CHAPTER 4

ACTIVE CONTOUR MODELS

4.1 OVERVIEW OF THE CONTOUR MODELS

Following the localization of the OD region, the segmentation scheme should accurately demarcate the boundary of the OD to help track the progression of eye diseases. Naive segmentation algorithms that do not take the edge smoothness and continuity properties into account falls short of accurately detecting the OD boundary. On the contrary, an Active Contour Model (ACM) otherwise known as the snake is a promising alternative, as it relies on the gradient as well as the spatial distribution at a specific point to verify the presence of an edge. The snake is adapted to capture even concavities on the OD boundary.

The snake in general is an energy minimizing deformable spline that satisfies the Euler equation. In the sequel, the Euler equation is viewed as a force balance equation, \( F_{int} + F_{ext}^{(p)} = 0 \), where the internal force \( F_{int} \) discourages the snake to stretch and bend, while the external potential force \( F_{ext}^{(p)} \) pulls the snake toward an image edge. However, the traditional snake inherently suffers from the failure to progress into (image) boundary concavities, and possesses only a limited capture range. To overcome these drawbacks, Xu and Prince (2000) replaced \( F_{ext}^{(p)} \) in the force balance equation with a static external force field \( \nu(x;y) \), namely, GVF field, on the premise of the Helmholtz theorem (Xu,C and Prince,J.L (1998)).
An attractive consequence is that while \( v \) minimizes an energy functional, the snake is endowed with a large capture range and the ability to move into boundary concavities. In the energy functional, if the gradient of an edge map \(|\nabla f|\) is small, the smoothing term corresponding to the sum of the squares of the partial derivatives of \( v \) is predominant. As a result, the energy minimization yields a slowly varying field in homogeneous image regions. By contrast, for a large \(|\nabla f|\), the energy is minimized by bringing \( v \) almost close to \(|\nabla f|\), which is a desired effect. Furthermore, a regularization term \( \mu \) in the energy functional maintains a good balance between both scenarios, and its choice remains directly proportional to the amount of noise.

In the present setting, the snake is initialized with the approximate OD boundary generated by the CHT. Subsequently, the minimization of the GVF-based energy functional forces the snake to undergo dynamic adaptation to the edges of the OD. To assess the merit of our approach, the segmentation results from the Xu.C and Prince.J.L (1998) have been quantitatively compared with an exhaustive list of the following popular or recently reported ACM techniques: (i) Caselles-ACM (Jun. 1997), (ii) Patrick-ACM (Sep. 2000), (iii) Chan-ACM (Feb. 2001), (iv) Shi-ACM (May 2008), (v) Li2008-ACM (Oct. 2008), (vi) Lankton-ACM (Nov. 2008), (vii) Bernard-ACM (Jun. 2009), (viii) Li2010-ACM (Dec. 2010), and (ix) Wu-ACM (Jun. 2013). In order to measure the contour distance between the ACM and the five expert’s contour, a performance metric called Hausdorff distance is investigated in the subsequent section.

4.2 OVERVIEW OF THE HAUSDORFF DISTANCE

The Hausdorff Distance is a mathematical construct to measure the “closeness” of two sets of points that are subsets of a metric space. Such a measure is used to assign a scalar score to the similarity between any sets of points. Two sets are close
in the Hausdorff distance, if every point of either set is close to some point of the other set. The Hausdorff distance is the longest distance that is forced to travel by an adversary, that chooses a point in one of the two sets, from where it then must travel to the other set. It is the greatest of all the distances from a point in one set to the closest point in the other set.

This is a fast, reliable method for comparing binary images based on the generalized Hausdorff measure. The generalized Hausdorff measure provides a means of determining the resemblance of one point set to another, by examining the fraction of points in one set that lie near points in the other set (and perhaps vice versa). There are two parameters used to decide whether or not two point sets resemble one another using this measure: (i) the maximum distance that points are separated and still be considered close together, and (ii) what fraction of the points in one set are at a distance away from points of the other set.

Object detection often relies on metrics that describe the degree of difference between two shapes. The one-sided Hausdorff distance, $H(P, Q)$, in this context is a measure between the set of feature points defining a model, $A$, and the set of points defining a target image, $B$, where

**Definition 1** (Hausdorff distance). Given two finite sets containing points $P = \{p_1, p_2, \ldots, p_m\}$ and $Q = \{q_1, q_2, \ldots, q_n\}$, the Hausdorff distance is defined as

$$H(P, Q) = \max (h(P, Q), h(Q, P))$$

where

$$h(P, Q) = \max_{p \in P} \min_{q \in Q} ||p - q||$$

and $|| \cdot ||$ denotes the underlying norm on the points belonging to $P$ and $Q$.

Intuitively, if $h(P, Q) = d$, then every point in $P$ must lie within distance $d$ from a point in $Q$. Therefore, $H(P, Q)$ measures the distance between the point...
in $P$ that is the farthest from any point in $Q$ and vice versa, thereby implying a “degree of mismatch” between the points in $P$ and $Q$. Since this measure is adopted to compare a detected OD boundary with the one drawn by an expert, the sets $P$ and $Q$ contain the coordinates of the boundary pixels, and $|| \cdot ||$ is considered as the Euclidean norm ($L^2$ norm). The HD has been deployed in the design of a robust template matching technique in order to detect OD boundaries (Lalonde et al. (2001)).

In the absence of ground truth, a viable alternative to validate image segmentation methods is to rely on a variability measure. While the intraobserver variability reflects the reproducibility of a single observer’s results, the interobserver variability predicts the differences between the results of multiple observers. The interobserver variability is preferred to the intraobserver one, provided a database is supplied with the results from several observers. The RIM-ONE database, consists of contour drawn by five individual experts. Therefore, a resort to the former measure in our experiments is done. Performance measure for each of the Active Contour model, is carried out by using the Hausdorff distance. The experimental results of the Gradient Vector Flow Active Contour Model is investigated in the subsequent section. The section summarizes, the advantages and disadvantages of this model, and the effect of the model in retinal fundus images.

4.3 GRADIENT VECTOR FLOW ACTIVE CONTOUR MODEL

The Gradient Vector Flow (GVF) Active Contour Model is a traditional model. The traditional active contour model is limited to capture range and poor convergence to boundary concavities. GVF (Xu.C and Prince.J.L (1998)) produces a new external force for active contour model to overcome these issues. It is a dense vector field, generated by diffusing the gradient vectors of a gray-level or
binary edge map derived from an image. The GVF field is defined as a vector field
\[ V(x,y) = [u(x,y), v(x,y)] \]
that minimizes the following energy functional:
\[ E(u,v) = \int \int \mu(U_x^2 + U_y^2 + V_x^2 + V_y^2)dxdy + \int \int |\nabla f|^2 |V - \nabla f|^2 dxdy \] (4.1)
where \( f \) is the edge map, \( |\nabla f| \) is high near the edges and nearly zero in homogeneous regions and \( \mu \) is a positive weight to control the balance between smoothness energy and edge energy. By the calculus of variation, the minimization of Equation (4.1) reduces to solving the following Euler-Lagrange equation:
\[ \mu \nabla^2 V - (V - \nabla f)(f_x^2 + f_y^2) = 0 \] (4.2)

The Euler-Lagrange equations evolving Eq. 4.2, embedded into a dynamic scheme by treating \( V(x,y) \) as the function of \( t, x \) and \( y \), formally are
\[ \frac{\partial u}{\partial t} = \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0 \]
\[ \frac{\partial v}{\partial t} = \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0 \] (4.3)
where \( \nabla^2 \) is the Laplacian operator. The first term is called as the diffusion term and the second term, the data attraction term. The active contour model with \( V(x,y) \) as external force is called GVF active contour model.

Particular advantages of the GVF snake over a traditional snake are its insensitivity to initialization and its ability to move into boundary concavities. Its initializations are found inside, outside, or across the objects boundary. Unlike pressure forces, the GVF snake does not need prior knowledge about whether to shrink or expand toward the boundary. The GVF snake has a large capture range, which means that, barring interference from other objects, it is initialized far away from the boundary. This increased capture range is achieved through a diffusion process that does not blur the edges themselves, so multi-resolution methods are not needed.
This variational formulation follows a standard principle, that of making the result smooth when there is no data. In particular, it is observed that when $|\nabla f|$ is small, the energy is dominated by sum of the squares of the partial derivatives of the vector field, yielding a slowly varying field. On the other hand, when $|\nabla f|$ is large, the second term dominates the integrand, and is minimized by setting $v = \nabla f$. This produces the desired effect of keeping $v$ nearly equal to the gradient of the edge map when it is large, but forcing the field to be slowly-varying in homogeneous regions. The parameter $\mu$ is a regularization parameter governing the tradeoff between the first term and the second term in the integrand. This parameter should be set according to the amount of noise present in the image (more noise, increase $\mu$). It has recently been shown that this term corresponds to an equal penalty on the divergence and curl of the vector field. Therefore, the vector field resulting from this minimization is expected to be neither entirely irrotational nor entirely solenoidal.

Using the calculus of variations, the GVF field is found by solving the following Euler equations

\[
\mu \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0 \\
\mu \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0
\] (4.4)

where $\nabla^2$ is the Laplacian operator. These equations provide further intuition behind the GVF formulation. It is noted that in a homogeneous region [where $I(x,y)$ is constant], the second term in each equation is zero because the gradient of $f(x,y)$ is zero. Therefore, within such a region, $u$ and $v$ are each determined by Laplaces equation, and the resulting GVF field is interpolated from the regions boundary, reflecting a kind of competition among the boundary vectors. This explains why GVF yields vectors that point into boundary concavities. First, convergence is made to be faster on coarser images i.e., when $\Delta x$ and $\Delta y$ are larger. Second, when $\mu$ is large and the GVF is expected to be a smoother field, the convergence rate
will be slower. Figure 4.1 shows how snake evolves to capture the OD boundary.

![Figure 4.1: The evolution of the snake to capture the OD boundary (a) 0 iterations (b) 100 iterations (c) 200 iterations (d) 500 iterations](image)

The experimental results of Xu-ACM shows that the Xu-ACM has a closer contour drawn for the normal and the diseased eyes. From the Figure 4.2, investigations reveal that the detection of ONH deformation from the contours drawn, helps the ophthalmologists to detect the early glaucoma. The gold standard of the experts’ contour is drawn in black colour and the Xu-ACM’s contour is drawn in blue colour. The detection of glaucoma at an early stage helps in stopping the progression of the disease at an early stage. The Xu-ACM has less computa-
Figure 4.2: Average Contours drawn by experts, Xu-ACM and Shi-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image
tional complexity. The subsequent section discusses the results of the Geodesic Active Contour model and a comparison of the experimental results of Caselles-ACM, with the Xu-ACM.

### 4.4 GEODESIC ACTIVE CONTOUR MODEL

The Caselles-ACM, combines the Geodesic Active Contour model and the classical energy based snakes (Caselles. V et al. (1997)). This approach is a geodesic formulation of active contours by introducing a term to the curve evolution models with the following consequences: improvement in the detection of boundaries, whose gradients differ significantly; no requirement to estimate crucial parameters. Experimental study by Caselles. V et al. (1997) with various imagery demonstrates its capability to simultaneously detect several objects and both interior and exterior boundaries. The geodesic active contour model provides a substitute model for edge detection that is derived from the conventional active contour model. This approach is equivalent to finding geodesic distance of two points on a carefully chosen Riemannian space.

This model does not suffer from some of the limitations of the conventional active contour model. Because of the inherent nature of this model, changes in geometry of contours are handled automatically; hence multiple objects could be recognized concurrently. Furthermore, the approach requires only two parameters, the iteration count and the convergence criteria. So the need for pre-processing the image is minimized. This approach is less inclined to variation on the edges, allowing object with non-ideal edges to be predicted. In the image processing related works, there are two main methods of representing curves: intrinsic and extrinsic. In many cases, working with an intrinsic representation of a curve is more desirable.
This is mainly because the intrinsic representation of a curve allows us to use functions that are defined on all points of an image. However, the extrinsic representation of a curve only keeps track of values of functions at boundary points (Caselles.V et al. (1997)). The purpose of an edge detector function is to halt the evolution of curves when they reach the edges of objects. Edge detector functions use the deviations in the values of the intensity between neighbouring pixels as a way of determining object boundaries. Two of the most common edge detector functions used in image processing applications are \( g(I) = 1/(1 + |\nabla I|) \) and \( g(I) = 1/(1 + |\nabla I|^2) \), where \( \hat{I} \) is the smoothed version of image \( I \) computed using Gaussian filters. Note that the value of \( |\nabla I| \) at the edge of an object is large, so \( g(I) \) is adjacent to zero at the boundaries.

Riemannian space, is a curved spaced that is locally analogous enough to a linear space. Because of this, description of local notions of angles, curvature, and length in a Riemannian space are carried out. In particular, the distance in Riemannian space is called geodesic distance. Geodesic distance is a generalization of the notion of a ”straight line” in ”curved spaces”. In a Riemannian space, geodesic distance between two points is defined as length of the shortest path between them. The signed distance function value at a point is equal to the signed value of the distance between that point and its closest point on the curve. Note that for a signed distance function \( f, |\nabla f| = 1 \). The level-set representation of geodesic active contours is very different from the extrinsic definition of classical active contours. Nevertheless, the geodesic active contour model is based on a specific variation of active contours. The evolution equation of geodesic active contours is derived from the classical active contour model and it is equivalent to a geodesic computation on a specific Riemannian space.

The classical snakes approach (Kass.M et al. (1987b)) associates the curve
$C$ with an energy given by

$$E(C) = \alpha \int_1^0 |C'(q)|^2 dq + \beta \int_1^0 |C''(q)|^2 dq - \lambda \int_1^0 |\nabla I(C(q))| dq \quad (4.5)$$

where $\alpha, \beta,$ and $\lambda$ are real positive constants. The first two terms control the smoothness of the contours to be detected (internal energy), while the third term is responsible for attracting the contour towards the object in the image (external energy). Solving the problem of snakes amounts to finding, for a given set of constants $\alpha, \beta,$ and $\lambda,$ the curve $C$ that minimizes $E$. Another possible problem of the energy based models is the need to select the parameters that control the trade-off between smoothness and proximity to the object. The curve smoothing will be obtained by taking $\beta = 0$. Assuming this Equation (4.5) reduces to

$$E(C) = \alpha \int_1^0 |C'(q)|^2 dq - \lambda \int_1^0 |\nabla I(C(q))| dq \quad (4.6)$$

A general energy function is determined from the above equation.

$$E(C) = \int_1^0 (E_{int}(C(q)) + E_{ext}(C(q)))dq \quad (4.7)$$

The above equation is transformed by minimizing Equation (4.6) into a problem of geodesic computation in a Riemannian space, according to a new metric.

$$\text{Min} \int_1^0 g(|\nabla I(C(q))| |C'(q)| dq. \quad (4.8)$$

Therefore, when trying to detect an object, interest need not be taken in finding the path of minimal classical length but the one that minimizes a new length definition which takes into account image characteristics. Therefore, the theory of boundary detection based on geodesic computations given above is applied to any general edge detector functions $g$. In order to minimize Equation (4.8), the Euler-Lagrange equation (Strang G (1986)) is solved. According to the solution of Euler-Lagrange equation, the steepest descent method would deform the curve $C$ using the equation
\[
\frac{\partial C(t)}{\partial t} = g(I)k\vec{N} - (\nabla g(I))\cdot \vec{N})\vec{N}
\]  

(4.9)

where \(\vec{N}\) is the unit normal vector to the curve. This curve evolution is equivalent to the following level-set evolution.

\[
\frac{\partial u}{\partial t} = kg(I) |\nabla u| + \nabla u \cdot \nabla g(I)
\]  

(4.10)

which is the level-set representation of the geodesic active contours.

The average mean of the Hausdorff distance obtained for the 169 images is listed below. The Hausdorff distance is calculated for the ONH contour drawn manually by the five experts. The RIM-ONE database gives a gold standard by combining the contours of all the experts. The Table 4.1 gives the results of the Caselles-ACM and the Xu-ACM method. From the table, it is observed that Xu-ACM produces better results than the Caselles-ACM. The Figure 4.3 illustrates the better performance of the Xu-ACM. Experimental results shows that the Xu-ACM has drawn a contour closer to expert 1’s contour. Both the Xu-ACM and Caselles-ACM have the highest contour deviation for the contour of expert 5.

Table 4.1: Hausdorff distance by Xu-ACM and Caselles-ACM

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<tr>
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<td>39.29</td>
</tr>
</tbody>
</table>

From the Figure 4.3, it is inferred that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM. Figure 4.4 illustrates the contours drawn by Xu-ACM (blue colour) and Caselles-ACM (green colour) against the five experts contour (black colour). The average of the five experts contour is taken as the gold standard (drawn in black colour), Xu-ACM contour is drawn in blue colour and the Caselles-ACM contour is drawn in green colour. The Xu-ACM-Contour is at a minimum Hausdorff distance to the gold standard than the Caselles-ACM contour.
Figure 4.3: Hausdorff Distance between Xu-ACM and Caselles-ACM

Figure 4.4: Average Contours drawn by experts, Xu-ACM and Caselles-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image
The Caselles-ACM contours are deformed and is farther away than the Xu-ACM in the early glaucoma and moderate glaucoma cases. Therefore, the Caselles-ACM will not fit for the early glaucoma and moderate glaucoma cases. Experimentation reveals that the Xu-ACM is better than the Caselles-ACM. Therefore in our approach, the Xu-ACM approach has been utilized for accurate OD segmentation. Investigations are carried out in the subsequent section to analyse the results of the chosen Xu-ACM with the Patrick-ACM, a B-Spline snake model.

4.5 B-SPLINE ACTIVE CONTOUR MODEL

This section gives a brief overview of B-Spline snakes (Brigger et al. (2000)). B-Spline Snakes is reviewed in order to justify the use of splines for solving snake problems. This gives an introduction to the approach and describes it as an improvement for a good fit for any region of high curvature.

A snake is a deformable model whose shape is controlled by internal forces and external forces using energy minimization. Let \( s(t) = (x(t), y(t)) \) represent the parametric position of a snake. Its energy function is written as shown in equation

\[
E_{\text{snake}} = \int_{\text{snake}} E(s(t)) \, dt = \int_{\text{snake}} [E_{\text{int}}(s(t)) + E_{\text{ext}}(s(t))] \, dt
\] (4.11)

The internal force of the snake defines its physical properties, and the external force deforms the contour from the initial position to the feature of interest in an image. Then, in B-Spline snakes, \( s(t) \) is partitioned into segments. Each curve segment is approximated by the piecewise polynomial that is obtained by the basic function of B-Spline, \( b_m^n(t - k) \), and a set of control points \((c_k = (x_k, y_k))\), where \( k \) is the
number of control points and \( n \) is the degree of polynomial. Then,

\[
s(t) = \sum_{k=1}^{N} c(k) b_1^3(t - k) \quad (4.12)
\]

The total energy of the snake shown in Equation (4.12) has to be minimized. To minimize this energy methods such as a multistage and optimal snake approach were selected such that the B-Spline snakes fits into any region of high curvature and improves the speed of convergence.

B-Spline snakes, their control points (knots) correspond to the B-Spline coefficients, \( c(k) = (x_k, y_k) \) \( k = 1, 2, ..., N \), where \( N \) is the number of control points. Fundamentally, each curve segment is defined by four control points. Each control point affects four curve segments. Moving a control point in a given direction will affect the four curve segments that move in the same direction. However, other curve segments are not influenced. The energy of the snake is minimized by iteratively adjusting the two most effective control points. Note that this solution is a snake function \( s(t) \) that is continuously defined over, even though our data are discrete.

A smoothness constraints is imposed in a simpler and more economical fashion, and to give an intuitive B-spline snake formulation useful for images. A variable knot spacing between the knot points is introduced. An increased knot spacing will essentially have the same smoothing effect on the solution. The simplified optimization problem is considered for further processing.

\[
s^*(x) = \text{argmin}_{s_h(t)} \sum_{k \in Z} V(k, s_h(k)) \quad (4.13)
\]

which is now constrained indirectly in the sense \( s_h(x) \) that with \( h > 1 \) is a coarser spline with knot spacing \( h \)

\[
s_h(x) = \sum_{k \in Z} c_h(k) \beta^3(x/h - k) \quad (4.14)
\]
Hence our new parameter is $h$. Typically, $h$ will be taken to be an integer $m$, which will reduce the number of degrees of freedom (B-spline coefficients) in the same proportion. If the same substitution is performed as before, it is found that in the case of a quadratic potential function the new solution corresponds to a weighted least square spline approximation of the unconstrained curve $f(x)$. In the general case where $V(x,y)$ is not quadratic, it still have some form of minimum error approximation, except that the metric is no longer Euclidean. To differentiate this new solution from the previous one, it is called a parametric spline.

This terminology is justified by the fact that the smoothness constraint is entirely implicit and that the number of degrees of freedom is lesser than the number of contour points. The advantages of this parametric formulation are as follows. First, the number of parameters are reduced, which simplifies the implementation, but accelerates computation. Second, it is relatively easy to get an intuitive feeling for the smoothing effect of the parameter $h$. Another way to understand the nature of this smoothing is to use the close relation that exists between spline and band-limited approximations. With this interpretation, $s^*_h$ is more or less equivalent to the band-limited version of $f(x)$ with a cutoff frequency at $w_o = \pi/h$. Specifically, a general B-spline snake is represented as follows:

$$s_h(t) = (s_x(t), s_y(t)) \quad (4.15)$$

where $s_x(t)$ and $s_y(t)$ are the $x$ and $y$ spline components, respectively; these are both parameterized by the curvilinear variable $t$.

The node points are part of the snake and correspond to the knots of the spline curve. By positioning them appropriately, a very direct and intuitive way of controlling the shape of the curve is found. For B-splines of degree zero and one, the control points are identical to the curve points at that location. For higher degree splines, however, the control points are significantly distinct from the actual coordinates of the spline curve, especially for large values of $h$. The problem
consists in evaluating Equation (4.15) at \( M \) discrete points. Such an evaluation is necessary for the computation of the energy function and for the display of the curve (where \( M \) is typically chosen larger). Therefore, the continuous variable \( t \) is replaced by a discrete variable \( i, 0 \leq i \leq M \). The value of \( M \) and the number \( N \) of given node points directly determines the knot spacing \( h \). The discrete B-spline snake with \( N \) node points and \( M \) curve points is given as

\[
s(i) = \sum_{k \in \mathbb{Z}} c(k) \cdot \beta_n\left(\frac{i}{h} - k\right), h = \frac{M}{N}
\]  

(4.16)

The main computational drawback of this procedure is that the function needs to be evaluated for each term in the sum. The above described algorithm works for any combination of values of \( M \) and \( N \). If it is imposed on \( M \) such that \( h \) is an integer value, a much more efficient algorithm evolves. In general, this requirement is easily met, since \( M \) is not critical and is loosely chosen. The simplification is based on a convolution property for B-splines. It states that any spline of \( n \) degree and knot spacing \( h \) (integer) is represented as the convolution of \( n + 1 \) moving average filters of size \( h \) followed by a spline of knot spacing one. Hence, the curve points are obtained by three successive steps:

1. upsampling of the B-spline coefficients;
2. averaging \( n + 1 \) by moving average filters of size \( h \);

This algorithm is implemented with as few instructions and hence generally, it is faster than method one. In all of the applications, the user had to specify an initial contour. For a specific application, it is advantageous to perform dedicated pre-processing so as to obtain an automatic initialization. Moreover, its formulation obviates the need for internal energies. It works well with poorly defined contours, e.g., biomedical images, and requires good initialization of the contours.
The average mean of the Hausdorff distance obtained for the 169 images by the chosen Xu-ACM, Caselles-ACM and Patrick-ACM are listed in the Table 4.2. From the table it is observed that Xu-ACM produces better results than the Caselles-ACM and Patrick-ACM. The Figure 4.6 illustrates the better performance of the Xu-ACM than these two methods.

![Average Contours drawn by experts, Xu-ACM and Patrick-ACM](image)

(a) (b) (c) (d)

Figure 4.5: Average Contours drawn by experts, Xu-ACM and Patrick-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image

From the Figure 4.6, it is inferred that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM and Patrick-ACM. The Figure 4.5 shows the contours of the Xu-ACM (blue colour) and Patrick-ACM (green colour) for
Table 4.2: Hausdorff distance by Xu-ACM and Patrick-ACM

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Figure 4.6: Hausdorff Distance between Xu-ACM and Patrick-ACM.

a normal eye, early glaucoma affected eye, moderate glaucoma affected eye and a deep glaucoma affected eye. The experts’ contour are drawn in black colour. The figure illustrates that the Xu-ACM has minimum Hausdorff distance than the Patrick-ACM in all the expert’s contour (black colour). The Patrick-ACM performs better for the normal eyes than the diseased eyes. Therefore, Patrick-ACM can be used efficiently, only for the normal eyes. Investigations are carried out in the subsequent section to analyse the results of the Xu-ACM with the Chan-ACM, which is an efficient energy-based segmentation method.
4.6 GLOBAL REGION-BASED ACTIVE CONTOUR MODEL

The Chan-ACM model detects objects whose boundaries are not necessarily defined by gradient, but is a minimization of an energy based segmentation. In the level set formulation, the problem becomes a mean-curvature flow-like evolving the active contour, which will stop on the desired boundary. However, the stopping term does not depend on the gradient of the image, as in the classical active contour models, but is instead related to a particular segmentation of the image. The numerical algorithm uses finite differences for its contour evolution. The initial curve can be placed anywhere in the image, and interior contours are automatically detected (Chan and Vese (2001)).

The basic idea in active contour models or snakes is to evolve a curve, subject to constraints from a given image $u_0$, in order to detect objects in that image. For instance, starting with a curve around the object to be detected, the curve moves toward its interior normal and has to stop on the boundary of the object. In the classical snakes and active contour models an edge-detector is used, depending on the gradient of the image $u_0$, to stop the evolving curve on the boundary of the desired object.

Chan-ACM adopts the minimization of an energy based segmentation, for the accurate segmentation of the OD. Assume that the image $u_0$ is formed by two regions of approximately piecewise-constant intensities, of distinct values $u^i_0$ and $u^o_0$. Assume further that the object to be detected is represented by the region with the value $u^i_0$. Let the boundary be denoted by $C_0$.

$$F_1(C) + F_2(C) = \int_{inside(C)} |u_0(x,y) - c_1|_2 dxdy + \int_{outside(C)} |u_0(x,y) - c_2|_2 dxdy$$

(4.17)

where $C$ is any other variable curve, and the constants $c_1, c_2$ depending on $C$ are
the averages of $u_0$ inside $C$ and respectively outside $C$. In this simple case, it is obvious that $C_0$, the boundary of the object, is the minimizer of the fitting term. This is observed easily. For instance, if the curve $C$ is outside the object, then $F_1(C) > 0$ and $F_2(C) \approx 0$. If the curve is inside the object, then $F_1(C) \approx 0$ but $F_2(C) > 0$. If the curve $C$ is both inside and outside the object, then $F_1(C) > 0$ and $F_2(C) > 0$. Finally, the fitting energy is minimized if $C = C_0$, i.e., if the curve $C$ is on the boundary of the object.

This region-based segmentation scheme, is acclaimed to be a powerful and flexible method and is applied to segment ”difficult” images, i.e., detecting objects with boundaries not necessarily defined by the gradient or with very smooth boundaries. By contrast, this category of images would normally pose challenges to a classical thresholding or a gradient based method. Furthermore, the Chan-ACM does not require the initial curve to surround the object of interest, and particularly when employed for the OD segmentation, it proceeds efficiently by first producing a first-order estimation of the PPA region.

Chan-ACM utilizes the MumfordShah segmentation techniques and the level set method, for the movement of the contour towards the OD boundary. This model is not based on an edge-function to stop the evolving curve on the desired boundary. It does not need to smooth the initial image, even if it is very noisy and in this way, the locations of boundaries are very well detected and preserved. It detects objects whose boundaries are not necessarily defined by gradient or with very smooth boundaries, for which the classical active contour models are not applicable. Finally, it automatically detects interior contours starting with only one initial curve. The position of the initial curve is found anywhere in the image, and it does not necessarily surround the objects to be detected.

The average mean of the Hausdorff distance obtained for the 169 images, for the chosen Xu-ACM, Caselles-ACM, Patrick-ACM and Chan-ACM, are listed in the Table 4.3. From the Table 4.3, it is observed that Xu-ACM produces bet-
Figure 4.7: Hausdorff Distance between Xu-ACM and Chan-ACM.

Table 4.3: Hausdorff distance by Xu-ACM and Chan-ACM

<table>
<thead>
<tr>
<th>Method</th>
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<th>Expert 2</th>
<th>Expert 3</th>
<th>Expert 4</th>
<th>Expert 5</th>
<th>Average</th>
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</thead>
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<tr>
<td>Caselles-ACM</td>
<td>37.65</td>
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<td>40.95</td>
<td>44.35</td>
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</tr>
<tr>
<td>Patrick-ACM</td>
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</tr>
<tr>
<td>Chan-ACM</td>
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<td>55.90</td>
<td>60.42</td>
<td>62.93</td>
<td>58.32</td>
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Figure 4.7 illustrates the better performance of the Xu-ACM than the Caselles-ACM, Patrick-ACM and Chan-ACM methods.

Experimental results from the Figure 4.7 show that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM, Patrick-ACM and Chan-ACM. Of all the four methods, the Chan-ACM contours are at a maximum Hausdorff distance from the contours drawn by the five different experts. Figure 4.8 illustrates the Xu-ACM (blue colour) and the Chan-ACM (green colour) for the normal eyes, early glaucoma, moderate glaucoma and deep glaucoma. The experts’ contour are drawn in black colour. Investigations are carried out and the results indicate that the Xu-ACM has a minimum Hausdorff distance than the Chan-ACM and the Chan-ACM has poorer results in the early and moderate cases of glaucoma. Therefore, the Chan-ACM will not be suitable for the early and moderate
Figure 4.8: Average Contours drawn by experts, Xu-ACM and Chan-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image
glaucoma cases. We investigated the energy based Chan-ACM model and observed that the Xu-ACM performs better in detecting the defective eyes at an early stage. In the subsequent section investigations are carried out to analyse the results of the chosen Xu-ACM with the Shi-ACM, which uses level-set based curve evolution method.

4.7 LEVEL-SET-BASED ACTIVE CONTOUR MODEL

This level-set-based curve evolution has been developed to achieve a significant computational speedup by requiring only integer operations and refraining from solving partial differential equations (PDEs). Besides the method being suitable for real-time applications, the performance of Shi-ACM remains on par with PDE-based implementations (Shi and Karl (2008)]. The basic elements of the representation and the curve evolution is represented here. The data structure of the algorithm is simple and consists of

- an integer array $\phi$ for the level-set function;
- an integer array $\tilde{F}$ for the speed function;
- two lists of grid points adjacent to the evolving curve $C$: $L_{in}$ and $L_{out}$.

Interior points are the grid points that are inside $C$, but not in $L_{in}$ and exterior points are the grid points that are outside $C$, but not in $L_{out}$. For faster computation, the value of the level-set function from a limited set of integers $\{-3, -1, 1, 3\}$ is chosen. This function locally approximates the signed distance function and is
defined as follows:

\[
\phi(x) = \begin{cases} 
3 & \text{if } x \text{ is an exterior point} \\
1 & \text{if } x \in L_{\text{out}} \\
-1 & \text{if } x \in L_{\text{in}} \\
-3 & \text{if } x \text{ is an interior point}
\end{cases}
\]

Only the sign of the evolution speed \( F \) is used and represented as an integer-valued array \( \hat{F} \) with values +1, 0 or −1. Since, the speed is independent of the geometric properties of the curve, the selection of an integer-valued level-set function will not affect the accuracy in the evaluation of \( F \). A procedure SwitchIn() effectively moves the boundary outward by one pixel. For a point \( x \in L_{\text{out}} \) it is defined as follows. The first step in SwitchIn() procedure switches \( x \) from \( L_{\text{out}} \) to \( L_{\text{in}} \). With \( x \in L_{\text{in}} \), all its neighbors that were exterior points become neighboring grid points and are added to \( L_{\text{out}} \) in the second step. By applying the SwitchIn() procedure to any point in \( L_{\text{out}} \), the boundary is moved outward by one grid point at that location. Similarly, the procedure SwitchOut() effectively moves the boundary inward by one pixel. With \( x \in L_{\text{out}} \), all its neighbors that were interior points become neighboring grid points and are added to \( L_{\text{in}} \) in the second step.

At every iteration, the speed is computed at each point in \( L_{\text{out}} \) and \( L_{\text{in}} \) and the sign of the speed is stored in the array \( \hat{F} \). After that, the two lists sequentially to evolve the curve. More specifically, a scan through the list \( L_{\text{out}} \) is carried out and a SwitchIn() procedure applied at a point if \( \hat{F} > 0 \). This scan takes care of those parts of the curve with positive speed and moves them outward by one grid point. After this scan, some of the points in \( L_{\text{in}} \) become interior points due to the newly added inside neighboring grid points, so they are deleted from \( L_{\text{in}} \). Then the list \( L_{\text{in}} \) is scanned and a SwitchIn() procedure applied at a point if \( \hat{F} < 0 \). This scan moves those parts of the curve with negative speed inward by one grid point. Similarly, points that have become exterior points are deleted from \( L_{\text{out}} \) after this scan. After
a scan through both lists, a stopping condition is checked. If it is satisfied, the evolution is stopped; otherwise, this iterative process is continued.

The curve evolution algorithm stops if either of the following conditions is satisfied. (a) The speed at all the neighboring grid points satisfies

\[
\begin{align*}
\widehat{F} &\leq 0 \forall x \in L_{\text{out}} \\
\widehat{F} &\geq 0 \forall x \in L_{\text{in}}
\end{align*}
\]

(b) A prespecified maximum number of 200 iterations is reached.

Figure 4.9: Average Contours drawn by experts, Xu-ACM and Shi-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image
The average mean of the Hausdorff distance obtained for the 169 images by the chosen Xu-ACM, Caselles-ACM, Patrick-ACM, Chan-ACM and Shi-ACM are listed in the Table 4.4. Experimental results indicate that Xu-ACM produces better results than the Caselles-ACM, Patrick-ACM, Chan-ACM and Shi-ACM. The Figure 4.10 illustrates the better performance of the Xu-ACM than Caselles-ACM, Patrick-ACM, Chan-ACM and Shi-ACM methods.

Table 4.4: Hausdorff distance by Xu-ACM and Shi-ACM

<table>
<thead>
<tr>
<th>Expert</th>
<th>Expert 1</th>
<th>Expert 2</th>
<th>Expert 3</th>
<th>Expert 4</th>
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<td>Xu-ACM</td>
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<td>55.90</td>
<td>60.42</td>
<td>62.93</td>
<td>58.32</td>
</tr>
<tr>
<td>Shi-ACM</td>
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<td>87.50</td>
<td>85.54</td>
<td>89.91</td>
<td>87.26</td>
</tr>
</tbody>
</table>

Figure 4.10: Hausdorff Distance between Xu-ACM and Shi-ACM.

From the Figure 4.10, it is inferred that in all the expert’s contour, the Xu-ACM outcasts the Caselles’s ACM, Patrick-ACM, Chan-ACM and Shi-ACM. But Chan-ACM is better than the Shi-ACM, but poorer in its contour formations. Investigations were carried out to evaluate the performance of the Xu-ACM for the different cases of glaucoma. Experimental results are shown in Figure 4.9. The experts’ contour are drawn in black colour, the Xu-ACM’s in blue colour and the
Shi-ACM in green colour. It is inferred that for the moderate glaucoma case, the Hausdorff distance is more than the Hausdorff distance of the normal eyes, early glaucoma eyes and the deep glaucoma eyes and its contour is deformed. The Shi-ACM does not give better results for the moderate glaucoma cases. It gives better results in the early and deep glaucoma stages. Therefore Shi-ACM does not fit for the moderate glaucoma cases. In the subsequent section, investigations are carried out to analyse the performance of the Xu-ACM model with the Li2008-ACM, a region-based active contour model.

4.8 GRADIENT DESCENT ACTIVE CONTOUR MODEL

This Li2008-ACM has been developed to cater to segmenting images with intensity inhomogeneities and weak object boundaries. This contour model (Li et al. (2008)) exploits the intensity information in local regions at a controllable scale. The regularity of the level set function is intrinsically preserved by means of a regularization term, thereby ensuring an accurate computation and precluding an expensive initialization procedure. A region-based active contour model that draws upon intensity information in local regions at a controllable scale is discussed here.

A data fitting energy is defined in terms of a contour and two fitting functions that locally approximate the image intensities on the two sides of the contour. This energy is then incorporated into a variational level set formulation with a level set regularization term, from which a curve evolution equation is derived for energy minimization. Due to a kernel function in the data fitting term, intensity information in local regions is extracted to guide the motion of the contour, which thereby enables our model to cope with intensity inhomogeneity. In addition, the regularity of the level set function is intrinsically preserved by the level set regularization term to ensure accurate computation and avoids expensive reinitialization.
of the evolving level set function.

Let $C$ be the closed contour in the image domain, which separates the image domain into two regions: one outside and one inside the contour. An approximation function is used in the image domain to approximate image intensities both inside and outside the contour. A local intensity fitting energy function is used to control the contour.

$$
\varepsilon_{x}^{\text{Fit}}(C, f_1(x), f_2(x)) = \sum_{i=1}^{2} \lambda_i \int_{\Omega_i} K(x-y)|I(y)-f_i(x)|^2 dy \quad (4.18)
$$

The choice of the kernel function $K$ is flexible and depends on the three basic properties. The kernel function chosen is a Gaussian kernel.

- $K(-u) = K(u)$
- $K(u) \geq K(v)$, if $|u| < |v|$, and $\lim_{|u| \to \infty} K(u) = 0$;
- $\int K(x) dx = 1$

The Gaussian Kernel is given by

$$
K_{\sigma}(u) = \frac{1}{(2\pi)^{n/2} \sigma^n} e^{-\frac{|u|^2}{2\sigma^2}} \quad (4.19)
$$

with a scale factor of $\sigma > 0$. First $\varepsilon_{x}^{\text{Fit}}$ is a weighted mean square error of the approximation of the image intensities $I(y)$ outside and inside the contour $C$ by the fitting values $f_1(x)$ and $f_2(x)$, respectively, with $K(x-y)$ as the weight assigned to each intensity $I(y)$ at $y$. Second, due to the localization property of the kernel function, the contribution of the intensity $I(y)$ to the fitting energy $\varepsilon_{x}^{\text{Fit}}$ decreases and approaches to zero as the point $y$ goes away from the center point $x$. Therefore, the energy $\varepsilon_{x}^{\text{Fit}}$ is dominated by the intensities $I(y)$ of the points $y$ in the neighborhood of $x$. In particular, the Gaussian kernel $K_{\sigma}(x-y)$ decreases drastically to zero as $y$ goes away from $x$. The fitting energy $\varepsilon_{x}^{\text{Fit}}$ is localized around the point $x$. The
fitting energy in Equation (4.18) is region scalable one.

The fitting values $f_1(x)$ and $f_2(x)$ approximate the image intensities in a region centered at the point $x$, whose size is controlled by the scale parameters $\sigma$. The fitting energy with a small $\sigma$ only involves the intensities within a small neighborhood of the point $x$, while the fitting energy with a large $\sigma$ involves the image intensities in a large region centered at $x$. However, it is more appropriate to call the fitting energy a region-scalable fitting energy, since the intensities for the fitting energy are not restricted to a small local region. In fact, the intensities $I(y)$ for the fitting energy is in a region of any size: from a small neighborhood to the entire image domain. This region-scalability is a unique and desirable feature of this method.

Given a center point $x$, the fitting energy $\epsilon_{\text{Fit}}^x$ is minimized when the contour $C$ is exactly on the object boundary and the fitting values $f_1$ and $f_2$ optimally approximate the local image intensities on the two sides of $C$. To obtain the entire object boundary, a contour $C$ that minimizes the energy $\epsilon_{\text{Fit}}^x$ for all $x$ in the image domain have to be found. This is achieved by minimizing the integral of $\epsilon_{\text{Fit}}^x$ over all the center points $x$ in the image domain. In addition, it is necessary to smooth the contour by penalizing its length $|C|$, as in most of active contour models.

To preserve the regularity of the level set function $\phi$, which is necessary for accurate computation and stable level set evolution, a level set regularization term is introduced in the variational level set formulation.

$$P(\phi) = \int \frac{1}{2}(|\nabla \phi(x)| - 1)^2 dx$$

(4.20)

which characterizes the deviation of the function $\phi$ from a signed distance function. Therefore, to minimize the energy functional

$$F(\phi, f_1, f_2) = \epsilon_\phi(\phi, f_1, f_2) + \mu P(\phi)$$

(4.21)
where $\mu$ is a positive constant. A standard gradient descent (or steepest descent) method to minimize the energy functional is used. For a fixed level set function $\phi$, the functional $F(\phi, f_1, f_2)$ in Equation (4.21) with respect to the functions $f_1(x)$ and $f_2(x)$ is minimized.

A region-based active contour model is executed that draws upon intensity information in local regions at a controllable scale. The model is able to segment images with intensity inhomogeneity, and has desirable performance for images with weak object boundaries. The level set regularization term, preserves the regularity of the level set function to ensure accurate segmentation of the OD. From the Figure 4.11, it is inferred that in all the expert’s contour, the

<table>
<thead>
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<th>Table 4.5: Hausdorff distance by Xu-ACM and Li2008-ACM</th>
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<td>---------</td>
</tr>
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<td>Xu-ACM</td>
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<td>Chan-ACM</td>
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<tr>
<td>Shi-ACM</td>
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<tr>
<td>Li2008-ACM</td>
</tr>
</tbody>
</table>

Figure 4.11: Hausdorff Distance between Xu-ACM and Li2008-ACM.
Figure 4.12: Average Contours drawn by experts, Xu-ACM and Li2008-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image
Xu-ACM outperforms the Caselles’s ACM, Patrick-ACM, Chan-ACM, Shi-ACM and Li2008-ACM. Figure 4.12 illustrates the active contours drawn by the Xu-ACM (blue colour) and the Li2008-ACM (green colour) for the early, moderate and deep cases of the glaucoma. The experts’ contour are drawn in black colour. Investigations are carried out and the results indicate that the Li2008-ACM has not performed better for the normal eyes and moderate cases of glaucoma. The Xu-ACM, has outperformed the Li2008-ACM for the normal eyes, early glaucoma, moderate glaucoma and deep glaucoma as observed from the Table 4.5. Investigations are carried out in the subsequent section, to analyse the results of the chosen Xu-ACM with the Lankton-ACM, which uses local region statistics for computing the points along the evolving curve.

4.9 LOCAL REGION-BASED ACTIVE CONTOUR MODEL

This Lankton-ACM considers only local image statistics while evolving a contour, so that the framework befits well with segmenting objects having heterogeneous feature profiles. The benefits of localization amount to a robust and accurate segmentation of “challenging” images, where the approaches using global region-based energies tend to encounter a failure (Lankton and Tannenbaum (2008)). The downside of this technique is its increased sensitivity to initialization with regard to global schemes.

In cases where multiple foreground objects exist, simple separation into foreground and background is not sufficient. The localized region-based framework allows simultaneous segmentation of multiple objects. This is based on the idea of competing regions. In a standard single level set evolution scheme, the energy update equation is thought of as having two competing components: advance and retreat. The advance component, $a$ is always positive and tries to move the curve...
outward along its normal. Alternatively, the retreat component, \( r \) is always negative and tries to move the curve inward along its normal. The relative magnitudes of \( a \) and \( r \) govern curve evolution. Hence, the update equation for \( \phi \) is expressed as

\[
\frac{\partial \phi}{\partial t}(x) = \delta(x)(a + r) \tag{4.22}
\]

where \( a \) and \( r \) are given by

\[
a = \int_{\Omega} B(x,y) \delta \phi(y). \frac{(I(y) - v_x)^2}{A_v} + \frac{\lambda}{2} \text{div} \left( \frac{\nabla \phi(x)}{|\nabla \phi(x)|} \right) \tag{4.23}
\]

\[
r = \int_{\Omega} B(x,y) \delta \phi(y). \frac{(I(y) - u_x)^2}{A_u} + \frac{\lambda}{2} \text{div} \left( \frac{\nabla \phi(x)}{|\nabla \phi(x)|} \right) \tag{4.24}
\]

The length penalty used for curve regularization is included in both the \( a \) and \( r \) terms. Inclusion of this term ensures that all forces acting on the curve are represented solely by \( a \) and \( r \). The inherent competition of \( a \) and \( r \) in this formulation of curve evolution allows multiple signed distance functions to interact. Consider \( n \) signed distance functions, \( \phi(i) \) representing \( n \) evolving curves. The goal is to evolve every \( \phi_i \) such that every point in the domain is eventually in the interior of exactly one curve. To accomplish this, each \( \phi_i \) moves to come closer towards the boundary.

This scheme compares the retreat portion of the typical evolutions and causes all of the curves interacting at a point to move together according to the strongest retreat force. When only one curve is present, it advances toward the uninhabited region. The simplicity of this method is appealing, and it is capable of producing complete segmentation of a scene very naturally. This behavior is well suited for global region-based energies, but a modification of this scheme is needed when used with the proposed techniques because of their local nature.

The localized techniques are capable of segmenting heterogeneous objects. If they are used directly in the framework to produce complete segmentation,
the localized active contours could easily capture very different objects within the same contour. Specifically, the tendency of curves to advance into uninhabited areas could cause local-looking contours to move far from the intended object and fail to capture it correctly. The method is modified to work more appropriately with this technique by allowing regions of the domain to remain uncovered, but continuing to prevent overlaps.

The goal is to allow multiple contours to compete with each other at an interface, but allow them to compete with themselves when no other contours are nearby. This allows contours to stop either as they would in a single-contour framework or by competing with adjacent contours. In this formulation, the advance force of the current contour is compared to the corresponding retreat forces of all adjacent contours. Similarly, the retreat force is compared to adjacent advance forces. By choosing the strongest candidates in each case, all contours at an interface will move together in order to find the best joint solution, and lone contours will continue to evolve as before.

Local region statistics must be computed for each of the points along the evolving curve. This increases the complexity of the algorithm, and the computation time beyond that of standard global methods. Computation of local statistics is separated into two parts: initialization and updates. The proposed local region-based method begins by initializing every pixel in the narrow band with the local interior and exterior statistics. The nature of this operation varies depending on the energy implemented. Computation of local means, for instance, is simpler than computation of local histograms. An additional cost occurs whenever the narrow band moves to include an uninitialized pixel. In this case, the local statistics of this new pixel must be initialized as well. The number of initialization operations performed is, therefore, dependent on how far from its final position the contour is initialized. The initialization operation is only performed once for each pixel and, therefore, adds a constant complexity increase. However, depending on the size of
the local radius, these computations are significant.

![Figure 4.13: Average Contours drawn by experts, Xu-ACM and Lankton-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image](image)

The update step occurs when any initialized pixel is crossed by the contour moving it from the interior to the exterior or vice versa. In the implementation, the local statistical models are kept in memory for every initialized pixel. When the interface crosses a pixel, the statistical models of all pixels within the neighborhood are updated. When local means are used, each pixel must maintain the number of pixels in the local regions both inside and outside of the curve as well as the sums of pixel intensities in those two regions. Updating this model consists
of transferring values from the inside groups, the outside groups or vice versa.

For the histogram separation energy, a full histogram is kept of the local interior and exterior regions for each initialized pixel. Although this requires significantly more memory to maintain than the means model, updates are just as simple: pixel intensities are subtracted from bins of the interior histogram and added to the same bin of the associated exterior histogram or vice versa. A global region-based method would perform statistical updates (one for each pixel), whereas the corresponding local region-based flow would perform updates where is the number of pixels that exist within the neighborhood.

<table>
<thead>
<tr>
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<th>Expert 1</th>
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Table 4.6: Hausdorff distance by Xu-ACM and Lankton-ACM

Figure 4.14: Hausdorff Distance between Xu-ACM and Lankton-ACM.
From the Figure 4.14, it is inferred that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM, Patrick-ACM, Chan-ACM, Shi-ACM, Li2008-ACM and Lankton-ACM, but Lankton-ACM is a bit closer to Xu-ACM. The Li2008-ACM is better than the Chan-ACM and the Shi-ACM, but not so good as the chosen Xu-ACM method. Figure 4.13 shows the contours of Xu-ACM (blue colour) and the Lankton-ACM (green colour). The experts’ contour are drawn in black colour. Investigations on the results taken from the Lankton-ACM for the normal and diseased eyes shows closer results to Xu-ACM, but Xu-ACM is exemplary for all the cases than Lankton-ACM, as observed from the Table 4.6. Investigations are carried out in the subsequent section to analyse the results of the chosen Xu-ACM with the Bernard-ACM, which uses B-spline coefficients for minimizing the energy criterion.

4.10 RADIAL BASIS ACTIVE CONTOUR MODEL

The energy criterion is minimized in the Bernard-ACM with respect to the B-spline coefficients in contrast with other level-set-based ACM approaches. This formulation provides an overall control of the level set and avoids the reinitialization step of the level set. The advantages claimed for this approach (Bernard et al. (2009)) are:

- simplified derivations of the cost function and an exact calculation of its gradient;
- avoidance of arbitrary choices as the computations are exact. Due to the intrinsic smoothing of this approach, it efficiently deals with the additive noise during the segmentation tasks.

In the course of propagation, the level-set function develops steep or flat gradients, which, in turn, yield inaccuracies in the numerical approximation. The classical implementations, reshape the level-set through periodical re-initializations. This scheme has two drawbacks:
• it increases the computational cost of the method;

• it reduces the topological flexibility of the method since it prevents the level-set from creating new contours (i.e., new zero-level components) far away from the initial interface.

The bounding of the level-set function prevents steep gradients and the re-initialization step. Moreover, due to the linearity of the expansion, bounding the level-set is easily performed through a simple re-normalization of the expansion coefficients. This feature is used in the case of Radial Basis Function (RBF) by using the $\ell_1$-norm of the expansion coefficients. The advantage of the B-spline formulation is taken and re-normalization of the level-set by constraining the $\ell_\infty$-norm of the expansion. Such a procedure has the following advantages:

• it has a modest computational cost;

• it does not prevent the creation of new zero level components, making the solution topologically more flexible.

The level-set function is normalized to the range $[1, 1]$ by

$$c^{(i+1)} \leftarrow \frac{c^{(i+1)}}{\|c^{(i+1)}\|_\infty} \quad \text{(4.25)}$$

Re-normalization imposes a bound on the gradient norm of the level-set. The gradient-descent algorithm minimizes the energy with feedback step adjustment. At each step $c^{(i)}$ is used to compute the candidate update $c^{(i+1)}$. If this update decreases, then the step is considered successful and the corresponding B-spline coefficients $c^{(i+1)}$ are accepted. A normalization procedure is applied every time the $c^{(i)}$ is accepted. This strategy makes the traditional level-set re-initialization unnecessary. Since the gradient-descent algorithm described above requires the
Figure 4.15: Average Contours drawn by experts, Xu-ACM and Bernard-ACM
(a) Normal Image
(b) Early Glaucoma Image
(c) Moderate Glaucoma Image
(d) Deep Glaucoma Image
computation of the energy through the sampled version, it requires the evaluation of the level-set itself.

By a similar analysis, the total cost of evaluating the level-set function corresponds again to \((n + 1)^N d\) operations. The B-spline formulation offers specific features. The level-set evolution is expressed as a sequence of 1-D convolutions, yielding an efficient algorithm. Moreover, this evolution corresponds to a smoothing-filter operation, the amount of smoothing being explicitly controlled via the scale and the degree of the selected B-spline basis.

### Table 4.7: Hausdorff distance by Xu-ACM and Bernard-ACM

<table>
<thead>
<tr>
<th>Method</th>
<th>Expert 1</th>
<th>Expert 2</th>
<th>Expert 3</th>
<th>Expert 4</th>
<th>Expert 5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xu-ACM</td>
<td>30.64</td>
<td>32.13</td>
<td>31.39</td>
<td>34.73</td>
<td>38.58</td>
<td>33.49</td>
</tr>
<tr>
<td>Caselles-ACM</td>
<td>37.65</td>
<td>37.31</td>
<td>36.17</td>
<td>40.95</td>
<td>44.35</td>
<td>39.29</td>
</tr>
<tr>
<td>Patrick-ACM</td>
<td>35.52</td>
<td>35.50</td>
<td>34.42</td>
<td>38.78</td>
<td>42.67</td>
<td>37.38</td>
</tr>
<tr>
<td>Chan-ACM</td>
<td>55.70</td>
<td>56.66</td>
<td>55.90</td>
<td>60.42</td>
<td>62.93</td>
<td>58.32</td>
</tr>
<tr>
<td>Shi-ACM</td>
<td>85.64</td>
<td>87.72</td>
<td>87.50</td>
<td>85.54</td>
<td>89.91</td>
<td>87.26</td>
</tr>
<tr>
<td>Li2008-ACM</td>
<td>41.92</td>
<td>46.90</td>
<td>46.19</td>
<td>44.32</td>
<td>48.34</td>
<td>45.53</td>
</tr>
<tr>
<td>Lankton-ACM</td>
<td>36.08</td>
<td>35.56</td>
<td>33.91</td>
<td>39.64</td>
<td>43.03</td>
<td>37.60</td>
</tr>
<tr>
<td>Bernard-ACM</td>
<td>144.30</td>
<td>145.99</td>
<td>144.76</td>
<td>145.26</td>
<td>143.61</td>
<td>144.79</td>
</tr>
</tbody>
</table>

![Figure 4.16: Hausdorff Distance between Xu-ACM and Bernard-ACM](image-url)
From the Figure 4.16 and the Table 4.7, it is inferred that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM, Patrick-ACM, Chan-ACM, Li2008-ACM, Lankton-ACM, Bernard-ACM and Shi-ACM. The Shi-ACM and Chan-ACM are better than Bernard-ACM. Xu -ACM is very much better than the Bernard-ACM as observed in the figure. The Figure 4.15 shows that the Bernard-ACM performs better only for the Early Glaucoma cases. In all the other cases the contours drawn are very poor. Therefore, the Bernard-ACM is not advisable to be used for the normal eyes, moderate eyes and deep glaucoma eyes. Investigations are carried out in the subsequent section to analyse the results of the chosen Xu-ACM with the Li2010-ACM that uses the level-set based method with a regularization term for energy minimization.

### 4.11 PARAMETRIC ACTIVE CONTOUR MODEL

This Li2010-ACM model (Li et al. (2010)) efficiently counteracts the irregularities faced by a conventional level-set-based ACM by introducing a distance regularization term, which maintains the desired shape of the level set function. As a consequence, the level set evolution is subject to a forward-and-backward diffusion effect, thereby intrinsically maintaining the regularity of the level set function in the course of its evolution. The Li2010-ACM has been claimed to offer a significant computational advantage by allowing relatively large time steps, besides preserving the numerical accuracy in image segmentation implementations.

A desirable advantage of level set methods is that they represent contours of complex topology and are able to handle topological changes, such as splitting and merging, in a natural and efficient way, which is not allowed in parametric active contour models unless extra indirect procedures are introduced in the implementations. Another desirable feature of level set methods is that numerical computations are performed on a fixed Cartesian grid without having to
parametrize the points on a contour as in parametric active contour models. These desirable features, among others, have greatly promoted further development of level set methods and their applications in image segmentation.

Figure 4.17: Average Contours drawn by experts, Xu-ACM and Li2010-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image

Although level set methods have been used to solve a wide range of scientific and engineering problems, their applications have been plagued with the irregularities of the level set function that are developed during the level set evolution. In conventional level set methods, the level set function typically develops irregularities during its evolution, which cause numerical errors and eventually destroy the stability of the level set evolution. To overcome this difficulty, a numerical
remedy, commonly known as reinitialization, is introduced to restore the regularity of the level set function and maintain stable level set evolution. Reinitialization is performed by periodically stopping the evolution and reshaping the degraded level set function as a signed distance function.

In level set methods, a contour (or more generally a hyper surface) of interest is embedded as the zero level set of an level set function. Although the final result of a level set method is the zero level set of the level set function, it is necessary to maintain the level set function in a good condition, so that the level set evolution is stable and the numerical computation is accurate. This requires that the level set function is smooth and not too steep or too flat (at least in a vicinity of its zero level set) during the level set evolution. This condition is well satisfied by signed distance functions for their unique property $|\phi = 1|$, which is referred to as the signed distance property.

For the 2-D case as an example, a signed distance function $z = \phi(x, y)$ as a surface is considered. Then, its tangent plane makes an equal angle of $45^\circ$ with both the $xy$-plane and the $z$-axis, which are easily verified by the signed distance property $|\phi = 1|$. For this desirable property, signed distance functions have been widely used as level set functions in level set methods. In conventional level set formulations, the level set function is typically initialized and periodically reinitialized as a signed distance function. In this section, a level set formulation that has an intrinsic mechanism of maintaining this desirable property of the level set function is proposed.

The energy functional $\varepsilon(\phi)$ by

$$
\varepsilon(\phi) = \mu \mathcal{R}_p(\phi) + \varepsilon_{ext}(\phi)
$$

(4.26)

where $\mathcal{R}_p(\phi)$ is the level set regularization term defined in the following, $\mu > 0$ is a constant, and $\varepsilon_{ext}(\phi)$ is the external energy that depends upon the data of interest.
The level set regularization term $\mathcal{R}_p(\phi)$ is defined by

$$\mathcal{R}_p(\phi) = \int_{\Omega} p(|\nabla \phi|) dx \quad (4.27)$$

where $p$ is a potential (or energy density) function. The energy is designed such that it achieves a minimum when the zero level set of the level set function $\phi$ is located at desired position (e.g., an object boundary for image segmentation applications). An external energy will be defined in such a way that it paves a way to the application of the general formulation to image segmentation. The minimization of the energy is achieved by solving a level set evolution equation.

<table>
<thead>
<tr>
<th>Table 4.8: Hausdorff distance by Xu-ACM and Li2010-ACM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Hausdorff Distance for 169 images</strong></td>
</tr>
<tr>
<td>Expert 1</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Xu-ACM</td>
</tr>
<tr>
<td>Caselles-ACM</td>
</tr>
<tr>
<td>Patrick-ACM</td>
</tr>
<tr>
<td>Chan-ACM</td>
</tr>
<tr>
<td>Shi-ACM</td>
</tr>
<tr>
<td>Li2008-ACM</td>
</tr>
<tr>
<td>Lankton-ACM</td>
</tr>
<tr>
<td>Bernard-ACM</td>
</tr>
<tr>
<td>Li2010-ACM</td>
</tr>
</tbody>
</table>

A naive choice of the potential function is $p(s) = s^2$ for the regularization term $\mathcal{R}_p$, which forces $|\nabla \phi|$ to be zero. Such a level set regularization term has a strong smoothing effect, but it tends to flatten the level set function and finally make the zero level contour disappear. In fact, the purpose of imposing the level set regularization term is not only to smooth the level set function $\phi$, but to maintain the signed distance property $|\nabla \phi| = 1$, at least in a vicinity of the zero level set, in order to ensure accurate computation for curve evolution. This goal is achieved by using a potential function $p(s)$ with a minimum point $s = 1$, such that the level set regularization term $\mathcal{R}_p(\phi)$ is minimized when $|\nabla \phi| = 1$. Therefore, the potential
Figure 4.18: Hausdorff Distance between Xu-ACM and Li2010-ACM.

function should have a minimum point at $s = 1$ (it has other minimum points). The definition of the potential $p$ for the distance regularization is given by

$$p = p_1(s) = \frac{1}{2} (s - 1)^2$$  \hspace{1cm} (4.28)

A unique minimum point is achieved when $s = 1$. With the potential $p = p_1(s)$, the level set regularization term $\mathcal{R}_p(\phi)$ is expressed as

$$\mathcal{R}_p(\phi) = \frac{1}{2} \int_\Omega (|\nabla \phi - 1|)^2 \, dx$$  \hspace{1cm} (4.29)

which characterizes the deviation of $\phi$ from a signed distance function. A new potential function $p$, in the distance regularization term $\mathcal{R}_p$, is defined to overcome the side effects. This new potential function is aimed to maintain the signed distance property $|\nabla \phi| = 1$ only in a vicinity of the zero level set, while keeping the level set function as a constant, with $|\nabla \phi| = 0$, at locations far away from the zero level set. To maintain such a profile of the LSF, the potential function $p(s)$ must have minimum points at $s = 1$ and $s = 0$. Such a potential is a double-well potential as it has two minimum points (wells). A double-well potential $p = p_2$ not only avoids the side effect that occurs in the case of $p = p_1$, but offers some appealing theoretical and numerical properties of the level set evolution.
From the Figure 4.18, it is inferred that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM, Patrick-ACM, Chan-ACM, Shi-ACM, Li2008-ACM, Lankton-ACM, Bernard-ACM and Li2010-ACM. The Li2010-ACM is a bit closer to Chan-ACM in its performance as observed from the Table 4.8. Experimental results from the Figure 4.17, illustrates that the Li2010 is better only for the Normal eye cases and the experts’ contour are drawn in black colour, Xu-ACM in blue colour, Li2010-ACM in green colour. In all the other cases with defective eyes, the contours drawn are very poor. Therefore, the Li2010-ACM is not advisable to be used for diseased eyes. Investigations are carried out in the subsequent section and the analysis are done on the results of the chosen Xu-ACM with the Wu-ACM, a geometric and parametric active contour model.

4.12 GEOMETRIC AND PARAMETRIC CONTOUR MODELS

According to the representation and implementation, active contour models are classified into two categories: the parametric active contour models and the geometric active contour models. The focus is on the parametric active contour models, and the approach is integrated into geometric active contour models. Since the external force plays a leading role in driving the active contours to approach objects boundaries in the parametric active contour models, designing a novel external force field. A refinement of the GVF model has recently been proposed to rectify the susceptibility of the Xu-ACM model to weak edges as well as deep and narrow boundary concavities. An external force is incorporated into the model to adjust the diffusion process of the flow field adaptively in accordance with image characteristics. The salient advantage of Wu-ACM (Wu et al. (2013b)) is that the active contours are driven into deep and narrow concave regions of objects.

The advantages of this method are as follows:
1. A hyper surface minimal functional to substitute smoothness energy term in the original Gradient Vector Flow is used. It tends to degenerate to a uniformly elliptic equation having strong regularizing properties in all directions at locations where the variation of the intensity is weak. And, in a neighborhood of an edge presenting a strong gradient, the hyper surface minimal functional is preferable to diffuse along tangent direction of an edge so as to preserve the weak edge efficiently.

2. The $p(x)$ harmonic maps in which $p(x)$ ranges from 1 to 2 is utilized, such that the diffusion process of the flow field is adjusted adaptively according to image characteristics.

3. An infinity Laplace functional to guarantee the ADF diffusion mainly along normal direction in the homogeneous of an image is introduced so that it drives the active contours onto deep and narrow concavity. The ADF is able to efficiently suppress the influence of noise because the diffusion along tangent direction is inclined to smooth the noise while preserving edges.

The GVF field $V(x,y) = [u(x,y), v(x,y)]$ is simplified as

$$E(u,v) = \int \int \mu \cdot |\nabla V|^2 dxdy + \int \int |V - \nabla f|^2 dxdy \quad (4.30)$$

In this equation, the first term is the smoothness energy and the second term is the edge energy. An isotropic smoothing effect in the field $V(x,y)$ which is desired for homogeneous regions, but for edge regions is being used. Hence, it provides an equivalent framework between GVF diffusion process and image restoration model. Better diffusion properties are needed, while preserving the weaker edges in the image. The locations where the image gradients are low, the function $\phi(|\nabla v|)$ has an isotropic smoothing property. This function is called the hyper surface minimal function. To obtain a better adaptive diffusion, a more effective functional, in a departure from minimal surface and the $p(x)$ harmonic maps is built.
Table 4.9: Hausdorff distance by Xu-ACM and Wu-ACM

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Expert 1</th>
<th>Expert 2</th>
<th>Expert 3</th>
<th>Expert 4</th>
<th>Expert 5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xu-ACM</td>
<td>30.64</td>
<td>32.13</td>
<td>31.39</td>
<td>34.73</td>
<td>38.58</td>
<td>33.49</td>
</tr>
<tr>
<td>Caselles-ACM</td>
<td>37.65</td>
<td>37.31</td>
<td>36.17</td>
<td>40.95</td>
<td>44.35</td>
<td>39.29</td>
</tr>
<tr>
<td>Patrick-ACM</td>
<td>35.52</td>
<td>35.50</td>
<td>34.42</td>
<td>38.78</td>
<td>42.67</td>
<td>37.38</td>
</tr>
<tr>
<td>Chan-ACM</td>
<td>55.70</td>
<td>56.66</td>
<td>55.90</td>
<td>60.42</td>
<td>62.93</td>
<td>58.32</td>
</tr>
<tr>
<td>Shi-ACM</td>
<td>85.64</td>
<td>87.72</td>
<td>87.50</td>
<td>85.54</td>
<td>89.91</td>
<td>87.26</td>
</tr>
<tr>
<td>Li2008-ACM</td>
<td>41.92</td>
<td>46.90</td>
<td>46.19</td>
<td>44.32</td>
<td>48.34</td>
<td>45.53</td>
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<tr>
<td>Lankton-ACM</td>
<td>36.08</td>
<td>35.56</td>
<td>33.91</td>
<td>39.64</td>
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<td>37.60</td>
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<tr>
<td>Bernard-ACM</td>
<td>144.30</td>
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<td>144.76</td>
<td>145.26</td>
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<tr>
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<td>Wu-ACM</td>
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</table>

Figure 4.19: Hausdorff Distance between Xu-ACM and Wu-ACM.
Figure 4.20: Average Contours drawn by experts, Xu-ACM and Wu-ACM (a) Normal Image (b) Early Glaucoma Image (c) Moderate Glaucoma Image (d) Deep Glaucoma Image
From the Figure 4.19, it is inferred that in all the expert’s contour, the Xu-ACM outperforms the Caselles’s ACM, Patrick-ACM, Chan-ACM, Shi-ACM, Li2008-ACM, Lankton-ACM, Bernard-ACM, Li2010-ACM and Wu-ACM. The Wu-ACM is better than the Shi-ACM and the Bernard-ACM, but it is worse than all the other methods as observed from the Table 4.9. Investigations on the results from Figure 4.20, shows that the contours for the normal and diseased eyes are deformed and at a greater Hausdorff distance than Xu-ACM. The experts’ contour are drawn in black colour, the Xu-ACM in blue colour and the Wu-ACM in green colour. Investigations are carried out and the contributions and conclusions of this chapter is described in the subsequent section.

4.13 CONCLUSION

From all these discussions, it is observed that the Xu-ACM performs better than all the other nine ACM methods. It has also been observed that the average Hausdorff distance is minimum with the early glaucoma cases. Experimental analysis show that the Xu-ACM method detects the early glaucoma faster and stops the progression of the disease. The contours are perfect in the case of Xu-ACM for the normal and diseased eyes.

The Xu-ACM method provides a minimum Hausdorff distance for both the normal and the diseased eyes. It is acclaimed to be best than all the other nine models. The Chan-ACM contours do not smoothly capture the boundary as performed by the Xu-ACM. The Patrick-ACM needs more iterations and hence requires more time to capture the boundary. The Patrick-ACM have a greater Hausdorff distance from the expert’s contour, than the Xu-ACM. The Chan-ACM is not advisable for the moderate glaucoma cases. The Chan-ACM also have a greater Hausdorff distance from the expert’s contour, than the Xu-ACM.
Shi-ACM contours are very poor. Li2008-ACM do not perform well for the normal and moderate glaucoma cases. Lankton-ACM falls a bit closer to Xu-ACM, but Xu-ACM takes lesser time in segmenting the OD than the Lankton-ACM. Observing the results of Bernard-ACM, the contours are very poorly drawn for the normal and deep glaucoma cases and it could not segment the OD for the moderate glaucoma case. Therefore, Xu-ACM outperforms the Bernard-ACM with a minimum Hausdorff distance in all the cases. The contours of Li-2010-ACM are deformed in all the cases and it does not capture the boundary correctly. When comparing the Wu-ACM with the Xu-ACM, the Hausdorff distance is minimum for the Xu-ACM in both the normal and diseased eyes.

The contributions in this chapter are as follows:

- Ten active contour models that fine tunes the contour initialized by the CHT is investigated and their results are analysed for ONH segmentation.
- Gradient vector flow model is acclaimed to be a fast and hybrid level set method that accurately fits the boundary of the OD with less computational overhead and detecting Early Glaucoma effectively.
- An optic nerve head segmentation algorithm that reliably detects the optic disc (OD) boundaries with less computational overhead is proposed.

Having chosen the Xu-ACM as the model for accurate OD segmentation, the subsequent chapter analyses the results for the different types of glaucoma in the pre-processing stage, the localization of the OD and the segmentation of the OD.