p_2(t) + \alpha_3 p_3(t) \quad \ldots \text{(2.76)}
\]

\[
\frac{d}{dt} \beta_1 p_i(t) = \alpha_1 p_i(t) \quad i = 8, 9, 10, 11, \quad j = 2, 3, 4, 5 \quad \ldots \text{(2.77)}
\]

\[
\frac{d}{dt} \beta_2 p_i(t) = \alpha_2 p_i(t) \quad i = 15, 12, 13, 14, 16, \quad j = 1, 2, 3, 4, 5 \quad \ldots \text{(2.78)}
\]

\[
\frac{d}{dt} \beta_3 p_i(t) = \alpha_3 p_i(t) \quad i = 17, 18, 19, 20, \quad j = 2, 3, 4, 5 \quad \ldots \text{(2.79)}
\]

\[
\frac{d}{dt} \beta_4 p_i(t) = \alpha_4 p_i(t) \quad i = 21, 22, 23, 24, \quad j = 2, 3, 4, 5 \quad \ldots \text{(2.80)}
\]

\[
\frac{d}{dt} \beta_5 p_i(t) = \alpha_5 p_i(t) \quad i = 29, 25, 26, 27, 28, \quad j = 1, 2, 3, 4, 5 \quad \ldots \text{(2.81)}
\]

\[
\frac{d}{dt} \beta_6 p_i(t) = \alpha_6 p_i(t) \quad i = 34, 30, 31, 32, 33, \quad j = 1, 2, 3, 4, 5 \quad \ldots \text{(2.82)}
\]

Initial conditions

\[ p_0(0) = 1 \]

\[ p_i(0) = 0, \quad i = 1 \text{ to } 34 \]
In process industry, systems are required to run for long time, so the long-run availability of the system is calculated by taking \( \frac{d}{dt} = 0 \) as \( t \to \infty \). \( p_i(t) = p \) in each equation (2.69) to (2.82); we have steady-state probabilities as follows

\[
p_1 = p_0(\alpha + \beta_k z_4 + \beta_z z_3) / k_5 \quad p_2 = p_0(z_4 / z_2) \quad p_3 = p_0(k_{10} z_4 / z_2 + k_{11}) \quad p_4 = p_0 z_3 
\]

\[
p_5 = p_0 z_4
\]

where

\[
z_1 = k_{13} z_4 + k_{14} \quad z_2 = k_{12} - \frac{k_1 k_{10}}{k_5} \quad z_3 = k_1 z_4 + k_{12} - \frac{k_1 k_{10}}{k_5}
\]

\[
z_5 = \frac{k_5 x_5 + \alpha \beta}{k_6 x_5} \quad z_4 = \left[ k_7 z_4 + \frac{k_7 z_3 + k_9}{k_7} \right] + k_7 z_4 + \frac{k_9 z_4}{x_3}
\]

where

\[
k_1 = \alpha_i + \beta + \beta_n \frac{\alpha_i \beta_i}{\beta_i + \beta_n} \quad k_2 = \alpha_i + \beta + \beta_n \frac{\alpha_i \beta_i}{\beta_i + \beta_n}
\]

\[
k_3 = \alpha_i + \alpha_n + \beta - \frac{\alpha_i \beta_i}{\beta_n + \beta_i} \quad k_4 = \alpha_i + \alpha_n + \beta - \frac{\alpha_i \beta_i}{\beta_n + \beta_i}
\]

\[
k_5 = \alpha_i + \alpha_n + \beta
\]

\[
k_6 = \frac{\alpha_i (k_5 x_5 + \alpha i \beta_i)}{x_3 x_5}
\]

\[
k_7 = k_3 + \frac{\beta_n \alpha_i}{k_6}
\]

\[
k_11 = \alpha_i + \frac{\beta_k x_5}{k_6}
\]

\[
k_13 = x_5 + \frac{\beta_n \alpha_i (k_5 x_5 + \alpha i \beta_i)}{k_6 x_3}
\]

\[
x_1 = \frac{\alpha_i \beta_i}{\beta_i + \beta_n}
\]

\[
x_3 = k_5 x_5 - \alpha_i \beta_i
\]
Using normalizing condition $\sum p_i = 1$ we have

$$p_0 = \left[ \left( 1 + \sum_{i=2}^{5} \frac{\alpha_i}{\beta_i} \right) \left( 1 + \alpha_6 \frac{z_4 + \beta_7 z_3}{k_5} + \frac{z_4}{k_9} + \frac{1}{k_9} (k_{11} + k_{10} \frac{z_1}{z_2}) + z_3 \right) \right]^{-1}$$

$$+ \left( \frac{\alpha_3}{\beta_3 + \beta_4} \right) z_4 + \left( \frac{\alpha_3}{\beta_3 + \beta_4} \right) z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} \frac{1}{k_9}$$

The long-run availability of the system ($A_v$) is given as

$$A_v = p_0 + p_1 + p_2 + p_3 + p_4 + p_5$$

$$A_v = \frac{(\alpha + \beta_6 z_4 + \beta_7 z_3)}{k_5} + \frac{z_4}{k_9} + \frac{1}{k_9} (k_{11} + k_{10} \frac{z_1}{z_2}) + z_3 + z_4$$

$$(1 + \sum_{i=2}^{5} \frac{\alpha_i}{\beta_i}) \left[ 1 + \alpha_6 \frac{z_4 + \beta_7 z_3}{k_5} + \frac{z_4}{k_9} + \frac{1}{k_9} (k_{11} + k_{10} \frac{z_1}{z_2}) + z_3 + z_4 \right] + \frac{\alpha_3}{\beta_3 + \beta_4} z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} \frac{1}{k_9}$$

2.7 Profit Analysis

Let $K_1$ be the total revenue per unit up time of the system and $K_2$ be the total repair cost then profit incurred to the system model in steady state is obtained by using equation (2.83) as

$$\text{Profit} = K_1 A_v - K_2$$

$$= \frac{(\alpha + \beta_6 z_4 + \beta_7 z_3)}{k_5} + \frac{z_4}{k_9} + \frac{1}{k_9} (k_{11} + k_{10} \frac{z_1}{z_2}) + z_3 + z_4$$

$$(1 + \sum_{i=2}^{5} \frac{\alpha_i}{\beta_i}) \left[ 1 + \alpha_6 \frac{z_4 + \beta_7 z_3}{k_5} + \frac{z_4}{k_9} + \frac{1}{k_9} (k_{11} + k_{10} \frac{z_1}{z_2}) + z_3 + z_4 \right] + \frac{\alpha_3}{\beta_3 + \beta_4} z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} z_4 + \frac{\alpha_3}{\beta_3 + \beta_4} \frac{1}{k_9}$$
Chapter II

where

\[ A_v = \text{Steady state availability of the system.} \]

2.8 Numerical analysis

The effect of failure and repair rates on Availability and Profit of feeding system in sugar industry is given in tables 2.1, 2.2, 2.3 and 2.4 as follows:

Effect of failure rates of subsystems on availability of the feeding system

Table 2.1

| \( \alpha \) | \( \beta = 0.2 \), \( \beta = 0.21 \), \( \beta = 0.24 \), \( \beta = 0.25 \), \( \beta = 0.27 \), \( \beta = 0.3 \), \( \beta = 0.22 \) |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.633523 | 0.614971 | 0.61723 | 0.617866 | 0.618999 | 0.63197 | 0.632495 |
| 0.02 | 0.630841 | 0.612443 | 0.614684 | 0.615315 | 0.616438 | 0.629302 | 0.629822 |
| 0.03 | 0.6267 | 0.60854 | 0.610752 | 0.611374 | 0.612484 | 0.625181 | 0.625695 |
| 0.04 | 0.621339 | 0.603483 | 0.605659 | 0.606271 | 0.607362 | 0.619845 | 0.620351 |
| 0.05 | 0.614971 | 0.597474 | 0.599606 | 0.600206 | 0.601276 | 0.613507 | 0.614002 |
| 0.06 | 0.607783 | 0.590687 | 0.592771 | 0.593357 | 0.594402 | 0.606354 | 0.606837 |
| 0.07 | 0.599939 | 0.583276 | 0.585308 | 0.58588 | 0.586899 | 0.598547 | 0.599018 |
| 0.08 | 0.591583 | 0.575374 | 0.577352 | 0.577908 | 0.578899 | 0.590229 | 0.590687 |
| 0.09 | 0.582836 | 0.567097 | 0.569017 | 0.569557 | 0.57052 | 0.581521 | 0.581966 |
| 0.1 | 0.573802 | 0.558541 | 0.560404 | 0.560928 | 0.561862 | 0.572528 | 0.572959 |
Effect of failure rates of subsystems on profit of the feeding system

Table 2.2

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_2=0.02$, $\alpha_3=0.03$, $\alpha_4=0.04$, $\alpha_5=0.05$, $\alpha_6=0.025$, $\alpha_7=0.035$</td>
<td>$\beta_1=0.2$, $\beta_2=0.21$, $\beta_3=0.24$, $\beta_4=0.25$, $\beta_5=0.27$, $\beta_6=0.3$, $\beta_7=0.22$, $K_1=500, K_2=100$</td>
</tr>
<tr>
<td>0.01</td>
<td>216.7614</td>
</tr>
<tr>
<td>0.02</td>
<td>215.4206</td>
</tr>
<tr>
<td>0.03</td>
<td>213.3501</td>
</tr>
<tr>
<td>0.04</td>
<td>210.6695</td>
</tr>
<tr>
<td>0.05</td>
<td>207.4853</td>
</tr>
<tr>
<td>0.06</td>
<td>203.8914</td>
</tr>
<tr>
<td>0.07</td>
<td>199.9697</td>
</tr>
<tr>
<td>0.08</td>
<td>195.7915</td>
</tr>
<tr>
<td>0.09</td>
<td>191.4179</td>
</tr>
<tr>
<td>0.1</td>
<td>186.9011</td>
</tr>
</tbody>
</table>
Effect of repair rates of subsystems on availability of feeding system

Table 2.3

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>$\alpha_1=0.01$, $\alpha_2=0.02$, $\alpha_3=0.02$, $\alpha_4=0.03$, $\alpha_5=0.03$, $\alpha_6=0.035$, $\alpha_7=0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2$, $\beta_3$, $\beta_4$, $\beta_5$, $\beta_6$, $\beta_7$</td>
<td>Availability ($A_v$)</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0755564</td>
</tr>
<tr>
<td>0.15</td>
<td>0.758386</td>
</tr>
<tr>
<td>0.20</td>
<td>0.759415</td>
</tr>
<tr>
<td>0.25</td>
<td>0.759901</td>
</tr>
<tr>
<td>0.30</td>
<td>0.760168</td>
</tr>
<tr>
<td>0.35</td>
<td>0.760331</td>
</tr>
<tr>
<td>0.40</td>
<td>0.760437</td>
</tr>
<tr>
<td>0.45</td>
<td>0.760511</td>
</tr>
<tr>
<td>0.50</td>
<td>0.760563</td>
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</table>
Effect of repair rates of subsystems on profit of the feeding system

Table 2.4

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2=0.2$, $\beta_2=0.3$, $\beta_2=0.4$, $\beta_2=0.5$, $\beta_2=0.6$, $\beta_2=0.7$</td>
<td></td>
</tr>
<tr>
<td>$\beta_3=0.2$, $\beta_3=0.3$, $\beta_3=0.4$, $\beta_3=0.5$, $\beta_3=0.6$, $\beta_3=0.7$</td>
<td></td>
</tr>
<tr>
<td>$\beta_4=0.2$, $\beta_4=0.3$, $\beta_4=0.4$, $\beta_4=0.5$, $\beta_4=0.6$, $\beta_4=0.7$</td>
<td></td>
</tr>
<tr>
<td>$\beta_5=0.2$, $\beta_5=0.3$, $\beta_5=0.4$, $\beta_5=0.5$, $\beta_5=0.6$, $\beta_5=0.7$</td>
<td></td>
</tr>
<tr>
<td>$\beta_6=0.25$, $\beta_6=0.3$, $\beta_6=0.35$, $\beta_6=0.4$, $\beta_6=0.5$, $\beta_6=0.6$</td>
<td></td>
</tr>
<tr>
<td>$\beta_7=0.35$, $\beta_7=0.3$, $\beta_7=0.35$, $\beta_7=0.3$, $\beta_7=0.35$, $\beta_7=0.35$</td>
<td></td>
</tr>
</tbody>
</table>

$\alpha_1=0.01$, $\alpha_2=0.02$, $\alpha_3=0.02$, $\alpha_4=0.03$, $\alpha_5=0.03$, $\alpha_6=0.035$, $\alpha_7=0.04$, $K_1=500$, $K_2=100$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>277.7822</td>
</tr>
<tr>
<td>0.15</td>
<td>279.193</td>
</tr>
<tr>
<td>0.20</td>
<td>279.7073</td>
</tr>
<tr>
<td>0.25</td>
<td>279.9504</td>
</tr>
<tr>
<td>0.30</td>
<td>280.0841</td>
</tr>
<tr>
<td>0.35</td>
<td>280.1655</td>
</tr>
<tr>
<td>0.40</td>
<td>280.2187</td>
</tr>
<tr>
<td>0.45</td>
<td>280.2553</td>
</tr>
<tr>
<td>0.50</td>
<td>280.2816</td>
</tr>
</tbody>
</table>
2.10 Conclusion

The effect of failure rates of subsystems on availability and profit of the feeding system has been examined for arbitrary values of various parameters and cost. The results for availability and profit are shown numerically in tables 2.1 and 2.2 respectively. It is observed that availability and profit of the feeding system go on decreasing with the increase of failure rates of the subsystems A, B, C, D, E and T. However, the effect of failure rates of subsystems B, C, D and E is much more as compare to the failure rates of subsystems A and T. Therefore, there is need to control the failure rates of subsystems B, C, D and E in order to make the feeding system of sugar industry more profitable.

On the other hand the availability and profit of the feeding system keep on moving up with the increase of repair rates of all the subsystem as shown in tables 2.3 and 2.4 respectively. The effect of repair rates of subsystems B, C, D and E on availability and profit is more or less same but it is different for repair rates of subsystems A and T. Thus the study reveals that feeding system of a sugar industry can play vital role in improving availability and profit of whole sugar industry provided subsystems B, C, D and E have to be given more attention to get their repair and failure.

Finally, it is concluded that the subsystems B, C, D and E are key components of the feeding system which require more attention to make the sugar industry more profitable.