CHAPTER – 3

MIXED CONVECTIVE HEAT TRANSFER OF A PARTICULATE SUSPENSION OVER A SEMI-INFINITE FLAT PLATE WITH ELECTRIFICATION OF PARTICLES
3.1 Introduction

Heat transfer between a particulate suspension and a solid body is a problem whose solution involves the consideration of the equations of motion of a two-phase system. Both Marble[2] and Soo[13] have developed the conservation laws of mass, momentum and energy for two-phase flow. These equations are sufficiently complex to preclude the possibility of exact solutions except in much idealized cases. Most closed form solutions presently known are discussed by Soo[13] and Marble[2].

Singleton[10] has considered compressible laminar boundary-layer flow of a dusty gas over a semi-infinite flat plate and obtained asymptotic solutions using the series expansion method for both small slip region (where the particle slip velocity is small) towards downstream of the leading edge of the plate and the large slip region (where the slip velocity between two phases is large) close to the leading edge of the plate. He has assumed that the densities of both phases are not constant and that the fluid viscosity is proportional to the square root of its temperature. Wang and Glass[34] have also studied compressible laminar boundary layer flows of a dusty gas over a semi-infinite flat plate considering a moderate slip region (a non-equilibrium transition region) in addition to the large and small slip regions. They have obtained asymptotic series expansion results in all these regions as well as a finite difference solution over the whole plate. Marbel[2] has considered dynamics of a gas combining small solid particles and obtained the solutions valid for far downstream region of the plate assuming zero particulate velocity on the surface. Soo[13] has derived momentum integrals for the fluid & particle phases and solved the same by using linear profiles for both gas and particle phase velocities and quadratic profile for particulate density. Tabakoff & Hammed [16,17] have studied boundary layer flow of particulate gas and pointed out that particle velocity decreases linearly and particle density increases continuously along the plate length. Their study leads to a surface particle velocity zero and particle density to infinity at a distance along the plate length $x = 1$. Jain & Ghosh [111] have investigated the
structure and surface property of the boundary layer of a gas particulate flow over a flat plate by employing momentum integral method. They have pointed out that the third degree profile for velocity and particle density gives results which is valid for far downstream stations on the plate. Dutta & Mishra [18] have studied Boundary Layer Flow of a Dusty Fluid over a Semi-Infinite Flat Plate. Mishra & Tripathy[30] have investigated the two-phase boundary layer flow over a flat plate to study the boundary layer flow characteristics by using momentum integral method. Wang & Glass [33,34] have considered compressible laminar boundary layer flows of a Dusty Gas over a Semi-infinite Flat Plate.

The aspects like electrification of SPM, force due to gravity (buoyancy force) and viscous dissipation have not been considered simultaneously by the previous investigators to study their impact on the flow and heat transfer of a fluid-particle suspension over a semi-infinite flat plate. Further, The fluid is in neutral medium, whereas the particles are electrically charged (ionised) due to particle-particle and particle-wall interactions producing an effective drag force on the ions, Soo[11], which affect the dynamics of flow of fluid-particle suspension to a greater extent. The present study deals with the steady, laminar, incompressible boundary layer mixed convective flow and heat transfer of a fluid-particle suspension over a semi-infinite flat plate in presence of electrification of particles. The force due to electrification of particles are included in the momentum equations of both phases, whereas the energy sources due to electrification of particles are included in the energy equations of both phases for better understanding of the boundary layer characteristics and heat transfer phenomena.

3.2 Description of the Problem

A steady two-dimensional laminar boundary layer mixed convective flow of an incompressible viscous fluid with SPM past a semi-infinite flat plate is considered with the influence of electrification of particulate matter, shown in fig.3.1. The $x$-axis is along the plate and in the opposite direction of gravity, while $y$-axis is normal to the plate surface, Kaviany [113]. The plate being coincident with the plane $y=0$ and is placed in the flow of uniform stream parallel to $x$-$z$ plane. Far away from the plate, both fluid and particle phases are in equilibrium and move with a uniform velocity $U$ in the $x$-direction. Let the free stream suspension temperature be $T_\infty$. 

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Under these above assumptions, the governing equations of the flow and energy fields are given by

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \quad (3.2.1) \\
\frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) &= 0 \quad (3.2.2) \\
(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) &= \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{1 - \nu} \frac{\partial}{\partial y} \left( \frac{1}{\rho_p} \left( u - u_p \right) - g \beta^* (T - T_\infty) \right) + \frac{1}{1 - \nu} \frac{\partial}{\partial y} \left( \frac{e_p}{\rho} \right) E \quad (3.2.3) \\
\rho_p \left( u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) &= \varphi \mu_s \frac{\partial^2 u_p}{\partial y^2} + \tau_p \left( u - u_p \right) + \varphi \left( \rho_s - \rho \right) g + \rho_p \left( \frac{e_p}{\mu} \right) E \quad (3.2.4) \\
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= \kappa \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\varphi}{1 - \nu} \frac{\partial}{\partial y} \left( \tau_p - T \right) + \frac{\varphi}{1 - \nu} \rho_s \left( \frac{e_p}{\mu} \right) E u_p \quad (3.2.5) \\
\rho_p c_s \left( u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right) &= \varphi k_s \frac{\partial^2 T_p}{\partial y^2} + \rho_p c_s \frac{1}{\tau_p} \left( T - T_p \right) + \rho_p \left( \frac{e_p}{\mu} \right) E u_p \quad (3.2.6)
\end{align*}
\]

Subject to the boundary conditions

At \( y = 0 : u = 0, v = 0, u_p = a_2(x), v_p = 0, \rho_p = a_3(x), T = T_w, \quad T_p = a_4(x) \) (3.2.7)

At \( y = \delta : u = U, \quad u_p = U, \quad \rho_p = \rho_p, \quad T = T_\infty, \quad T_p = T_\infty \) (3.2.8)

Though in general, \( \delta > \delta_t \) and \( \delta > \delta_p \), but in this problem, the thickness of the thermal boundary layer (\( \delta_t \)), particle velocity boundary layer (\( \delta_p \)), particle thermal boundary layer (\( \delta_{p_t} \)) are assumed to be same as that of the velocity boundary layer (\( \delta \)).

This assumption is justified in the scene that it simplifies the computational work and the results obtained are very close to the experimental results and to the exact solutions.
On integration, equations (3.2.3) to (3.2.6) w. r. t. \( y \) from \( y = 0 \) to \( y = \delta \) and introducing the non-dimensional quantities

\[
x^* = \frac{x}{L}, \quad y^* = \frac{y}{\delta}, \quad u^* = \frac{u}{U}, \quad u_p^* = \frac{u_p}{U}, \quad \rho_p^* = \frac{\rho_p}{\rho_p^o}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_p^* = \frac{T_p - T_\infty}{T_p - T_\infty} \tag{3.2.9}
\]

The resulting equations [Ref.: Appendix I & II] are

\[
\frac{\partial}{\partial x^*}\left[\frac{\delta}{L} \int_0^1 u^*(1 - u^*) dy^*\right] = \frac{\mu}{\rho U \delta} \frac{\partial u_p^*}{\partial y^*} \bigg|_{y^* = 0} + \frac{1}{1 - \varphi} \frac{F \rho_p \delta}{\rho} \int_0^1 \rho_p^*(1 - u_p^*) dy^*
\]

\[
- \frac{1}{1 - \varphi} F \frac{\rho_p \delta}{\rho} \int_0^1 \rho_p^* dy^* + \frac{\delta}{L} \int_0^1 \frac{Gr}{Re} \frac{T^*}{T_w - T_\infty} dy^* - \frac{1}{1 - \varphi} \frac{E}{U^2} \frac{\rho_p \delta}{\rho} \int_0^1 \rho_p^* dy^*
\]

\[
\frac{\partial}{\partial x^*} \int_0^1 (\rho_p^* u_p^*)(1 - u_p^*) dy^* = \frac{\mu L^2}{\rho \delta} \frac{\partial u_p^*}{\partial y^*} \bigg|_{y^* = 0} - \frac{\delta}{U \delta} \int_0^1 \rho_p^* (u^* - u_p^*) dy^*
\]

\[
- \frac{\delta}{Fr} \left(1 - \frac{u}{\rho_p}\right) \int_0^1 \rho_p^* dy^* - M \delta \int_0^1 \rho_p^* dy^* \tag{3.2.10}
\]

\[
\frac{\partial}{\partial x^*} \int_0^1 u^* T^* dy^* = - \frac{UL}{\delta \mu} \frac{\partial T^*}{\partial y^*} \bigg|_{y^* = 0} + \frac{U L}{\mu \rho_c \delta (T_w - T_\infty)} \int_0^1 (\frac{\partial u_p^*}{\partial y^*})^2 dy^*
\]

\[
- \frac{1}{1 - \varphi} \frac{c_p}{c_p} \frac{\rho_p \delta}{\rho} \left(\frac{\partial}{\partial x^*} \left[\delta \int_0^1 \rho_p^* u_p^* T_p^* dy^*\right] - \frac{L^2}{\delta} \frac{\epsilon \partial T_p^*}{\partial y^*} \bigg|_{y^* = 0}\right)
\]

\[
+ \frac{3}{2} Pr Ec M \delta \int_0^1 \rho_p^* u_p^* dy^*
\]

\[
\tag{3.2.11}
\]

\[
+ \frac{1}{1 - \varphi} \frac{c_p}{c_p} \frac{\rho_p \delta}{\rho} \left(\frac{E \delta L}{U \delta (T_w - T_\infty)} \int_0^1 \rho_p^* u_p^* dy^*\right)
\]

\[
\tag{3.2.12}
\]

and the boundary conditions (3.2.7) & (3.2.8) are reduced to

\[
y^* = 0: \quad u^* = 0, \quad v^* = 0, \quad u_p^* = a_2(x^*), \quad v_p^* = 0, \quad \rho_p^* = a_3(x^*), \quad T^* = 1, \quad T_p^* = a_4(x^*) \tag{3.2.13}
\]

\[
y^* = 1: \quad u^* = u_p^* = \rho_p^* = 1, \quad T^* = 0, \quad T_p^* = 0 \tag{3.2.14}
\]

### 3.3 Method of Solution

For consistency, the auxiliary condition is used, i.e. the flux of particulate mass across any control volume is zero.

Thus \( \rho_p U \delta = \int_0^\delta \rho_p u_p dy \) \tag{3.3.1}

which gives rise in non-dimensional form as
The following third degree profiles for \( u^*, u_p^*, \rho_p^*, T^* \) & \( T_p^* \) satisfying the boundary conditions (3.2.13 & 3.2.14) are considered.

\[
\begin{align*}
u^* &= 1 - (1 - y^*)^3 \\
u_p^* &= 1 - (1 - a_2)(1 - y^*)^3 \\
\rho_p^* &= 1 - (1 - a_3)(1 - y^*)^3 \\
T^* &= (1 - y^*)^3 \\
T_p^* &= a_4 T^*
\end{align*}
\]

As these profiles give results to far downstream station on the plate (Jain & Ghosh[111]), so using these in the equations (3.2.10) to (3.2.12) and by suppressing the term due to the frictional heat (2\textsuperscript{nd} term of R.H.S. in eqn. (3.2.12)), it is obtained as [Ref.: Appendix-III]

\[
\frac{dA}{dx} = \frac{56\mu}{\rho \beta L} = \frac{2}{3} \frac{1}{\varphi U} FL A A a_2 (4a_3 + 3) + \frac{14 Gr}{3 \kappa} A - \frac{14}{3} \frac{1}{\varphi} aAM(a_3 + 3) \tag{3.3.4}
\]

\[
\frac{dA}{dx} = \left\{ \frac{\frac{dA}{dx}}{2A} (12 + 28a_3) \frac{dA}{dx} \right\} + 2A \frac{4x(a_3 + 3)}{x(a_3 + 3)} + 140 A \frac{1}{Re}(1-a_2) \frac{dA}{dx} \tag{3.3.5}
\]

\[
\frac{dA}{dx} = \frac{4a_3 + 3 dA}{4a_3 + 3 dx} \tag{3.3.6}
\]

\[
\frac{dA}{dx} = \frac{-3 dA}{6dx} + \frac{3 dA}{Pr Re} + \frac{9Ec}{5Re} \frac{1}{1-\varphi 105 Pr} \frac{dA}{dx} + \frac{1}{2} \frac{dA}{dx} + \frac{1}{2} \frac{dA}{dx} \tag{3.3.7}
\]

When frictional heat is considered, using a sixth degree profile of \( T^* \) as

\[
T^* = \left(1 - \frac{1}{2} Ec\right) (1 - y^*)^3 + \frac{1}{2} Ec (1 - y^*)^6 \tag{3.3.8}
\]

along with the profiles of \( u^*, u_p^*, \rho_p^* \) & \( T_p^* (= a_4 T^*) \) as discussed in (3.3.3), the equation (3.2.12) yields
Calculation of Skin Friction & Nusselt Number

(i) The shearing stress on the plane boundary layer is given by

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$$  \hspace{1cm} (3.4.1)

In the present case, using non-dimensional quantities (3.2.9) and after dropping stars,

$$\tau_w = \mu \frac{U}{\delta} \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

Using the non-dimensional third degree profile of $u^*$ from (3.3.3) and dropping stars,

$$\tau_w = \mu \frac{U}{\delta} \left( \frac{\partial u}{\partial y} \right)_{y=0} \left\{ 1 - (1 - y)^3 \right\} = 3 \mu \frac{U}{\delta} = \frac{3 \rho U^2}{\Re \delta} , \text{ where } \Re \delta = \frac{\rho U \delta}{\mu}$$

Hence the skin friction coefficient, $c_f$ is given by

$$c_f = \frac{\tau_w}{\rho U^2/2} = \frac{6}{\Re \delta}$$  \hspace{1cm} (3.4.2)

(ii) The coefficient of heat transfer ($Nu$) is given by

$$Nu_x = \frac{\left( \frac{\partial T}{\partial y} \right)_{y=0} x}{\tau_w - T_x}$$  \hspace{1cm} (3.4.3)

Using the profiles (3.3.3) & (3.3.8) with the help of non-dimensional quantities (3.2.9), after dropping stars
3.5 Discussion of Results

In the present study here the basic features like particle velocity, density, temperature, skin friction and heat transfer in the boundary layer flow of particulate suspension over a flat plate have been studied by following Von Karman-Pohlhausen method. The following data is considered for generating the dimensionless velocity and normalized temperature profiles:

\[
\rho = 0.9752 \text{ kg/m}^3; \mu = 1.5415 \times 10^{-5} \text{ kg/ms}; L = 0.3048 \text{ m}; M = 0.0, 0.5, 1.0; \\
D = 50 \mu \text{m}, 100 \mu \text{m}; \rho_s = 800 \text{ kg/m}^3; \alpha = 0.1; \epsilon = 0.05; U = 60.96, 160.96 \text{ m/s}; \\
g = 9.8 \text{ m/s}^2; \text{ Ec} = 0.0, 1.0; Pr = 0.71, 1.0, 7.0; \text{ Gr/Re}^2 = 0.2, 0.5, 1.0.
\]

Runge-Kutta 4th order scheme has been employed to integrate the equations (3.3.4) to (3.3.7) and (3.3.9) for different values of Prandtl number (Pr), Buoyancy parameter (Gr/Re²), electrification parameter(M), Eckret number(Ec), particle size (D), diffusion parameter (ε) with uniform plate temperature. The variations of non-dimensional velocity profiles, normalized temperature profiles, density of particle phase, coefficient of skin friction and rate of heat transfer w.r.t. all the above influencing parameters are depicted through figs. 3.2 to 3.14 as well as through tables 3.1 to 3.5 after dropping stars.

It is observed from fig 3.2 that the non-dimensional fluid velocity (u) satisfies the no slip condition. It is also seen that the normalized fluid temperature (T) gradually reduces from unity near the wall to zero towards the edge of boundary layer, a characteristic feature of thermal boundary layer of pure fluid flow.

The variation of non-dimensional particle velocity (u_p) and particle density (ρ_p) are shown in figs 3.3 and 3.4 respectively. The non-dimensional particle velocity (u_p) does not satisfy the no slip condition and is found to be negative near the wall indicating that the particles move in a direction opposite to that of the flow. Particles tend to attain a zero value towards the edge of the boundary layer with a decreasing trend as they move from the leading edge towards the downstream (fig 3.3). Fig 3.4 displays the profiles of non-dimensional particle density (ρ_p) along the plate. From this figure it can be noted

\[
Nu_x = \begin{cases} 
\frac{1}{2} Re x c_f, \text{ when frictional heat is not considered} \\
\left( \frac{1}{2} + \frac{1}{4} Ec \right) Re x c_f, \text{ when frictional heat is considered}
\end{cases}
\]
that the particle density is decreasing in the direction normal to flow and approaching unity from the leading edge of the plate towards the downstream.

From fig 3.5 it is seen that the normalized particle temperature $T_p$ on the plate goes on increasing with $x$, i.e. towards downstream of the plate. The physical boundary condition of $T_p$ becoming zero for any downstream point as $y$ approaches the edge of boundary layer is clearly established through this figure.

The effect of particle size is considered next by generating the particle phase profiles of non-dimensional velocity and normalized temperature by varying the particle size (figs 3.6 and 3.7). Fig 3.6 shows that the finer particles move faster than the coarser particles. This leads to the conclusion that the finer particles are carried away by the fluid with higher velocities near the plate w.r.t. the general flow direction, gradually approaching a zero value across the flow, whereas the coarser particles tend to move in a direction opposite to the general flow direction near the plate and attain equilibrium gradually across the boundary layer. The particle temperature becomes negative in case of coarser particles indicating the particles are hotter than the fluid and heat flows from particle to fluid as shown in fig 3.7.

In order to understand the effect of the state of the fluid on the heat transfer to the particle phase, the normalized temperature profiles are obtained by maintaining three different values of Prandtl number ($Pr$) viz., 0.71, 1.0 and 7.0 which physically correspond to air, electrolyte solution and water respectively (fig 3.8). From this figure it is clear that the magnitude of the normalized particle temperature corresponding to water is very low as compared to that of air and electrolyte solution.

From the foregoing discussions it is noted that the particle phase profiles are more sensitive to the variations of the parameters considered in the present investigation. The effect of Eckret number ($Ec$) and the electrification parameter ($M$) on the dimensionless particle velocity and normalized particle temperature are depicted in figs 3.9 to 3.12. It is observed from figs. 3.9 and 3.10 that the numerical values of non-dimensional particle velocity and normalized particle temperature with frictional heat ($Ec = 1.0$) is less than that of without frictional heat ($Ec = 0$).

From figs 3.11 and 3.12 it is observed that, the magnitudes of non-dimensional particle velocity and normalized particle temperature increase with the increase of the
electrification parameter $M$. Both velocity and temperature profiles of particle phase are seen to be reducing towards the edge of boundary layer and approaching more or less an asymptotic value of unity in the free stream for any value of $M$.

The variation of non-dimensional particle velocity ($u_p$) and normalized particle temperature ($T_p$) w.r.t. Buoyancy parameter($Gr/Re^2$) is shown in figs. 3.13 and 3.14 respectively. From fig.3.13 it is observed that the non-dimensional particle velocity profiles gradually reduce in the $y$- direction for any value of Buoyancy parameter. It is further observed that at any location within the boundary layer (for given value of $y$) the non-dimensional particle velocity decreases with the increase of Buoyancy parameter. The decrease in the particle velocity with Buoyancy parameter is found to be larger near the leading edge and smaller near the edge of boundary layer. From fig.3.14 it is seen that the normalized particle temperature varies with a decreasing trend with increase of Buoyancy parameter at any distance $y$ from the plate. As regards to the lower values of Buoyancy parameter($Gr/Re^2 = 0.2 \& 0.5$), $T_p$ decreases reaching an asymptotic value of zero in the free stream. However, for Higher Buoyancy parameter($Gr/Re^2 = 1.0$) it is seen that $T_p$ increases reaching the asymptotic value zero in the free stream. Again higher Buoyancy parameter($Gr/Re^2 = 1.0$) induces negative particle temperature near the leading edge indicating the particles are hotter than the fluid and heat flows from particles to fluid.

Further study has been carried out by computing the values of Nusselt number ($Nu$) for different values of electrification parameter $M$, Ecklet number $Ec$ and Buoyancy parameter. The skin friction coefficient ($c_f$) has also been computed along with the above study. These computations are shown in tables 3.1 to 3.5. From table 3.1, it is observed that the Nusselt number ($Nu$) along the plate is greater for flow with frictional heat ($Ec = 1.0$) compared to that without frictional heat ($Ec = 0.0$) for both the values of electrification parameters considered ($M = 0$ and 1). It is also noted that ($Nu$) increases with electrification parameter $M$, for flow either with or without frictional heat (Table 3.1). Tables 3.2 and 3.3 show that, both Nusselt number ($Nu$) and the skin friction coefficient ($c_f$) increase with the electrification parameter $M$, indicating that electrification of particles enhances the rate of heat transfer from plate to fluid. It is revealed through the tables 3.4 and 3.5 that as Buoyancy parameter increases, both the
Nusselt number \((Nu)\) and skin friction coefficient \((c_f)\) decrease at any given location along the plate and the decrease being drastic towards the trailing edge.

3.6 Conclusion

The important results of this investigation are highlighted below.

i. The non-dimensional particle velocity \((u_p)\) and particle density \((\rho_p)\) decrease, where as the normalized particle temperature \(T_p\) on the plate goes on increasing as the particles move from the leading edge of the plate towards the downstream of the plate.

ii. The finer particles move faster than the coarser particles. The finer particles are carried away by the fluid with higher velocities, whereas the coarser particles tend to move in a direction opposite to the general flow direction near the plate. The particle temperature becomes negative in case of coarser particles indicating the particles are hotter than the fluid and heat flows from particles to fluid.

iii. The magnitude of the normalized particle temperature for the Prandtl number \((Pr)\) corresponding to water phase is very low as compared to that of air and electrolyte solution.

iv. The magnitudes of non-dimensional particle velocity and normalized particle temperature with frictional heat \((Ec = 1.0)\) is less than that without frictional heat \((Ec = 0)\).

v. The magnitudes of non-dimensional particle velocity and normalized particle temperature increase with the increase of the electrification parameter \(M\).

vi. Higher Buoyancy parameter induces lower velocity and lower temperature of particle phase.

vii. Frictional heat \((Ec)\) and electrification of SPM \((M)\) enhance the rate of heat transfer \((Nu)\) from plate to fluid.

viii. Electrification of SPM \((M)\) enhances the coefficient of skin friction \((c_f)\).

ix. Higher Buoyancy parameter leads to a reduction in the values of the Nusselt number \((Nu)\) as well as skin friction coefficient \((c_f)\).
Fig. 3.2: Variation of non-dimensional fluid velocity ($u$) & normalized fluid temperature ($T$) w.r.t. $y$.

$$
\begin{align*}
\rho_s &= 800 \text{ kg/m}^3, \\
\phi &= 0.001, \alpha = 0.1, \\
D &= 100 \text{ \mu m}, \\
M &= 1.0, \varepsilon = 0.05, \\
Pr &= 0.71, Ec = 1.0, \\
Gr/Re^2 &= 0.20
\end{align*}
$$

Fig. 3.3: Variation of non-dimensional particle velocity ($u_p$) for different downstream stations ($x$).

$$
\begin{align*}
\rho_s &= 800 \text{ kg/m}^3, \\
\phi &= 0.001, \alpha = 0.1, \\
D &= 100 \text{ \mu m}, \\
M &= 1.0, \varepsilon = 0.05, \\
Pr &= 0.71, Ec = 1.0, \\
Gr/Re^2 &= 0.20
\end{align*}
$$
Fig. 3. 4: Variation of non-dimensional particle density \( \rho_p \) for different downstream stations \( x \).

\[ \rho_s = 800 \text{kg/m}^3, \quad \varphi = 0.001, \alpha = 0.1, \]
\[ D = 100 \mu\text{m}, \quad M = 1.0, \varepsilon = 0.05, \quad \Pr = 0.71, \Ec = 1.0, \]
\[ \Gr/\Re^2 = 0.20 \]

Fig. 3. 5: Variation of normalized particle temperature \( T_p \) for different downstream stations \( x \).

\[ \rho_s = 800 \text{kg/m}^3, \quad \varphi = 0.001, \alpha = 0.1, \]
\[ D = 100 \mu\text{m}, \quad M = 1.0, \varepsilon = 0.05, \quad \Pr = 0.71, \Ec = 1.0, \]
\[ \Gr/\Re^2 = 0.20 \]
Fig. 3. 6: Variation non-dimensional particle velocity ($u_p$) for different size of particles ($D$).

Fig.3. 7 : Variation of normalized particle temperature ($T_p$) for different size of particles ($D$).

$D = 50\mu m, 100\mu m$

$\rho_s = 800 kg/m^3$, 
$\varphi = 0.001, \alpha = 0.1$, 
$M = 1.0, \epsilon = 0.05$, 
$Pr = 0.71, Ec = 1.0$, 
$Gr/Re^2 = 0.20$
Fig. 3. 8: Variation of normalized particle temperature ($T_p$) for different Prandtl number ($Pr$).

Fig. 3. 9: Variation of non-dimensional particle velocity ($u_p$) for different Eckert number ($Ec$).
Fig.3. 10: Variation of normalized particle temperature ($T_p$) for different Eckert number ($Ec$).

$\rho_s = 800 \text{kg/m}^3$, $\varphi = 0.001, \alpha = 0.1$, $D = 100 \text{\mu m}, M = 1.0$, $\varepsilon = 0.05, \text{Pr} = 0.71$, $Gr / Re^2 = 0.20$

Fig.3. 11: Variation of non-dimensional particle velocity ($u_p$) for different electrification parameter ($M$).

$\rho_s = 800 \text{kg/m}^3$, $\varphi = 0.001, \alpha = 0.1$, $D = 100 \text{\mu m}, \varepsilon = 0.05$, $\text{Pr} = 0.71, Ec = 1.0$, $Gr / Re^2 = 0.20$
Fig. 3.12: Variation of particle temperature ($T_p$) for different electrification parameter ($M$).

Fig. 3.13: Variation of particle velocity ($u_p$) for different Buoyancy parameter ($Gr/Re^2$).
\[ \rho_s = 800 \text{ kg/m}^3, \]
\[ \varphi = 0.001, \alpha = 0.1, \]
\[ M = 1.0, D = 100 \mu\text{m}, \]
\[ \varepsilon = 0.05, Pr = 0.71, \]
\[ Ec = 1.0 \]

\[ Gr/Re^2 = 0.20, 0.50, 1.0 \]

Fig.3.14: Variation of particle Temperature \( (T_p) \) for different Buoyancy parameter \( (Gr/Re^2) \).

Table 3.1: Variation of Nusselt Number \( (Nu) \) along the plate \( (x) \) for different values of Eckert Number \( (Ec) \) and electrification parameter \( (M) \) at \( \rho_s = 800 \text{ kg/m}^3, D = 100 \mu\text{m}, \)
\[ \varphi = 0.001, \alpha = 0.1, \varepsilon = 0.05, Pr = 0.71, Gr/Re^2 = 0.20. \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Ec = 0, M = 0 )</th>
<th>( Ec = 0, M = 1 )</th>
<th>( Ec = 1, M = 0 )</th>
<th>( Ec = 1, M = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>7.53E+01</td>
<td>8.12E+01</td>
<td>1.26E+02</td>
<td>1.37E+02</td>
</tr>
<tr>
<td>5.00</td>
<td>2.17E+02</td>
<td>8.50E+02</td>
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Table 3.2: Variation of Nusselt number ($Nu$) along the plate with different electrification parameter ($M$) at $\rho_s = 800$ kg/m$^3$, $D = 100\mu m$, $\varphi = 0.001$, $\alpha = 0.1$, $\varepsilon = 0.05, Pr = 0.71$, Ec = 1.0, $Gr/Re^2 = 0.20$

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Table 3.3: Variation of Coefficient of Skin friction ($c_f$) along the plate with different electrification parameter ($M$) at $\rho_s = 800$ kg/m$^3$, $D = 100\mu m$, $\alpha = 0.1$, $\varphi = 0.001$, $\varepsilon = 0.05, Pr = 0.71$, Ec = 1.0, $Gr/Re^2 = 0.20$.

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Table 3.4: Variation of Nusselt Number (Nu) along the plate with different
       Buoyancy parameter \((Gr/Re^2)\) at \(\rho_s = 800 \text{ kg/m}^3\), \(D = 100\mu\text{m}\),
       \(\alpha = 0.1\), \(M = 1.0\), \(\varphi = 0.001\), \(\varepsilon = 0.05\), \(Pr = 0.71\), \(Ec = 1.0\).

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Table 3.5: Variation of Coefficient of Skin friction \((c_f)\) along the plate with different
       Buoyancy parameter \((Gr/Re^2)\) at \(\rho_s = 800 \text{ kg/m}^3\), \(D = 100\mu\text{m}\), \(\alpha = 0.1\),
       \(M = 1.0\), \(\varphi = 0.001\), \(\varepsilon = 0.05\), \(Pr = 0.71\), \(Ec = 1.0\).

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