Preface

The present thesis entitled “Solvability of certain classes of mixed equilibrium problems, variational-like inequality problems and fixed point problems” is an outcome of the studies made by the author at Department of Mathematics, Aligarh Muslim University, Aligarh, India.

Theory of variational inequalities and theory of equilibrium problems both have played fundamental and important roles in the study of a wide range of unrelated problems arising in optimization, transportation, economics equilibrium and other problems of practical interest.

The theory of variational inequalities was initiated independently by Fichera [57] and Stampacchia [148] in the early 1960’s to study the boundary value problems arising in the elasticity and potential theory, respectively. Since then variational inequalities have been extended and generalized in several directions, using new powerful and innovative techniques, to study a wide class of unrelated problems arising in physics, mechanics, optimization, control theory, management science, operations research, economics, transportation and other branches of mathematical and engineering sciences, in a unified and general framework.

Variational inclusion is an important generalization of variational inequality, introduced and studied by Brézis [18]. It includes quasi-variational inequality [11] and variational-like inequality [131] as special cases. A large number of books and research monographs for the applications of variational inequalities and variational inclusions are available in the literature.
Theory of equilibrium problems was introduced by Zuhovickii et al. [170] and Fan [54,55], perhaps motivated by minimax problems appearing in economic equilibrium. But, in 1994, the terminology of equilibrium problem was adopted by Blum and Oettli [14]. Since then various extensions and generalizations of equilibrium problem have been studied by many authors. The theory of equilibrium problems provides a natural, novel and unified framework for several problems arising in nonlinear analysis, optimization, economics, game theory, physics, operation research and engineering. The equilibrium problem includes many mathematical problems as particular cases for examples, mathematical programming problems, complementary problems, variational inequality problems, saddle point problems, Nash equilibrium problems in noncooperative games, minimax inequality problems, minimization problems and fixed point problems.

In recent years, much attention has been given for developing efficient and implementable iterative methods including projection method and its variant forms, extragradient method, linear approximation, auxiliary principle method, descent and Newton methods, Helpern iterative method, Mann and Ishikawa iterative methods, viscosity approximation method, hybrid iterative method for the variational inequalities and equilibrium problems.

There is a vast literature is available on iterative methods for studying variational inequalities, equilibrium problems and fixed point problems for nonlinear mappings, separately, and on iterative methods to approximate common solutions of these problems. But to develop and study the iterative methods for approximating common solutions of new generalizations of these problems in the settings of Hilbert spaces and Banach spaces, is still an unexplored field.

The objective of this thesis is to develop and study some iterative methods for split equality monotone variational inclusion, split equality generalized equilibrium problem, Combination of split general variational-like inequality problem, split monotone variational inclusion, mixed equilibrium problem, and fixed point problems for a (family of) nonlinear mapping(s) in the setting of Hilbert spaces; and to develop and study some iterative methods for the mixed equilibrium problem, system of unrelated generalized
mixed variational-like inequality problem, and fixed point problems for a (family of) nonlinear mapping(s) in the setting of Banach space.

The thesis comprises of seven chapters.

In Chapter 1, we review various notations, known definitions and results which are required in carrying out the research work presented in the thesis. Further, we give brief survey of some classes of variational inequalities and equilibrium problems. Furthermore, we give brief survey of some iterative methods for solving fixed point problems, variational inequalities and equilibrium problems.

In Chapter 2, we suggest and analyze an iterative method based on hybrid iterative method without extrapolating step for solving mixed equilibrium problem. Further, we obtain a strong convergence theorem for the sequences generated by the proposed iterative algorithm. Furthermore, we derive a consequence from the main result. The result and method presented in this chapter extend and generalize some known results and iterative methods.

In Chapter 3, we suggest and analyze an iterative method without extrapolating step for finding a common solution of equilibrium problem and a fixed point problem for an asymptotically quasi-$\phi$-nonexpansive mapping in intermediate sense in a uniformly smooth and strictly convex Banach space. We prove a strong convergence theorem for this method. The method and result presented here generalize and unify the previously known related methods and results.

In Chapter 4, we consider a generalized mixed variational-like inequality problem and prove a Minty-type lemma for its related auxiliary problems in real Banach space. We prove the existence of solution of these auxiliary problems. Further, we prove some properties of solution set of generalized mixed variational-like inequality problem. Furthermore, we use a hybrid projection method to find a common solution of a system of unrelated generalized mixed variational-like inequality problems for generalized relaxed $\alpha$-monotone mappings and a common fixed point problem for a family of generalized asymptotically quasi-$\phi$-nonexpansive mappings in reflexive, uniformly smooth and
strictly convex Banach space. The results presented here generalize and extend some known results in the literature.

In Chapter 5, we suggest and analyze an iterative method to approximate a common solution of a combination of split general variational-like inequality problems and a common fixed point problem for a family of generalized asymptotically nonexpansive nonself mappings by using a specific way of choosing the indexes of the involved mappings. Further, we prove that sequences generated by the proposed iterative method converges strongly to a common solution to combination of split general variational-like inequality problem and fixed point problem for a family of generalized asymptotically nonexpansive nonself mappings.

In Chapter 6, we suggest and analyze a hybrid-extragradient iterative method to approximate a common solution of split monotone variational inclusion, mixed equilibrium problem and fixed-point problem for a nonexpansive mapping. Further, we establish a strong convergence theorem for the sequences generated by the proposed iterative algorithm. Furthermore, we derive some consequences from our main result. Finally, a preliminary numerical example is given to support our main result.

In Chapter 7, we suggest and analyze a viscosity iterative method for a split equality monotone variational inclusion problem, a split equality generalized equilibrium problem and a split equality common fixed point problem in real Hilbert spaces. Further, we prove that the sequences generated by the proposed algorithm converge strongly to a common solution of these problems, which also solves the system of variational inequality problems. Finally, we derive a consequence from the main result.

A comprehensive list of references of books, monographs, proceedings and research papers is provided at the end of the thesis.

It has been observed that, in some particular cases our results reduce to some of the existing work available in the literature, which shows that our results and methods are more general than the existing results.
The published/ accepted/ communicated research papers based on the work of this thesis are as follows:


3. Common solution to an equilibrium problem and a fixed point problem for an asymptotically quasi-\(\phi\)-nonexpansive mapping in intermediate sense, Communicated.

4. Combination of split general variational-like inequality problems and a family of generalized asymptotically nonexpansive nonself mappings, Communicated.

5. A hybrid-extragradient iterative method for split monotone variational inclusion, mixed equilibrium problem and fixed point problem for a nonexpansive mapping, Communicated.

6. Viscosity approximation method for split equality monotone variational inclusion problem, split equality generalized equilibrium problem and split equality common fixed point problem, Communicated.

Some results of this thesis have been presented in the following International conferences:

1. International Conference on Analysis and its Applications at Department of Mathematics, Aligarh Muslim University, Aligarh, India, December 19-21, 2015.

2. International Conference on Recent Advances on Mathematical Biology, Analysis and Applications at Department of Applied Mathematics, Aligarh Muslim University, Aligarh, India, June 04-06, 2015.