Chapter-8

Summary and Future Scope

8.1 SUMMARY

In this thesis, the mathematical analysis of some nonlinear autonomous models representing some ecological and eco-epidemiological systems is provided. In **chapter 1**, the main aspects of mathematical modelling of ecological systems, a brief history of research works focussing the stability of nonlinear autonomous models and a brief collection of basic concepts are presented.

In **chapter 2**, a detritus based model with harvesting of detritus as a time function, time delay in the predator's growth equation and random noise effect is considered. The conditions for feasibility of equilibrium points are obtained. The local stability of the system without time delay is guaranteed only if the population densities of detritivores and predator of detritivores is above the obtained level. The global stability of non-delayed system is verified. The optimal tax and harvesting levels are identified. The stability in the presence of delay is analysed by estimating the analytical threshold for time delay parameter. The condition for Hopf-bifurcation is derived and identified time delay as bifurcation parameter. The stability of stochastic model is analysed through the estimates of population variances. The dynamics with and without delay is explored in the graphs. The periodic nature of solutions is well exhibited graphically if the time-delay parameter exceeding the threshold value. The stochastic dynamics with various strength of noise is simulated and compared with each other.

In **chapter 3**, a four species model with special interactions predation, commensalism, mutualism and neutralism is considered with prey and predator harvesting and stochastic effect. The interior steady state and the condition for the existence of positivity are obtained. The local stability is verified. The parametric domain for global stability is identified. The bionomic equilibrium and its existence conditions are derived. The stability of the model with random perturbations is analysed by calculating the mean square fluctuations of population densities. The dynamics of the model with and without noise is visualised and compared with each

other. It is clearly observed that the increase in the amplitude of noise resulted in the increase in the population variances.

In **chapter 4**, a plankton based ecosystem is considered with stage structure for predator, gestation delay and random perturbations. The existence conditions and ecologically meaningful equilibrium points are obtained. The local stability is verified and the parametric domain of global stability is obtained. The dynamics of the system is exhibited in the graphs both in local and global sense. The analytical threshold of time lag is constructed and the condition for existence of periodic solutions is derived. The dynamics of delayed system for different time lag levels is portrayed and the periodic oscillation of the system for the time lag level beyond the threshold value is confirmed graphically. The population variances are calculated for the stochastic system. The time series evolution of the system in the presence of noise is exploring the positive correlation between the amplitude of noise and mean square fluctuations of population densities.

In **chapter 5**, a prey-predator fishery model is analysed with prey reserve, harvesting of prey in the free zone as time function and stochastic perturbations. The possible steady states and conditions for existence of positive steady states are obtained. The local stability is verified. The global stability is guaranteed only when the prey in free zone maintained in the obtained range. The optimal equilibrium is derived. The optimal tax level is also identified. The negative correlation between harvesting effort and tax levels are clearly explored in the graphs. The population variances of the model with random noise effect are estimated analytically. The dynamics of deterministic and stochastic models for different sets of parameters is portrayed in the graphs. The variation in population densities for different strength of noises are visualised and clearly the bulk increase in noise leading to chaotic dynamics.

In **chapter 6**, an S-I epidemic model with two time lags representing the incubation period and immunity period is considered. All equilibrium points and the existence conditions for positivity are obtained. By assuming that the incubation and immunity periods are same, the threshold of time lag is estimated analytically. The condition for existence of periodic oscillations is also obtained. The periodic nature of solutions is verified in the graphs. The comparison between time lag parameters and

random parameters, are clearly convey that the noise effects more on the size of the epidemic than the time lag constraints.

In **chapter 7**, an S-I epidemic model with special intermediate incubation class is considered. Both continuous and discrete versions are analysed for stability. The local dynamics of disease free and endemic equilibriums of continuous model is predicted with the threshold value of removable rate from incubation class. The analytical findings of continuous model are visualised in the numerical illustrations. The stability behaviour of disease free and endemic equilibriums of discrete model is also analysed analytically and numerically. It is clearly observed that the dynamics of discrete model is very similar to that of continuous model.

8.2 FUTURE SCOPE

The fields of mathematical ecology and mathematical epidemiology are very significant to gain insight about the maintenance of ecological balance and preventive healthcare respectively. There are many possibilities to extend the current research and some of the possible future work is listed below:

- The discrete analogous models may be constructed and analysed, which is more realistic and gives rise to efficient computational methods
- The movement of species plays a major role in population dynamics. So the spatio-temporal study of the models can be done.
- In the delayed models, the existence of periodic solutions is visualized in numerical simulations, the periods of limit cycles can be estimated analytically. The time delay analysis is carried out in view of gestation only, but recycling and maturation time lags may be incorporated which will make the model more realistic.
- The classical Lotka-Volterra functional response of predator is considered in the analysis, but the Holling type functional responses may be considered.
- The study of the dynamics of living resources using economic models, that is, the models can be extended in the direction of mathematical bionomics.
- The environmental fluctuations are approximated by Gaussian white noise, other random noises may be used and results may be compared with the obtained results.

- The construction of analytical methods for finding the solutions of nonlinear systems is a developing field. The numerical methods can also be developed to deal with the nonlinear models. The comparative study of nonlinear behaviour with the linear approximations may be carried out using numerical simulations.
- The mathematical models can be validated and improved using real data to understand the discrepancy and to improve the models accordingly for better forecasting. There is a high scope in this direction of building a bridge between theory and reality.