CHAPTER – 4

BOUNDARY LAYER FLOW AND HEAT TRANSFER OF FLUID-PARTICLE SUSPENSION OVER A VERTICAL STRETCHING SHEET WITH RADIATION
4.1 Introduction

Momentum and heat transfer in a boundary layer over a linearly stretching/shrinking sheet has considerable interest in the recent and past years because of its over increasing industrial applications such as extrusion of plastic sheets, wire drawing, glass fiber production and important bearings on several technological processes. In particular, in the extrusion of a polymer in a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a thin sheet, and then solidified through quenching or gradual cooling by direct contact with water or coolant liquid. Viscous dissipation changes the temperature distribution by playing a role like an energy source, which leads to affect the rate of heat transfer. The merit of viscous dissipation depends on whether the sheet is being cooled or heated. Such processes occur when the effect of buoyancy forces in free convection becomes significant. If the temperature of the surrounding fluid becomes high, then the thermal radiation effect plays a vital role in the case of space technology. So the study of two-dimensional boundary layer viscous flow and heat transfer over a stretching surface with effect of buoyancy force and thermal radiation is very important as it finds a large scale of practical applications in different areas.

The boundary layer flow on a continuously solid stretching surface with various aspects has been first investigated by Sakiadis[42,114]. He has considered the boundary layer flow over a flat surface moving with a constant velocity and formulated a boundary layer equation for two dimensional, axisymmetric flows. Due to entertainment of ambient fluid, this phenomenon represents a different type of boundary layer problem having solution substantially different from that of boundary layer flow over semi-infinite flat plate. Crane[49] has extended the work of Sakiadis by considering a moving strip, the velocity of which is proportional to the distance from the slit and obtained closed form exponential solution. Subsequently, many investigators have taken the advantage of simplicity of geometry attempting the problem with variety of assumptions to obtain
exact solutions. Gupta and Gupta[115], Carragher and Crane[116], Dutta et al.[117] have studied the heat transfer in the flow over a stretching surface with different aspects. Pal and Mondal[118] have analyzed the effects of temperature dependent viscosity and variable thermal conductivity on mixed convection problem by considering the wall heating conditions namely prescribed surface temperature and prescribed wall heat flux over a stretching sheet. They have solved the equations of flow field numerically by using the fifth-order Runge-Kutta Fehlberg method with shooting technique.

In the context of space technology and in processes involving high temperature the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, power plants for interplanetary flight and gas cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer, and emphasize the need for improved understanding of radiative transfer in these processes. The interaction of radiation with laminar free convection heat transfer from a vertical plate has been investigated by Cess[119] for an absorbing, emitting fluid in the optically thick region using the singular perturbation technique. Arpaci[120] has considered a similar problem in both the optically thin and optically thick regions and used the approximate integral technique and first order profiles to solve the energy equation. Cheng and Ozisik[121] have considered a related problem for an absorbing, emitting and isotropically scattering fluid and treated the radiation part of the problem exactly with the normal mode expansion technique. Raptis[122] has analyzed the thermal radiation and free convection flow through a porous medium using perturbation technique. Hossain and Takhar[123] have studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method. Raptis and Perdikis[124] have studied the effects of thermal radiation past a moving vertical plate. Das et al.[125] have analyzed the radiation effects on the flow past an impulsively started infinite isothermal vertical plate solving the governing equations by Laplace transform technique. The natural convection flow with radiation effects past a semi-infinite plate has been studied by Chamakha et al.[126].

The presence of dust particles has significant effect on the flow of a viscous fluid. The dust particles tend to retard the flow and to decrease the fluid temperature. Such flows are encountered in a wide variety of engineering problems such as nuclear reactor cooling, rain erosion, paint spraying, transport, waste water treatment and combustion. Saffman[3] has initiated the study of dusty fluids and discussed the stability
of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Dutta and Mishra[18,19] have investigated the boundary layer flow of a dusty fluid over a semi infinite flat plate and an oscillating plate. Vajravelu et.al.[70] have studied hydro magnetic flow of a dusty fluid over a stretching sheet including the effects of suction. Nandkeolyar and Sibanda[127] have investigated the two-dimensional boundary layer flow of a viscous, incompressible and electrically conducting dusty fluid past a vertical permeable stretching sheet under the influence of transverse magnetic field with the viscous and joule dissipation. Gireesha et.al.[56] have studied the two-dimensional unsteady mixed convective flow of a dusty fluid over a stretching sheet with thermal radiation and space dependent internal heat generation and absorption. MHD flow and heat transfer of an incompressible dusty fluid over a stretching sheet has been investigated by Gireesha et.al. [128]. Heat transfer effects on dusty gas flow past a semi-infinite inclined plate has been studied by Palani et.al.[26]. Mishra and Tripathy[31,112] have studied the boundary layer flow and heat transfer of two phase flow over a flat plate and wedge.

Motivated by all these investigations, the present study at a time includes the influencing parameters like thermal radiation, buoyancy force, heat transfer due to fluid-particle interaction, viscous dissipation, heat conduction, effective volumetric force, the momentum equation for particulate phase in the direction normal to the flow to study their impact on the boundary layer flow and heat transfer of fluid-particle suspension over a vertical stretching sheet fulfilling the inadequacies of previous investigators. The buoyancy force, effective volumetric forces are included in the momentum equations of fluid/particle phases, whereas the radiation heat flux, heat due to conduction and viscous dissipation are included in the energy equations of both phases for better understanding of the boundary layer characteristics and heat transfer phenomena. The governing coupled, non-linear partial differential equations of the flow and heat transfer problem are transferred into non-linear coupled ordinary differential equations by using a similarity transformation. These coupled non-linear ordinary differential equations with variable coefficients subject to the appropriate boundary conditions are solved numerically by using Runge-Kutta fourth order scheme with shooting technique.
4.2 Description of the Problem

A steady two-dimensional laminar boundary layer flow of an incompressible viscous fluid with SPM past a vertical stretching sheet is considered in presence of radiation field, shown in fig.4.1. The $x$-axis is along the stretching surface and in a direction opposite to the direction of gravity $g$ and $y$-axis is normal to the surface. Two equal and opposite forces are applied along the $x$-axis so that the sheet is stretched keeping the origin fixed in the fluid of ambient temperature $T_\infty$. The flow is generated by stretching of the sheet from a slit with a linear velocity $U_w(x) = CX$, where $C$ being a positive constant and $x$ is the coordinate measured along the stretching surface.

![Fig. 4.1: Schematic diagram of the problem](image)

Under these above assumptions, the governing equations of the flow and energy fields are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.2.1}
\]

\[
\frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0 \tag{4.2.2}
\]

\[
(1 - \varphi)\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = (1 - \varphi)\mu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\tau_p} \varphi \rho_s (u - u_p)
- (1 - \varphi)\rho g \beta^* (T - T_\infty) \tag{4.2.3}
\]
\[ \varphi \rho_s \left( u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) = \varphi \mu_s \frac{\partial^2 u_p}{\partial y^2} + \frac{1}{\tau_p} \varphi \rho_s (u - u_p) + \varphi (\rho_s - \rho) g \] (4.2.4)

\[ \varphi \rho_s \left( u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} \right) = \varphi \mu_s \frac{\partial^2 v_p}{\partial y^2} + \frac{1}{\tau_p} \varphi \rho_s (v - v_p) \] (4.2.5)

\[ (1 - \varphi) \rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = (1 - \varphi) k \frac{\partial^2 T}{\partial y^2} + \frac{1}{\tau_T} \varphi \rho_s c_s (T_p - T) \]

\[ + \frac{1}{\tau_p} \varphi \rho_s (u - u_p)^2 + (1 - \varphi) \mu \left( \frac{\partial u}{\partial y} \right)^2 - (1 - \varphi) \frac{\partial q_{rf}}{\partial y} \] (4.2.6)

\[ \varphi \rho_s c_s \left( u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right) = \varphi k_s \frac{\partial^2 T_p}{\partial y^2} - \frac{1}{\tau_T} \varphi \rho_s c_s (T_p - T) \]

\[ - \frac{1}{\tau_p} \varphi \rho_s (u - u_p)^2 + \varphi \mu_s \left[ u_p \frac{\partial^2 u_p}{\partial y^2} + \left( \frac{\partial u_p}{\partial y} \right)^2 \right] - \varphi \frac{\partial q_{rf}}{\partial y} \] (4.2.7)

Using Rosseland approximation for thermal radiation, the radiation heat flux \( q_{rf} \) in the energy equation (4.2.6) of the fluid phase (Brewster [146], Shorin[106]) is

\[ q_{rf} = -\frac{4 \sigma \cdot \partial T^4}{3 \kappa} \] (4.2.8)

Hence with reference to equation (2.6.17),

\[ \frac{\partial q_{rf}}{\partial y} = -\frac{16 \tau_p \sigma \cdot \partial^2 T}{3 \kappa} \] (4.2.9)

Similarly in the energy equation (4.2.7) of the particle phase,

\[ \frac{\partial q_{rp}}{\partial y} = -\frac{16 \tau_p \sigma \cdot \partial^2 T_p}{3 \kappa} \] (4.2.10)

The boundary conditions of the flow problem are given by

\[ y = 0 \ : \ u = U_w(x), v = 0, T = T_w = T_\infty + A \left( \frac{x}{l} \right)^2 \] (4.2.11a)

\[ y = \infty \ : \ \rho_p = \omega \rho, u = 0, u_p = 0, v_p = v, T = T_p = T_\infty \] (4.2.11b)

Where \( \omega \) is the density ratio in the main stream, \( A \) is a positive constant and \( l = \sqrt{v/c} \) is a characteristic length. For most of the gases \( \tau_p \approx \tau_T \) if \( \frac{c_s}{c_p} = 2 \) \( \frac{3}{3 \tau} \) and \( k_s = k \frac{c_s \mu_s}{c_p \mu} \).
4.3 Method of Solution

The equation (4.2.1) is identically satisfied by introducing the stream function 
\( \psi(x, y) = \sqrt{c}vxf(\eta) \) such that \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \).

The following non-dimensional transformations are introduced to convert the governing equations (4.2.2) to (4.2.7) into a set of similarity equations.

\[
\begin{align*}
\theta(\eta) & = \frac{T-T_w}{T_w-T_\infty}, \quad \theta_p(\eta) = \frac{T_p-T_w}{T_w-T_\infty}, \\
T-T_\infty & = A \left( \frac{x}{L} \right)^2 \theta(\eta), \quad T_p-T_\infty = A \left( \frac{x}{L} \right)^2 \theta_p(\eta)
\end{align*}
\]

Where \( T = T_\infty \) in place of \( T_w \).

After substituting (4.2.9), (4.2.10) and (4.3.1) in the equations (4.2.2) to (4.2.7), the following non-linear ordinary differential equations are obtained [Ref: Appendix-IV].

\[
\begin{align*}
Hf + HG' + GH' & = 0 \quad (4.3.2) \\
f'' + ff'' - f^2 + \frac{1}{\varphi_h} \theta H(F - f') + Gr \theta & = 0 \quad (4.3.3) \\
P^2 + Gf' - \epsilon F'' - \Phi (f' - F) + \frac{1}{Fr} \left( 1 - \frac{1}{y} \right) & = 0 \quad (4.3.4) \\
GG' - \epsilon G'' + \Phi (f + G) & = 0 \quad (4.3.5) \\
(1 + Ra) \theta'' - Pr(2f' \theta - f \theta') + \frac{1}{3} \frac{1}{\varphi_h} \theta H(\theta_p - \theta) & \\
- \frac{1}{\varphi_h} \theta Pr Ec H(F - f')^2 - Pr Ec f''^2 & = 0 \quad (4.3.6) \\
\left( \frac{\epsilon}{Fr} + \frac{3 Ra}{2 \gamma} \right) \theta'' - 2F \theta_p - G \theta_p - \Phi (\theta_p - \theta) - \frac{3}{2} \theta Pr Ec (f' - F)^2 & \\
+ \frac{3}{2} \epsilon Pr Ec \left( FF'' + F^2 \right) & = 0 \quad (4.3.7)
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
\eta = 0 & : G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1, F'(\eta) = 0, \theta(\eta) = 1, \theta_p = 0 \quad (4.3.8a) \\
\eta = \infty & : f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta), H(\eta) = \omega, \theta(\eta) = 0, \theta_p(\eta) = 0 \quad (4.3.8b)
\end{align*}
\]

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Where prime denotes differentiation with respect to $\eta$.

### 4.3.1 Numerical Simulation

The system of coupled and highly non-linear ordinary differential equations (4.3.2) to (4.3.7) along with the boundary conditions (4.3.8) are solved by converting into initial value problems with the help of fourth order Runge-Kutta method along with Shooting technique as

\[
F_1 = f'
\]

\[
F_2 = f''
\]

\[
F_3 = f''''(\eta) = f'''' = -f f'' + f''^2 - \frac{1}{1-\varphi} \partial H(F - f) - Gr \theta
\]

\[
F_4 = G'
\]

\[
F_5 = G'' = [G G' + \theta(f + G)] / \varepsilon
\]

\[
F_6 = F'
\]

\[
F_7 = F'' = [F^2 + G F' - \theta(f' - F) + \frac{1}{Fr} (1 - \frac{1}{\gamma})] / \varepsilon
\]

\[
F_8 = H' = -(H F + H G') / G
\]

\[
F_9 = \theta'
\]

\[
F_{10} = \theta'' = \left[ Pr \left( 2 f' \theta - f \theta' \right) - \frac{1}{31-\varphi} \partial H(\theta_p - \theta) + \frac{1}{1-\varphi} \partial Pr. Ec. H(F - f^2) + Pr Ec f''^2 \right] / (1 + Ra)
\]

\[
F_{11} = \theta_p'
\]

\[
F_{12} = \theta_{p}'' = \left[ \frac{2 F \theta_p + G \theta_p}{1} + 3 \partial Ec Pr(f' - F)^2 - \frac{3}{2} \varepsilon Ec Pr \left( F F'' + F'^2 \right) \right] / \left( \frac{\varepsilon}{Pr} + \frac{3 \varepsilon}{2 \gamma} \right)
\]

With boundary conditions

\[
f'(\infty) = 0, \quad F(\infty) = 0, \quad G(\infty) = -f(\infty), \quad H(\infty) = \omega, \quad \theta(\infty) = 0, \quad \theta_p(\infty) = 0.
\]

In order to integrate (4.3.9) to (4.3.20) as initial value problems, the values of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are required. But no such values are given at the boundary. The most important factor of shooting method is to choose the appropriate
finite values for $\eta_{\infty}$. In order to determine $\eta_{\infty}$, it is started with some initial guess value for some particular set of physical parameters and obtained values for unknown boundary conditions which differ only by a specified significant digit. The last value of $\eta_{\infty}$ is finally chosen to be the appropriate value of $\eta_{\infty}$ for that particular set of parameters. The value of $\eta_{\infty}$ may be different for another set of physical parameters. Once the finite value of $\eta_{\infty}$ is determined then the integration is carried out. Then compare the calculated values for unknown boundary values at $\eta = 10$ (say) with the given boundary conditions (4.3.21) and adjust the estimate values for unknown values of boundary conditions, to give a better approximation for the solution, i.e. by supplying $f'(0) = \alpha_0$ and $f''(0) = \alpha_1$. The improved value of $f'(0) = \alpha_2$ is determined by utilizing a linear interpolation formula. Then the value of $f'(\alpha_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\alpha_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\alpha_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\alpha_0 = \alpha_1$ and $\alpha_1 = \alpha_2$ until a correct $\alpha_2$ is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta(0), \theta_p(0)$.

Series values for unknown boundary conditions are taken to apply fourth order Runge-Kutta method with step size $h = 0.01$. The above procedure is repeated until the results up to the desired accuracy with an error of $10^{-5}$ are obtained.

### 4.4 Calculation of Skin Friction and Nusselt Number

(i) The local skin friction coefficient $c_f = \frac{\tau_w}{\rho u_w^2}$

where the skin friction $\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$

Using the non-dimensional variables, the skin friction ($c_f$) at the plate is obtained in non-dimensional form as

$c_f \sqrt{Re_x} = f''(0)$

(ii) The wall heat transfer rate i.e. the local Nusselt number $Nu_x$ is defined as

$Nu_x = \frac{xq_w}{k(\theta_w - \theta_x)}$

Where the heat transfer from the sheet $q_w$ is given by
\[ q_w = -\kappa \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad \text{(4.4.5)} \]

and using the non-dimensional variables, the rate of heat transfer coefficient is obtained in non-dimensional form in terms of Nusselt number \((Nu)\) as

\[ \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) \quad \text{(4.4.6)} \]

Where \(Re_x = \frac{UL}{v}\) is the local Reynold’s number.

### 4.5 Discussion of Results

In order to get an insight into the problem, a parametric study is carried out by generating velocity and temperature profiles that depict the variations w.r.t. \(\eta\) for both fluid and particle phases by varying the influencing parameters in succession.

The following data is considered for generating the dimensionless velocity and normalized temperature profiles:

\[ \theta = 0.01; \gamma = 1.0; Gr = 1.0, 2.0, 3.0; Fr = 10.0; \epsilon = 1.0, 2.0; \varphi = 0.001, 0.01, 0.1; \]
\[ Ra = 0, 1, 3.5, 10; Ec = 0.05, 0.1, 0.5; Pr = 0.71, 1.0 \]

The non-dimensional velocity and normalized temperature profiles for both fluid and particle phases are depicted in figs. 4.2 to 4.5 for the variation of diffusion parameter \((\epsilon)\) w.r.t. \(\eta\). While the non-dimensional velocity profiles for both phases are shown in figs 4.2 and 4.3 respectively, the normalized temperature profiles for both phases are shown respectively in figs 4.4 and 4.5. It is observed that the diffusion parameter \((\epsilon)\) has no effect on both velocity and temperature profiles of fluid phase, which is clearly visible from figs 4.2 and 4.4. The profiles shown in these figures for \(\epsilon = 1.0 \& 2.0\) are almost coinciding with each other. On the other hand the particle phase profiles are seen to be influenced by the magnitude of \(\epsilon\) (figs 4.3 & 4.5). From these figures it can be seen that both non-dimensional velocity and normalized temperature are decreasing across the boundary layer for any value of \(\epsilon\). However for any value of \(\eta\), the non-dimensional particle velocity \(F\) for any value of \(\eta\) is decreasing as \(\epsilon\) is increasing (fig 4.3) and the normalized particle temperature for any value of \(\eta\) is increasing with increase of \(\epsilon\) (fig 4.5).
Figs 4.6 to 4.9 show the effect of Grashoff number ($Gr$) signifying buoyancy parameter on the non-dimensional velocity and normalized temperature profiles of fluid and particle phases w.r.t. $\eta$. Here, the positive buoyancy force acts like a favorable pressure gradient accelerating the fluid as well as particles in the boundary layer. While figs. 4.6 & 4.7 shown here depict the effect of Grashoff number ($Gr$) on the variation of non-dimensional velocity profiles of both phases respectively, figs. 4.8 & 4.9 show the effect of $Gr$ on the variation of normalized temperature profiles of both phases across the boundary layer. From these figures, it is observed that both velocity and temperature profiles of fluid and particle phases gradually reduce in the direction normal to the flow irrespective of the values of Grashoff number. Grashoff number($Gr$) has a mild effect on the velocity profile of fluid phase as the profiles shown in fig. 4.6 for $Gr = 1.0$, $2.0$, $3.0$ are hardly differing from each other. Fig. 4.7 shows that the velocity of the particle phase at any point across the boundary layer decreases as Grashoff number increases.

Again from figs. 4.8 and 4.9 it is seen that the normalized temperature profiles of both phases decrease with increase of Grashoff number at any distance $\eta$ from the sheet and that the profiles for any value of $Gr$ decrease gradually towards the edge of the boundary layer. From the fig. 4.8 it is concluded that the decrease in the normalized temperature of fluid w.r.t. Grashoff number is found to be small near the surface of the sheet and larger near the edge of boundary layer, whereas the temperature of particle phase falls more rapidly for higher value of Grashoff number ($Gr = 3.0$) than that for lower values of $Gr = 1.0$, $2.0$ across the flow (fig. 4.9).

It is further considered to study the effect of the remaining parameters, viz. Eckret number($Ec$), radiation parameter($Ra$) and Prandtl number($Pr$) on the normalized temperature profiles. Figures 4.10 to 4.15 depict the above profiles for the parameters indicated.

The effect of Eckret number($Ec$), which signifies the viscous dissipation of the fluid on the heat transfer of both phases is exhibited in figs. 4.10 & 4.11. It is observed from fig. 4.10 that an increase in viscous dissipation of the fluid tends to increase the normalized fluid temperature in the direction of increasing $\eta$. The reason for this effect is that the viscosity of the fluid takes energy from motion of the fluid and transforms it into the internal energy of the fluid which results in the heating of the normalized fluid temperature. The thermal boundary layer gets thicker with the increase in the viscous
dissipation. Physically it means that the heat energy is stored in the fluid due to the frictional heating. Whereas due to two-phase interaction, the normalized particle temperature decays when the frictional heating is more across the boundary layer (fig. 4.11).

The variations of thermal boundary layer of both fluid & particle phases with respect to the effect of radiation parameter($Ra$) are studied in figs. 4.12 & 4.13. The thermal boundary layer thickness of fluid phase increases, but the normalized particle temperature decreases with the increase of radiation parameter($Ra$) in the direction of increasing$\eta$. This is in agreement with the physical fact that the effect of radiation is to intensify the heat transfer.

Figs. 4.14 & 4.15 depict the normalized temperature profiles of both the carrier fluid phase $\theta$ and the particle phase $\theta_p$ versus $\eta$ for different values of Prandtl number($Pr$). It is inferred from these figures that the normalized temperature of fluid and particle phases decrease with the increase of $Pr$, which implies momentum boundary layer is thicker than the thermal boundary layer. The normalized fluid temperature decays asymptotically and approaches to zero in the free stream region.

Table 4.1 shows the computations of the skin-friction coefficient ($c_f$) and Nusselt number($Nu$) for various physical parameters in terms of $f''(0)$ and $-\theta'(0)$ respectively. The magnitude of shear stress increases with increase of finite volume fraction($\varphi$), diffusion parameter($\epsilon$), Grashoff number($Gr$), Eckret number($Ec$) and radiation parameter ($Ra$) and decreases with the increase of the Prandtl number($Pr$) throughout the flow field. Similarly the rate of wall heat transfer significantly increases with the increase of diffusion parameter ($\epsilon$), Grashoff number($Gr$), Prandtl number($Pr$) and decreases with increase of Eckret number ($Ec$) and radiation parameter ($Ra$), but fluctuates with increase of finite volume fraction ($\varphi$) throughout the flow field.
4.6 Conclusion

The main results of this investigation are briefly summarized as follows:

i. The non-dimensional particle velocity is decreasing and the normalized particle temperature is increasing across the boundary layer with increase of diffusion parameter ($\epsilon$).

ii. An increase of Grashoff number ($Gr$) results in lower velocity of the particle phase and in decreasing the normalized temperatures of fluid and particle phases across the boundary layer.

iii. Viscous dissipation given in terms of Eckret number ($Ec$) tends to increase the normalized fluid temperature and decrease the normalized particle temperature across the boundary layer.

iv. The normalized temperature of fluid phase increases, whereas the normalized particle temperature decreases across the boundary layer when radiation ($Ra$) is more.

v. The increase in Prandtl number ($Pr$) results in decreasing the normalized temperature of fluid and particle phases across the boundary layer.

vi. The skin-friction coefficient ($c_f$) increases with the increase of diffusion parameter ($\epsilon$), Grashoff number ($Gr$), Eckret number ($Ec$), radiation parameter ($Ra$) and finite volume fraction ($\varphi$), but decreases with the increase of Prandtl number ($Pr$) throughout the flow field.

vii. The rate of wall heat transfer in terms of Nusselt number ($Nu$) increases with the increase of diffusion parameter ($\epsilon$), Grashoff number ($Gr$), Prandtl number ($Pr$), but decreases with the increase of Eckret number ($Ec$) and radiation parameter ($Ra$). Nusselt number ($Nu$) fluctuates with increase of finite volume fraction ($\varphi$) throughout the flow field.
Fig. 4. 2: Variation of non-dimensional fluid velocity ($f'$) for different diffusion parameter ($\varepsilon$).

Fig. 4. 3: Variation of non-dimensional particle velocity ($F$) for different diffusion parameter ($\varepsilon$).
Fig. 4.4: Variation of normalized fluid temperature ($\theta$) for different diffusion parameter ($\varepsilon$).

Fig. 4.5: Variation of normalized particle temperature ($\theta_p$) for different diffusion parameter ($\varepsilon$).
Fig. 4.6: Variation of non-dimensional fluid velocity ($f'$) for different Grashoff number ($Gr$).

Fig. 4.7: Variation of non-dimensional particle velocity ($F$) for different Grashoff number ($Gr$).
Fig. 4.8: Variation of normalized fluid temperature (θ) for different Grashoff number (Gr).

Fig. 4.9: Variation of normalized particle temperature (θₚ) for different Grashoff number (Gr).
Fig. 4.10: Variation of normalized fluid temperature ($\theta$) for different Eckret number ($Ec$).

Fig. 4.11: Variation of normalized particle temperature ($\theta_p$) for different Eckret number ($Ec$).
Fig. 4.12: Variation of normalized fluid temperature ($\theta$) for different Radiation parameter ($Ra$).

Fig. 4.13: Variation of normalized particle temperature ($\theta_p$) for different Radiation parameter ($Ra$).
Fig. 4.14: Variation of normalized fluid temperature ($\theta$) for different Prandtl number ($Pr$).

Fig. 4.15: Variation of normalized particle temperature ($\theta_p$) for different Prandtl number ($Pr$).
Table 4.1: Effect of Finite volume fraction, Diffusion parameter, Grashoff number, Prandtl number, Eckret number, Radiation parameter on Coefficient of Skin friction ($c_f$) and Nusselt number (Nu).

<table>
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<tr>
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<th>$\epsilon$</th>
<th>$Gr$</th>
<th>$Pr$</th>
<th>$Ec$</th>
<th>$Ra$</th>
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