Some useful attempts in distribution system planning started with defining several distribution system parameters and establishing the relationship between them. Mathematical models are formulated to represent the feeder cost, feeder loss cost and transformation loss cost, in terms of the variable system parameters like feeder voltage regulation, conductor size, substation size, number of feeders per substation etc. It was learnt that, the continuing growth in load density, load factor, cost of system capacities and cost of power and energy distributed, with time, greatly influence the cash flows in the system over the life of the feeder, though the feeder does not undergo major changes in its configuration during its life period. These changes in life time cash flows in feeder due to the growth factors have a considerable influence on the planning of optimal systems. Hence, the growing trends in load, load factor and cost of energy have been considered in the formulations to obtain realistic results. Mathematical models to represent feeder voltage regulation, feeder losses, transformation losses, substation cost and feeder cost are reviewed in this chapter.

2.1 REVIEW OF MATHEMATICAL MODELS

Voltage Regulation Constant (H)

H is defined [91] as the moment for a drop of 1% in the feeder, which is expressed as (see Appendix-A for details)

\[ H = \frac{10 \, kV^2}{(r \cos \theta + x \sin \theta)} \]  

(2.1)

It is a function of circuit voltage, line parameters and power factor angle and is independent of the percentage voltage regulation, feeder length and the load in the feeder.
It remains constant for a given conductor size, power factor and circuit voltage. It is known as *unit voltage regulation constant* and its unit is kVA-km. The line parameters and the values computed for H are given in the Appendix-B.

**Substation Cost**

The substation cost consists of three components, a fixed cost component and two variable cost components, one proportional to the substation capacity in kVA and the other proportional to the number of feeders [91]. It can be written as

\[
C_{ss} = e + h \cdot (kVA) + f \cdot n_f
\]

(2.2)

The values of \(e\) and \(h\) can be obtained from the linearized cost versus kVA capacity characteristic of the substation, excluding the bay cost and that of \(f\) is known independently.

**Feeder Cost**

The cost of a feeder is a function of its length \(L\) and the conductor size used [91]. For a given conductor size, the feeder cost can be expressed as

\[
C_{FF} = c_f L
\]

(2.3)

**Transformation Loss Cost**

Each of the two types of losses of a transformer, the core loss \(T_i\) and the copper loss \(T_c\) has two components; a fixed loss component and a variable loss component [91]. They can be individually represented as bearing a linear relation with the transformer kVA capacity as

\[
T_i = a' + b' \cdot (kVA_t)
\]

(2.4)

\[
T_c = c' + d' \cdot (kVA_t)
\]

(2.5)

\(a'\) and \(c'\) are the loss constants in kW and \(b'\) and \(d'\) are the loss constants in kW/kVA.

The energy loss in a transformer corresponding to the power loss given by the above relationships can be obtained as

\[
E_t = [T_i + T_c \cdot (UF)^2 \cdot (LLF)] \cdot 8760
\]

(2.6)
Assuming there are $N_t$ number of transformers at every substation and substituting for $kVA_t$, the energy loss in the substation can be expressed as

$$E_{ts} = a_1' + b_1' n_s^{-1} \quad (2.7)$$

Where,

$$a_1' = 8760 N_t \left[ a' + c' (UF)^2 (LLF) \right]$$

and

$$b_1' = \frac{8760 A D}{(DF) (pf) (UF)} \left[ b' + d' (UF)^2 (LLF) \right] \quad \cdots (2.8)$$

$$LLF = A(\text{LF})^2 + B(\text{LF}) \quad (2.9)$$

Where,

$$A + B = 1 \quad \text{and} \quad \text{LF} \text{ is the load factor.}$$

The guideline values of $A$ and $B$ is 0.8 and 0.2 respectively. The relationship of load factor growth with time was given by Scheer [78]. According to him, the system load factor grows, cutting the difference between an ultimate load factor $LF_u$ and the present load factor $LF_p$ into half over a period of 16 years, and the load factor at the $k^{th}$ year is obtained as

$$LF_k = LF_u - y_k (LF_u - LF_p) \quad (2.10)$$

Where,

$$y_k = (0.5)^{k/16}$$

Following his analyses in this respect, the ultimate load factor for a rural distribution system can be assumed as 45%. Considering the increase in cost of energy and $LLF$, the present worth cost of energy losses during the expected life of the substation can be obtained as

$$C_{els} = a_1 + b_1 n_s^{-1} \quad (2.11)$$

Where,

$$a_1 = 8760 N_t \left[ a' \sum_{k=1}^{N_{LS}} \frac{C_{ek}}{(1 + u)^k} + c' (UF)^2 \sum_{k=1}^{N_{LS}} \frac{C_{ek} (LLF_k)}{(1 + u)^k} \right]$$
Where, $C_{ek}$ is the cost of energy at the $k^{th}$ year and $u$ is the annual discount factor [91].

**Feeder Loss Cost**

It is known from the fundamentals that the peak power loss in a feeder segment between the nodes $i$ and $j$ can be obtained [91] as a function of power flow (in kVA) in the segment as

$$P_{L_{ij}} = \frac{0.001r}{Kv^2} L_{ij} W_{ij}^2 \quad (2.13)$$

The present worth of the annual energy loss costs of the feeder segment during its life corresponding to power loss given in (2.13) can be obtained as

$$E_p = \frac{8.76}{kV^2} L_{ij} \sum_{k=1}^{NLF} \frac{C_{ek}(LLF_k)}{(1 + u)^k} \quad (2.14)$$

### 2.2 DEFINITION OF THE DSP PROBLEMS STUDIED

#### 2.2.1 Substation Siting, Sizing and Network Routing Problem

The objective of this study is to place the substations in the load centers and to keep the feeders within their optimal level of loading and load distribution so that the overall investment and operational costs in the systems are minimum, subject to the given constraints. Thus, the problem of planning optimal systems in this case is to determine the optimal number and location of substations and optimal number of feeders and their routings from among the feasible ones, subject to a set of physical and technical constraints.

This problem was studied by M. Ponnaivakko and K.S.P. Rao [13] defining the objective function of the problem as second order nonlinear model and have contributed a
Quadratic Mixed-Integer solution procedure. The solution was obtained in two stages. In the first stage, the Quadratic Mixed-Integer Programming problem is solved following the procedure developed by Wolfe using Simplex method and treating all the variables as continuous variables. In the second stage, a procedure has been suggested to integerize the values of the decision variables. The model had considered the effects of growth of load, load factor and growth in cost of energy. The mathematical models developed by them are used in this research to develop GA and SA based new solution procedures for the problem.

**Problem Statement**

The objective function of the problem includes the following:

i) cost of the future substations as a function of substation capacity.

ii) costs associated with future feeder bays of the existing and future substations.

iii) cost of the future feeder segments.

iv) costs associated with capacity augmentation of existing feeders.

v) Present worth of the energy loss costs associated with the existing and future feeders.

The objective function to be is minimized is subjected to the usual technical constraints that represent the Kirchhoff’s laws, the voltage regulation constraints at the load points, radial constraints of the feeders and pre-specified possible sites for substations and routes for feeders. The objective function is minimized with the following assumptions:

- Every demand center is served
- The network has a radial configuration
- The nodal voltages are within the permissible limits
- The power flows in the feeder sections are within the feeder capacities
The objective function of the problem defined [91] above can be mathematically stated as

\[
\text{Minimize} \quad C(\delta, W) = \sum_{p=N_{E}+1}^{N_{S}} (C_{F_{S}})_{p} \delta_{p} + (C_{F_{B}}) \sum_{t=1}^{N_{e}} \sum_{j=1, j \in \lambda_{t}}^{N_{e}} \delta_{tj} \alpha_{tj} + (C_{F_{B}}) \sum_{p=N_{E}+1}^{N_{S}} \sum_{j=1}^{N_{p}} \delta_{p} \sum_{j=1, j \in \lambda_{p}} \delta_{p, j} \alpha_{p, j} + (C_{F_{F}}) \sum_{(i,j) \in F} L_{ij} \delta_{ij} \delta_{ij} \]

(2.15)

Subject to,

i) To satisfy the Kirchoff's current law

\[
\sum_{j=1, j \in \lambda_{t}}^{N_{e}} W_{tj} = P_{t} \quad t = N_{S} + 1 \ldots N_{T} \quad (2.16)
\]

ii) For a connected graph to be radial

\[
\sum_{(t,p) \in F} \delta_{tp} = N_{T} - N_{S} \quad t,p = 1 \ldots N_{T} \quad (2.17)
\]

iii) For the bus voltages within bounds

\[
V_{\text{Min}} \leq V_{t} \leq V_{\text{Max}} \quad t = N_{S} + 1 \ldots N_{T} \quad (2.18)
\]

iv) For the network to be reliable

\[
\sum_{j=1, j \in \lambda_{t}}^{N_{e}} \delta_{tj} \geq 1, \quad t = N_{S} + 1 \ldots N_{T} \quad (2.19)
\]

v) For the power flows to be within the feeder capacities

\[
W_{tp} \leq U_{tp} \quad t,p = 1 \ldots N_{T} \quad (2.20)
\]
2.2.2 Conductor Gradation Problem

The distribution feeders are mostly radial. In a radial feeder, the power flow is maximum in the sections, closure to the substation and minimum in the sections towards the far end of the feeder. Use of single conductor size for a radial distribution feeder is therefore not recommended. Hence the problem of grading the conductor sizes becomes an essential area of study. This problem was studied by M. Ponnavaikko and K.S.P. Rao [39, 40]. They have developed solution procedures using Dynamic Programming (DP) and Method of Local Variations (MLV) approaches. In this research the same problem is solved by developing solution procedures using GA and SA. Mathematical models to represent feeder voltage drop, feeder installation cost and energy loss cost are formulated as functions of conductor cross-section. The optimal profile of conductor gradation is considerably influenced by several other factors like growth in feeder load with time, growth in feeder load factor and increase in cost of energy per kWh distribution in the area with the system growth. Hence, the models incorporate the above growth factors. The cost function of the problem, thus, includes the following:

i) The present worth of the cost of energy losses in a three-phase radial distribution feeder (operational cost)

ii) The installation cost of the feeder (capital cost)

The main consideration in the feeder design is to keep the feeder voltage regulation within the prescribed limits. This limit is given by the system requirement to maintain the customer voltages within the declared limits. The solution procedure considers only those conductor sizes available in the inventory of the power industry. Therefore, another constraint is the conductor cross-section that can take only discrete values. Thus, the objective function is minimized subject to the following constraints.

i) Discrete values for conductor cross-sections

ii) The voltages at all load buses are within the pre-specified limits
Problem Statement

The problem can thus be mathematically stated [91] as

\[
\text{Minimize} \quad Z = \sum_{i=1}^{ND} \left( \frac{K_{4i}}{a_{c_i}} + K_{5i} a_{c_i} \right) \quad (2.21)
\]

Subject to,

\[
\sum_{i=1}^{ND} \left( \frac{K_{2i}}{a_{c_i}} \right) \leq \Delta v_m' \quad (2.22)
\]

and \( a_{c_i} \in a_{c1}, ..., a_{c_{ND}} \) \quad (2.23)

Where, \( K_{2i}, K_{3i}, K_{4i}, \) and \( K_{5i} \) as described in Appendix-C are reproduced below.

\[
\Delta v_m' = \frac{\Delta v_m}{(1 + g)^M} - \sum_{i=1}^{ND} K_{3i} \quad (2.24)
\]

\[
K_{2i} = \sqrt{3} I_m \ell_i \rho \cos \theta \quad (2.25)
\]

\[
K_{3i} = \sqrt{3} I_m \ell_i x \sin \theta \quad (2.26)
\]

\[
K_{4i} = 26.28 \rho \ell_i I_{m_i}^2 \left[ \sum_{k=1}^{M} \frac{(1 + g)^{2k} (LLF_k) C_{e_k}}{(1 + u)^k} \right. \\
\left. + (1 + g)^{2M} \sum_{k=M+1}^{NFL} \frac{(LLF_k) C_{e_k}}{(1 + u)^k} \right] \quad (2.27)
\]

\[
K_{5i} = f' \ell_i \quad (2.28)
\]

\( \Delta v_m \) is the maximum permissible voltage drop with the ultimate load in the feeder at the \( M^{th} \) year, the period up to which the load growth is allowed on the feeder.

2.2.3 Capacitor Placement Problem (CPP)

This section presents a brief review of the cost models and the problem formulation of reference [52]. M.Ponnavaikko et al, have expanded the earlier models to
represent the savings, the capacitor cost and the voltage rise during off-peak hours as a function of capacitive current flows against the inductive load currents in the feeder sections. The models incorporate the effects of growth in load, load factor and energy cost over the planning period. Present worth of energy loss cost is considered to account its foreseen changes. They had considered the allocation of fixed capacitors during light load periods subject to voltage rise constraint and the allocation of switched capacitors during peak load hours of a load cycle. The technique presented does not depend upon the circuit voltage, load distribution or other system and load characteristics and was applicable to all radial feeders in practice.

The Capacitor Placement Problem (CPP) in distribution feeder consists of determining the place, type (fixed or switched) and size of capacitors to be installed along the feeder length. It is assumed that voltage phase along the length of a feeder does not change appreciably such that reactive current requirements of each load along the length of the feeder may be directly added. It is further assumed that the loads on the feeder vary with time in a correlated manner. The daily load variations in the distribution system are approximated by discrete load levels. Fixed capacitors are allocated for the duration of the reactive load cycle denoted by $T$ with corresponding load factor $L_f$. Switched capacitors are allocated for the duration $T_s$ with corresponding load factor $f''$. $f''$ is the load factor over the interval $T_s$. $T_s$ is the duration in which the load reactive currents in the feeder is greater than the reactive currents due to all the optimal fixed capacitors allocated on the feeder.

The solution approach is organized into two stages. The first stage consists of achieving optimum allocations of fixed capacitors. It is followed by a second stage for optimum allocation of switched capacitors at the locations where fixed capacitors are allocated. The problem has been solved by M. Ponnavaikko and K.S.P. Rao developing solution procedures using DP and MLV methods [52]. In this research the same problem is solved using GA and SA.
Problem Statement

The objectives of the CPP problem are to increase the savings due to energy loss reduction and released system capacities against the capacitor costs. The models to represent these savings and cost functions and the voltage rise constraint during off peak hours use capacitive current flows in the feeder sections as unknown variables. The objective function $F$ is the energy saving function [91] given by

$$F = \text{cost saving due to energy loss reduction in the feeder}$$

$$+ \text{cost saving due to release in system capacities}$$

$$- \text{cost of capacitor banks} \quad (2.29)$$

Fixed Capacitor Allocation: The objective function for determining the optimal fixed capacitor allocation can be expressed using the models developed in Appendix-D, Appendix-E and Appendix-F, as a function of capacitor current flows in feeder sections as

$$F = \sum_{j=1}^{n} C_{SN_j}(l_{cj}) + C_{ro}(l_{c1}) - \sum_{j=1}^{n} \left\{ e'(l_{cj} - l_{c(j+1)}) + a d' \right\} \quad (2.30)$$

The main constraint is that the voltage rise $G(I_{cj})$, given by Appendix-F, during off-peak hours at any node, is kept at less than or equal to a given limit say zero so that the maximum voltage in the system is limited to the source voltage. This constraint can be stated as

$$G(l_{cj}) \leq 0 \quad j = 1, \ldots, n \quad (2.31)$$

Switched Capacitor Allocation: The objective function for determining the optimal switched capacitor banks in addition to fixed capacitors can be expressed using the models developed in Appendix-E, Appendix-G and Appendix-H as

$$F = \sum_{j=1}^{n} C'_{SN_j}(l'_{cj}) + C_{ro}(l'_{c1} + l_{c1}) - C_{ro}(l_{c1})$$

$$- \sum_{j=1}^{n} \left\{ e'(l'_{cj} - l'_{c(j+1)}) + a d' \right\} \quad (2.32)$$
Where, $I_{cj}'$ is the current due to switched capacitors alone. Since the capacitor current due to fixed capacitors is known in advance, the net inductive currents alone have to be substituted in (2.32). Hence, the term $I_{lj}(1 + g)^k$ in (2.32) has to be replaced by a term $[I_{lj}(1 + g)^k - I_{cj}]$ where, $I_{cj}$ is the capacitor current in the section $j$, corresponding to fixed capacitors already decided. Since the switched capacitors will not be in service during the off-peak hours, the voltage rise constraint need not be considered in this case.