Chapter 4

New Methods for Solving Fuzzy Transportation Problems
NEW METHODS FOR SOLVING FUZZY TRANSPORTATION PROBLEMS

This chapter provides a basic feasible solution for solving fuzzy transportation problems in which cost, demand and supply quantities are triangular fuzzy numbers. An efficient algorithm is given for finding the nearly optimum solution which requires least iterations. The degeneracy problem is also avoided by this method. The procedure for the solution is illustrated with an example.

4.1. INTRODUCTION

In chapter 2, several methods have been discussed for finding the initial basic feasible solution for fuzzy transportation problems. However, to the following questions are not being answered by the initial basic feasible solution:

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(i) Which initial basic feasible solution method is the best one?

(ii) Which initial basic feasible solution method gives a solution close to the optimum solution?

[29] have developed new techniques called Max(Min-Max) method and Min(Min-Max) method for finding the nearly optimum solution for transportation problems which require least iterations to reach optimality, compared to the existing methods available in the literature. The degeneracy problem is also avoided by these methods.

In this chapter, new methods namely, Max(Min-Max) method and Min(Min-Max) method are proposed for solving fuzzy transportation problems. These methods are found to provide a good starting solution for a given transportation problem in which the cost, demand and supply quantities are triangular fuzzy numbers. They are also found to require least iterations to reach optimality. The degeneracy problem is also avoided.
The algorithm developed in this chapter is found to have the following
major advantages:

(i) It is found to require less computation compared to the existing
methods in the literature.

(ii) The degeneracy problem is not found to arise in the fuzzy initial
basic feasible solution.

(iii) The algorithm is found to have the interesting property:
\[
\text{Max} \left( \text{Min-Max} \right) = \text{Min} \left( \text{Min-Max} \right) = \text{Optimal Solution}
\]

4.2. THE MAX(MIN-MAX) ALGORITHM

The Max(Min-Max) algorithm is proposed for solving a fuzzy
transportation problem.

The Max(Min-Max) algorithm proceeds as follows:
Step 1: The maximum ($\tilde{c}_{ij}$) cell ($i = 1, \ldots, m ; j = 1, \ldots, n$) is chosen. In case of a tie, the maximum is chosen arbitrarily.

Step 2:

(i) The maximum possible fuzzy quantity is allocated to the minimum cost cell.

(ii) The fuzzy total cost of this allocation is computed.

Step 3: Step 2 is repeated for the cell (column) containing the maximum ($\tilde{c}_{ij}$).

Step 4: The cell with the maximum cost is chosen for allocation. In case of a tie, the same is chosen arbitrarily.

Step 5: The concerned row or column is deleted, when the allocation is complete. In case of a tie, either the row or column is deleted. If both the row and column should be deleted, zero is assigned to the minimum cost cell which has not yet been deleted.
Step 6: When all $m+n-1$ cells are allocated, stop. Otherwise the algorithm is repeated from step 1.

4.3. THE MIN(MIN-MAX) ALGORITHM

The Min(Min-Max) algorithm is proposed for solving the fuzzy transportation problem.

The Min(Min-Max) algorithm proceeds as follows:

Step 1: The maximum $(c_{ij})$ cell ($i = 1, \ldots, m$; $j = 1, \ldots, n$) is chosen. In case of a tie, the maximum is chosen arbitrarily.

Step 2: (i) The minimum cost cell is chosen from the row containing the maximum $(\bar{c}_{ij})$.

(ii) The maximum possible fuzzy quantity is allocated to the minimum cost cell.

(iii) The fuzzy total cost of this allocation is computed.
Step 3: Step 2 is repeated for the cell (column) containing the maximum $(\bar{c}_y)$. 

Step 4: The cell with the minimum cost is chosen for allocation. In case of a tie, the same is chosen arbitrarily.

Step 5: The concerned row or column is deleted when the allocation is complete. In case of a tie, either the row or the column is deleted. If both the row and the column should be deleted, zero is assigned to the minimum cost cell which has not yet been deleted.

Step 6: When all $m + n - 1$ cells are allocated the algorithm is halted. Otherwise the algorithm is repeated from step 1.

4.4. SOLVED EXAMPLE

A company has three factories and four workhouses. The suppliers are transported from the factories to the workhouses, which are located at
varying distances from the factories. The workhouse requirements and
factory capacities are given as, $\tilde{d}_1=(1,2,3)$, $\tilde{d}_2=(5,10,12)$, $\tilde{d}_3=(3,5,7)$,
$\tilde{d}_4=(1,2,3)$ and $\tilde{s}_1=(3,5,7)$, $\tilde{s}_2=(3,6,9)$, $\tilde{s}_3=(4,8,9)$ respectively. The fuzzy
costs from the factory to the workhouses are given below:

\[
\begin{array}{c|cccc|c}
\hline
& W_1 & W_2 & W_3 & W_4 & S_i \\
\hline
F_1 & (1,5,6) & (0,1,2) & (1,4,7) & (1,2,3) & \tilde{s}_1 \\
F_2 & (2,6,7) & (0,2,4) & (2,5,8) & (1,2,6) & \tilde{s}_2 \\
F_3 & (2,7,9) & (1,3,5) & (0,1,2) & (0,1,2) & \tilde{s}_3 \\
\hline
\tilde{d}_1 & \tilde{d}_2 & \tilde{d}_3 & \tilde{d}_4 \\
\hline
\end{array}
\]

SOLVING USING THE MAX(MIN-MAX) ALGORITHM

\[
\begin{array}{ccc}
\text{Max} & \text{Min} & \text{Max} \\
\hline
(2,7,9) & \text{Row} & (0,1,2) \times (3,5,7) \text{ Omit Column 1} \\
\hline
\text{Column} & (1,5,6) \times (1,2,3) & \\
\hline
\end{array}
\]

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Max | Min  | Max
---|---|---
(2,5,8) | Row (0,2,4) x (3,6,9) | Omit Row 2
      | Column (1,4,7) x (0,0,0) |   
(1,3,5) | Row (0,1,2) x (1,2,3) | Omit Column 2
      | Column (0,1,2) x (0,3,6) |   
(1,2,3) | Row (1,2,3) x (0,0,0) |   
      | Column (1,2,6) x (0,0,0) |   

The following solution is obtained using the algorithm given above:

<table>
<thead>
<tr>
<th>(1,2,3)</th>
<th>(0,3,6)</th>
<th>(1,2,3)</th>
<th>(3,5,7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,5,6)</td>
<td>(0,1,2)</td>
<td>(1,4,7)</td>
<td></td>
</tr>
<tr>
<td>(2,6,7)</td>
<td>(3,6,9)</td>
<td>(2,5,8)</td>
<td>(3,6,9)</td>
</tr>
<tr>
<td>(2,7,9)</td>
<td>(-10,1,9)</td>
<td>(3,5,7)</td>
<td>(1,2,3)</td>
</tr>
</tbody>
</table>
<pre><code>     | (1,3,5) | (0,1,2) | (4,8,9) |
     | (1,2,3) | (5,10,12) | (3,5,7) |
                 |         | (1,2,3) |
</code></pre>
From the table above,

Fuzzy transportation cost for the given problem is:

\[
\begin{align*}
(0,1,2) \times (3,5,7) &+ (1,5,6) \times (1,2,3) &+ (0,2,4) \times (3,6,9) &+ (1,4,7) \times (0,0,0) \\
+ (0,1,2) \times (1,2,3) &+ (0,1,2) \times (0,3,6) &+ (1,2,3) \times (0,0,0) \\
+ (1,2,6) \times (0,0,0) &+ (1,3,5) \times (-10,1,9) \\
&= (0,5,14) &+ (1,10,18) &+ (0,12,36) &+ (0,0,0) &+ (0,2,6) \\
+ (0,3,12) &+ (0,0,0) &+ (0,0,0) &+ (-50,3,45) \\
&= (-49,35,131)
\end{align*}
\]

Thus, the Defuzzified Fuzzy Transportation Cost is 37.

**SOLVING USING THE MIN(MIN-MAX) ALGORITHM**

<table>
<thead>
<tr>
<th>Max</th>
<th>Min</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2,7,9)</td>
<td>Row (0,1,2) x (3,5,7)</td>
<td>Omit Row 3</td>
</tr>
<tr>
<td></td>
<td>Column (1,5,6) x (1,2,3)</td>
<td></td>
</tr>
<tr>
<td>(2,6,7)</td>
<td>Row</td>
<td>(0,2,4) x (3,6,9)</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>--------------------</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>(1,4,7) x (0,0,0)</td>
</tr>
<tr>
<td>(2,5,8)</td>
<td>Row</td>
<td>(1,2,6) x (0,0,0)</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>(1,4,7) x (0,0,0)</td>
</tr>
<tr>
<td>(1,2,3)</td>
<td>Row</td>
<td>(0,1,2) x (0,3,6)</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>(0,1,2) x (1,2,3)</td>
</tr>
</tbody>
</table>

The following solution is obtained using the algorithm given above:

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<td>(1,2,6)</td>
</tr>
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<td>(2,7,9)</td>
<td>(-10,1,9)</td>
<td>(3,5,7)</td>
<td>(1,2,3)</td>
</tr>
<tr>
<td></td>
<td>(1,3,5)</td>
<td>(0,1,2)</td>
<td>(0,1,2)</td>
</tr>
<tr>
<td>(1,2,3)</td>
<td>(5,10,12)</td>
<td>(3,5,7)</td>
<td>(1,2,3)</td>
</tr>
</tbody>
</table>

From the table above, Fuzzy transportation cost for the given problem is:

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(0,1,2) × (3,5,7) + (1,5,6) × (1,2,3) + (0,2,4) × (3,6,9) + (1,4,7) × (0,0,0) 
+ (0,1,2) × (1,2,3) + (0,1,2) × (0,3,6) + (1,2,3) × (0,0,0) 
+ (1,2,6) × (0,0,0) + (1,3,5) × (−10,1,9)

= (0,5,14) + (1,10,18) + (0,12,36) + (0,0,0) + (0,2,6) 
+ (0,3,12) + (0,0,0) + (0,0,0) + (−50,3,45)

= (−49,35,131)

Thus, the Defuzzified Fuzzy Transportation Cost is 37.

However, if the initial solution of this problem is found by Vogel’s method, the same solution is obtained, but this approach requires less number of computations and it avoids the problem of degeneracy, as the total number of basic cells is always m+n−1.