CHAPTER 11

MEAN TIME FOR RECRUITMENT IN A THREE GRADED MANPOWER SYSTEM HAVING CORRELATED INTER-DECISION TIMES AND SCBZ PROPERTY FOR THRESHOLD DISTRIBUTION

11.1 INTRODUCTION

For a two graded manpower system, Srinivasan and Mariappan (2001(b),(c)) have obtained the expected time for recruitment in manpower planning associated with correlated approach. In this chapter, an organization with three grades each having its own threshold level is considered. The recruitment becomes necessary if the loss of manpower crosses (i) maximum of three grades & (ii) minimum of three grades. The objective of this chapter is to find the mean time for recruitment for the organization under this univariate policy, assuming that the inter-decision times are exchangeable constantly correlated and the threshold distribution is continuous. The rest of this chapter is organised as follows: In section 11.2, Model 1 description of Model 1 is given, analytical expression for meantime to recruitment is obtained and are numerically
illustrated for Model 1. In section 11.3, description of Model 1 is given, analytical expression for meantime to recruitment is obtained and are numerically illustrated for Model 1.

MODEL 1
DESCRIPTION AND ANALYSIS OF THE MODEL

Assumptions:
1. An organization having three grades takes policy decisions at k epochs, and at every decision making epoch, a random number of persons quit the organization.
2. There is an associated loss of manhours to the organization if a person quits and it is linear and cumulative. If the total number of persons who leave the organization exceeds a threshold level in each grade the organization faces a breakdown and so the recruitment is necessary.
3. The inter-decision times are exchangeable and constantly correlated exponential random variables.
4. The mobility of manpower from one grade to the other is permitted
Notations:

$U_i$: time between the $(i-1)$th and $i$th decision epoch. $U_i$'s are exchangeable and constantly correlated exponential random variables.

$X_i$: a discrete random variable denoting the number of persons who leave the organization at the $i$th decision epoch. $i = 1, 2, \ldots$. $X_i$'s are independent and identically distributed random variables.

$Y_A, Y_B, Y_C$: exponential random variable denoting threshold level with SCBZ property for the grades A, B, C respectively.

$Y$: max $(Y_A, Y_B, Y_C)$

$T$: a continuous random variable denoting the time for recruitment in the organization.

$V_k(t)$: probability that there are exactly $k$ decision making epochs in $(0, t]$.

$L(t)$: cumulative distribution function of $T$.

$H_A(.)$: distribution function of $Y_A$ follows exponential distribution with the parameter $\mu_1$.

$H_B(.)$: distribution function of $Y_B$ follows exponential distribution with the parameter $\mu_2$.

$H_C(.)$: distribution function of $Y_C$ follows exponential distribution with the parameter $\mu_3$.

$H(.)$: distribution function of $Y$

$F_k(x)$: cumulative distribution function of $\sum_{i=1}^{k} U_i$.

*: Laplace – Stieltje’s transform.
R : correlation between any $U_i$ and $U_j$, $i \neq j$

$\phi(n,x) : \int_0^x e^{-\tau} \tau^{n-1} d\tau$

$a : \text{mean of } U_i, i=1,2,\ldots$

$b : a(1-R)$

$m : m = \frac{1}{1+bs}$

$E(T) : \text{meantime for recruitment}$

**MAIN RESULTS**

In this section an analytical expression is obtained for Meantime to recruitment is obtained and special cases are discussed

Assume that the thresholds $Y_a, Y_b, Y_c$ have SCBZ property, referring to chapter 2, their distribution functions are given by

$$H_a(x) = 1 - p_1 e^{-(\theta_1 + \mu_1)x} - q_1 e^{-\theta_1 x}$$

$$H_b(x) = 1 - p_2 e^{-(\theta_2 + \mu_2)x} - q_2 e^{-\theta_2 x}$$

and

$$H_c(x) = 1 - p_3 e^{-(\theta_3 + \mu_3)x} - q_3 e^{-\theta_3 x}$$

respectively.

where

$$p_1 = \frac{\theta_1 - \theta_2}{\mu_1 + \theta_1 - \theta_2}; q_1 = \frac{\mu_1}{\mu_1 + \theta_1 - \theta_2}; p_2 = \frac{\theta_3 - \theta_4}{\mu_2 + \theta_3 - \theta_4}; q_2 = \frac{\mu_2}{\mu_2 + \theta_3 - \theta_4}$$
where $\mu_1, \mu_2$ and $\mu_3$ are the parameters of the exponentially distributed truncated random variables.

Since $Y = \max(Y_A, Y_B, Y_C)$

As in chapter 4 (4.2.1), the distribution function for this model is given by,

\[
1 - H(x) = p_1 e^{-(\theta_1 + \mu_1)x} + p_2 e^{-(\theta_2 + \mu_2)x} + p_3 e^{-(\theta_3 + \mu_3)x} + q_1 e^{-(\theta_1)x} + q_2 e^{-(\theta_2)x} + q_3 e^{-(\theta_3)x} \\
- p_1 p_3 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_3)x} - p_1 q_2 e^{-(\theta_1 + \theta_2 + \mu_1)x} - p_1 q_3 e^{-(\theta_1 + \theta_3 + \mu_1)x} \\
- p_2 p_3 e^{-(\theta_2 + \theta_3 + \mu_2 + \mu_3)x} - p_2 q_1 e^{-(\theta_2 + \theta_3 + \mu_2)x} - p_2 q_3 e^{-(\theta_2 + \theta_3 + \mu_2)x} \\
- p_3 q_1 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_3)x} - p_3 q_2 e^{-(\theta_1 + \theta_3 + \mu_1 + \mu_3)x} - p_3 q_3 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_3)x} \\
+ p_1 p_3 q_1 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3)x} + p_1 p_3 q_2 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3)x} \\
+ p_1 p_3 q_3 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3)x} + p_1 p_3 q_2 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3)x} + q_1 q_2 q_3 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3)x}
\]

We shall find $L(t)$

The probability that the threshold level is not reached till 't' is,

\[
P[T > t] = \sum_{k=0}^{\infty} P\{ k \ \text{instants of exits in} \ (0,t) \ \text{and the cumulative loss of manpower in these decisions does not reach the threshold level} \}
\]

\[
= \sum_{k=0}^{\infty} V_k(t) P\left\{ \sum_{i=1}^{k} X_i < Y \right\}
\]

(11.2.3)
\[ F_k(t) = \sum_{k=0}^{\infty} P\left\{ \sum_{i=1}^{k} X_i < Y \right\} \]

(11.2.4)

For \( i = 1, 2, \ldots \), using the law of total probability and (11.2.2),

\[
P\left( \sum_{i=1}^{k} X_i < Y \right) = \sum_{r=0}^{\infty} P\left[ Y > \sum_{i=1}^{k} X_i / \sum_{i=1}^{k} X_i = r \right] P\left[ \sum_{i=1}^{k} X_i = r \right]
\]

\[
= \sum_{r=0}^{\infty} P\left[ Y > r \right] P\left[ \sum_{i=1}^{k} X_i = r \right]
\]

\[
= \sum_{r=0}^{\infty} p_1 e^{-\left(\theta_1 + \mu_1 + \mu_2\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} p_2 e^{-\left(\theta_2 + \mu_2\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} p_3 e^{-\left(\theta_3 + \mu_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} q_1 e^{-\left(\theta_1\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} q_2 e^{-\left(\theta_2\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} q_3 e^{-\left(\theta_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right)
\]

\[
- \sum_{r=0}^{\infty} p_1 p_2 e^{-\left(\theta_1 + \theta_2 + \mu_1 + \mu_2\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} p_1 p_3 e^{-\left(\theta_1 + \theta_3 + \mu_1 + \mu_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} p_2 p_3 e^{-\left(\theta_2 + \theta_3 + \mu_2 + \mu_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right)
\]

\[
- \sum_{r=0}^{\infty} p_1 q_1 e^{-\left(\theta_1 + \theta_2 + \mu_1\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} p_1 q_2 e^{-\left(\theta_1 + \theta_2 + \mu_2\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} p_2 q_3 e^{-\left(\theta_2 + \theta_3 + \mu_2 + \mu_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} q_1 q_2 e^{-\left(\theta_2 + \theta_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} q_1 q_3 e^{-\left(\theta_1 + \theta_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right) - \sum_{r=0}^{\infty} q_2 q_3 e^{-\left(\theta_2 + \theta_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right)
\]

\[
+ \sum_{r=0}^{\infty} p_1 p_2 p_3 e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right)r} P\left( \sum_{i=1}^{k} X_i = r \right)
\]

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\[ + \sum_{r=0}^{\infty} p_1 p_2 q_1 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2)} p \left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} p_1 p_2 q_3 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2)} p \left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} p_1 p_3 q_2 e^{-(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_3)} p \left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} p_2 p_3 q_2 e^{-(\theta_2 + \theta_3 + \theta_1 + \mu_2 + \mu_3)} p \left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} p_2 p_3 q_3 e^{-(\theta_2 + \theta_3 + \theta_1 + \mu_2 + \mu_3)} p \left( \sum_{i=1}^{k} X_i = r \right) + \sum_{r=0}^{\infty} q_1 q_2 q_3 e^{-(\theta_1 + \theta_2 + \theta_3)} p \left( \sum_{i=1}^{k} X_i = r \right) \]

(11.2.5)

Also as in chapter 4, one can write
\[ \sum_{r=0}^{\infty} p e^{-(\theta_1 + \mu_1)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_1 a_1^k \]
\[ \sum_{r=0}^{\infty} p e^{-(\theta_2 + \mu_2)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_2 a_2^k \]
\[ \sum_{r=0}^{\infty} p e^{-(\theta_3 + \mu_3)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_3 a_3^k \]
\[ \sum_{r=0}^{\infty} q e^{-(\theta_1)} p \left( \sum_{i=1}^{k} X_i = r \right) = q_1 a_4^k \]
\[ \sum_{r=0}^{\infty} q e^{-(\theta_2)} p \left( \sum_{i=1}^{k} X_i = r \right) = q_2 a_5^k \]
\[ \sum_{r=0}^{\infty} q e^{-(\theta_3)} p \left( \sum_{i=1}^{k} X_i = r \right) = q_3 a_6^k \]
\[ \sum_{r=0}^{\infty} p_1 p_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_1 p_2 a_7^k \]
\[ \sum_{r=0}^{\infty} p_1 p_3 e^{-(\theta_1 + \theta_3 + \mu_1 + \mu_3)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_1 p_3 a_8^k \]
\[ \sum_{r=0}^{\infty} p_2 p_3 e^{-(\theta_2 + \theta_3 + \mu_2 + \mu_3)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_2 p_3 a_9^k \]
\[ \sum_{r=0}^{\infty} q_1 q_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)} p \left( \sum_{i=1}^{k} X_i = r \right) = p_1 q_2 a_{10}^k \]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_2 p_3 a_1^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_2 q_1 a_2^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_2 q_3 a_3^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_3 q_1 a_4^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_3 q_2 a_5^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = q_1 q_2 a_6^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = q_1 q_3 a_7^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = q_2 q_3 a_8^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_1 p_2 p_3 a_9^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_1 p_2 q_1 a_{10}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_1 p_2 q_3 a_{11}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_1 p_3 q_1 a_{12}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_1 p_3 q_2 a_{13}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_1 p_3 q_3 a_{14}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_2 p_3 q_1 a_{15}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_2 p_3 q_2 a_{16}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = p_2 p_3 q_3 a_{17}^k
\]
\[
\sum_{r=0}^\infty \sum_{n=1}^k X_i = r \quad P \left[ \sum_{i=1}^k X_i = r \right] = q_1 q_2 q_3 a_{18}^k
\]

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\[
\sum_{r=0}^{\infty} q_1 q_2 q_3 e^{-(q_1+q_2+q_3)} p \left[ \sum_{i=1}^{k} X_i = r \right] = q_1 q_2 q_3 a_{26}^k 
\]

(11.2.6)

Using the results (11.2.5) in (11.2.4), we get

\[
P(T > t) = \sum_{k=1}^{\infty} \left( F_k(t) - F_{k+1}(t) \right)
\]

\[
\begin{align*}
&= \sum_{k=1}^{\infty} \left[ p_1 a_1^k + p_2 a_2^k + p_3 a_3^k + q_1 a_4^k + q_2 a_5^k + q_3 a_6^k + p_1 p_2 a_7^k + p_1 p_3 a_8^k + p_1 q_2 a_9^k + p_1 q_3 a_{10}^k + p_2 q_1 a_{11}^k + p_2 q_3 a_{12}^k + p_3 q_1 a_{13}^k + p_3 q_2 a_{14}^k + p_3 q_3 a_{15}^k + q_1 q_2 a_{16}^k + q_1 q_3 a_{17}^k + q_2 q_3 a_{18}^k + p_1 p_2 p_3 a_{19}^k + p_1 p_2 q_1 a_{20}^k + p_1 p_2 q_2 a_{21}^k + p_1 p_3 q_1 a_{22}^k + p_1 p_3 q_2 a_{23}^k + p_2 p_3 q_1 a_{24}^k + p_2 p_3 q_2 a_{25}^k + q_1 q_2 q_3 a_{26}^k \right]
\end{align*}
\]

(11.2.7)

As in Chapter 4, (4.2.13) we have

\[
L(t) = \sum_{k=1}^{\infty} F_k(t) = \left[ p_1 a_1^{k-1} + p_2 a_2^{k-1} + p_3 a_3^{k-1} + q_1 a_4^{k-1} + q_2 a_5^{k-1} + q_3 a_6^{k-1} + p_1 p_2 a_7^{k-1} + p_1 p_3 a_8^{k-1} + p_1 q_2 a_9^{k-1} + p_1 q_3 a_{10}^{k-1} + p_2 q_1 a_{11}^{k-1} + p_2 q_3 a_{12}^{k-1} + p_3 q_1 a_{13}^{k-1} + p_3 q_2 a_4^{k-1} + p_3 q_3 a_{15}^{k-1} + q_1 q_2 a_{16}^{k-1} + q_1 q_3 a_{17}^{k-1} + q_2 q_3 a_{18}^{k-1} + p_1 p_2 q_1 a_{19}^{k-1} + p_1 p_2 q_2 a_{20}^{k-1} + p_1 p_3 q_1 a_{21}^{k-1} + p_1 p_3 q_2 a_{22}^{k-1} + p_2 p_3 q_1 a_{23}^{k-1} + p_2 p_3 q_2 a_{24}^{k-1} + p_2 p_3 q_3 a_{25}^{k-1} + q_1 q_2 q_3 a_{26}^{k-1} \right]
\]

(11.2.8)

As in Chapter 4, (4.2.16)
\[ L'(s) = \sum_{k=0}^{\infty} \frac{g^k}{1 + \left( kR(1-g) \right) \left( 1-R \right)} \]

\[
\begin{align*}
&\quad [p_1a_1a_{1-k} + p_2a_2a_{2-k} + p_3a_3a_{3-k} + q_1a_4a_{4-k} + q_2a_5a_{5-k} + q_3a_6a_{6-k}] \\
&+ [p_1p_2a_7a_{7-k} + p_1p_3a_8a_{8-k} + p_1q_2a_9a_{9-k} + p_1q_3a_{10}a_{10-k}] \\
&+ [p_2q_1a_{11}a_{11-k} + p_2q_3a_{12}a_{12-k} + p_3q_1a_13a_{13-k} + p_3q_2a_{14}a_{14-k}] \\
&+ [p_3q_3a_{15}a_{15-k} + q_1q_2a_{16}a_{16-k} + q_1q_3a_{17}a_{17-k} + q_2q_3a_{18}a_{18-k}] \\
&+ [p_1p_2p_3a_{19}a_{19-k} + p_1p_2q_4a_{20}a_{20-k} + p_1p_3q_5a_{21}a_{21-k} + p_1p_3q_6a_{22}a_{22-k}] \\
&+ [p_1p_3q_7a_{23}a_{23-k} + p_2p_3q_8a_{24}a_{24-k} + p_2p_3q_9a_{25}a_{25-k} + q_1q_2q_{10}a_{26}a_{26-k}] \\
\end{align*}
\]

\[ (11.2.9) \]

\[ E(T) = -\frac{d}{ds} \left[ L'(s) \right]_{s=0} \quad (11.2.10) \]

As in Chapter 3, (3.2.12) one can found that
Equation (11.2.11) gives the mean time for recruitment where \( a_i, a_2, K, K, a_n \) are given by

\[
\begin{align*}
\sum_{r=0}^{\infty} e^{-(\beta_1+\mu) r} P(X_i = r) &= a_1; \\
\sum_{r=0}^{\infty} e^{-(\beta_2+\mu) r} P(X_i = r) &= a_2; \\
\sum_{r=0}^{\infty} e^{-(\beta_3+\mu) r} P(X_i = r) &= a_3; \\
\sum_{r=0}^{\infty} e^{-(\beta_4+\mu) r} P(X_i = r) &= a_4; \\
\sum_{r=0}^{\infty} e^{-(\beta_5+\mu) r} P(X_i = r) &= a_5; \\
\sum_{r=0}^{\infty} e^{-(\beta_6+\mu) r} P(X_i = r) &= a_6; \\
\sum_{r=0}^{\infty} e^{-(\beta_7+\mu) r} P(X_i = r) &= a_7; \\
\sum_{r=0}^{\infty} e^{-(\beta_8+\mu) r} P(X_i = r) &= a_8; \\
\sum_{r=0}^{\infty} e^{-(\beta_9+\mu) r} P(X_i = r) &= a_9; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_2+\mu) r} P(X_i = r) &= a_{10}; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_3+\mu) r} P(X_i = r) &= a_{11}; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_4+\mu) r} P(X_i = r) &= a_{12}; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_5+\mu) r} P(X_i = r) &= a_{13}; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_6+\mu) r} P(X_i = r) &= a_{14}; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_7+\mu) r} P(X_i = r) &= a_{15}; \\
\sum_{r=0}^{\infty} e^{-(\beta_1+\beta_8+\mu) r} P(X_i = r) &= a_{16};
\end{align*}
\]
\[
\sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2)r} P(X_i = r) = a_{17}; \sum_{r=0}^{\infty} e^{-(\theta_1+\theta_3)r} P(X_i = r) = a_{18};
\]
\[
\sum_{r=0}^{\infty} P_i e^{-(\theta_1+\theta_2+\theta_3+\mu_1+\mu_2)r} P(X_i = r) = a_{19}; \sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2+\theta_3+\mu_1+\mu_2)r} P(X_i = r) = a_{20};
\]
\[
\sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2+\theta_3+\mu_2)r} P(X_i = r) = a_{21}; \sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2+\theta_3+\mu_3)r} P(X_i = r) = a_{22};
\]
\[
\sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2+\theta_3+\mu_3+\mu_4)r} P(X_i = r) = a_{23}; \sum_{r=0}^{\infty} e^{-(\theta_2+\theta_3+\mu_2+\mu_3)r} P(X_i = r) = a_{24};
\]
\[
\sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2+\theta_3+\mu_4)r} P(X_i = r) = a_{25}; \sum_{r=0}^{\infty} e^{-(\theta_1+\theta_2+\theta_4)r} P(X_i = r) = a_{26}.
\]

**SPECIAL CASE:**

Suppose \(X_i, i = 1, 2, \ldots\) follows Poisson distribution with parameter \(\lambda\).

Then,
\[
a_i = \sum_{r=0}^{\infty} e^{-(\theta_1+\mu_1)r} P(X_i = r)
= \sum_{r=0}^{\infty} e^{-(\theta_1+\mu_1)} \frac{e^{-\lambda} \lambda^r}{r!}
= e^{-\lambda} \sum_{r=0}^{\infty} \frac{(\lambda e^{-(\theta_1+\mu_1)})^r}{r!}
= e^{-\lambda} e^{-\lambda e^{-(\theta_1+\mu_1)}}
= e^{-\lambda} e^{e^{-(\theta_1+\mu_1)}}
\]

Similarly,
\[
a_2 = e^{-\lambda e^{-(\theta_1+\mu_2)}}; \quad a_3 = e^{-\lambda e^{-(\theta_1+\mu_3)}}; \quad a_4 = e^{-\lambda e^{-(\theta_1+\theta_2+\mu_2)}}; \quad a_5 = e^{-\lambda e^{-(\theta_1+\theta_2+\mu_3+\mu_4)}};
\]
\[
a_6 = e^{-\lambda e^{-(\theta_1+\theta_3+\mu_2+\mu_3)}}; \quad a_7 = e^{-\lambda e^{-(\theta_1+\theta_3+\mu_4+\mu_3)}}; \quad a_8 = e^{-\lambda e^{-(\theta_2+\theta_3+\mu_2)}};
\]
\[
a_9 = e^{-\lambda e^{-(\theta_1+\theta_4+\mu_3+\mu_4)}}
\]

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Using (11.2.12) in (11.2.11), meantime for recruitment is given by

\[ E(T) = a \left[ \frac{P_1}{1 - e^{-\lambda T^3}} + \frac{P_2}{1 - e^{-\lambda T^2}} + \frac{P_3}{1 - e^{-\lambda T}} \right] \]

\[ + a \left[ \frac{q_1}{1 - e^{-\lambda T^3}} + \frac{q_2}{1 - e^{-\lambda T^2}} + \frac{q_3}{1 - e^{-\lambda T}} \right] \]

\[ + a \left[ \frac{P_1 P_2}{1 - e^{-\lambda T^3}} - \frac{P_1 P_3}{1 - e^{-\lambda T^2}} + \frac{P_1 q_2}{1 - e^{-\lambda T}} \right] \]

\[ + a \left[ \frac{P_2 P_3}{1 - e^{-\lambda T^2}} - \frac{P_2 q_1}{1 - e^{-\lambda T}} \right] \]
\[\begin{align*}
&+a \left[ \frac{p_1 p_2}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{p_1 p_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{p_1 q_2}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{p_1 q_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} \right] \\
&+a \left[ \frac{p_1 q_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{q_1 q_2}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{q_1 q_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{q_2 q_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} \right] \\
&+a \left[ \frac{p_1 p_3 p_4}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} + \frac{p_1 p_3 q_4}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} + \frac{p_1 p_4 q_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} + \frac{p_1 q_3 q_4}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} \right] \\
&+a \left[ \frac{p_1 p_3 p_4 q_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{q_1 q_2 p_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{q_1 q_3 p_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} - \frac{q_2 q_3 p_3}{1-e^{-(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)}} \right] \\
&(11.2.13)
\end{align*}\]

(11.2.13) gives the mean time for recruitment

**NUMERICAL ILLUSTRATION:**

In this section model 1 is numerically illustrated and relevant conclusions are made.

Fixing

\[\mu_1 = 0.5; \mu_2 = 0.4; \mu_3 = 0.1; \theta_1 = 0.5; \theta_2 = 0.3; \theta_3 = 0.2; \theta_4 = 0.1; \theta_5 = 0.8; \theta_6 = 0.7; b = 2\]

and varying \( R \) and \( \lambda \), the values of \( E(T) \) are computed and tabulated in **Table 11.2.1**
**Table 11.2.1**

<table>
<thead>
<tr>
<th>( R )</th>
<th>( \lambda )</th>
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<th>3</th>
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</table>

**CONCLUSION:**

From the above table, we make the following observations.

(i) The mean time for recruitment increases as \( \lambda \) decreases, keeping other parameters fixed. In other words, when the number of exits decreases on the average, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as \( R \) increases, keeping other parameters fixed.
(iii) When both \( \lambda \) and \( R \) varies, mean and Variance of time to recruitment decreases for negative correlation and increases for positive correlation.

**MODEL 2**

**DESCRIPTION AND ANALYSIS OF THE MODEL**

**Assumptions:**

1. An organization having three grades takes policy decisions at \( k \) epochs, and at every decision making epoch, a random number of persons quit the organization.

2. There is an associated loss of manhours to the organization if a person quits and it is linear and cumulative. Each grade has its own threshold level and the threshold distribution has SCBZ property.

3. If the total number of persons who leave the organization exceeds a threshold level in each grade the organization faces a breakdown and so the recruitment is necessary.

4. The inter-decision times are exchangeable and constantly correlated exponential random variables.

5. The mobility of manpower from one grade to the other is not permitted.
Notations:

$U_i$: time between the $(i-1)$th and $i$th decision epoch. $U_i$s are exchangeable and constantly correlated exponential random variables.

$X_i$: a discrete random variable denoting the number of persons who leave the organization at the $i$th decision epoch. $i = 1, 2, ..., X_i$'s are independent and identically distributed random variables.

$Y_A, Y_B, Y_C$: exponential random variable denoting threshold level with SCBZ property for the grades A, B, C respectively.

$Y$: min ($Y_A, Y_B, Y_C$)

$T$: a continuous random variable denoting the time for recruitment in the organization.

$V_k(t)$: probability that there are exactly $k$ decision making epochs in $[0, t]$.

$L(t)$: cumulative distribution function of $T$.

$H_c(.)$: distribution function of $Y_A$ follows exponential distribution with the parameter $\mu_3$.

$F_k(x)$: cumulative distribution function of $\sum_{i=1}^{k} X_i$.

* : Laplace – Stieltje’s transform.

$R$: correlation between any $U_i$ and $U_j$, $i \neq j$.

$\phi(n,x) = \int_0^x e^{-t} t^{n-1} dt$.

$a$: mean of $U_i$, $i = 1, 2, ....$
In this section an analytical expression for Meantime to recruitment is obtained and special cases are discussed for model 2.

Assume that the thresholds $Y_A, Y_B, Y_C$ have SCBZ property, referring to the chapter 2, their distribution functions are given by

$$H_A(x) = 1 - p_1 e^{-(\theta_1 + \mu_1 x)} - q_1 e^{-\theta_2 x}$$

$$H_B(x) = 1 - p_2 e^{-(\theta_3 + \mu_2 x)} - q_2 e^{-\theta_4 x}$$

and

$$H_C(x) = 1 - p_3 e^{-(\theta_3 + \mu_3 x)} - q_3 e^{-\theta_5 x}$$

respectively.

where

$$p_1 = \frac{\theta_1 - \theta_2}{\mu_1 + \theta_1 - \theta_2}; q_1 = \frac{\mu_1}{\mu_1 + \theta_1 - \theta_2}; p_2 = \frac{\theta_3 - \theta_4}{\mu_2 + \theta_3 - \theta_4}; q_2 = \frac{\mu_2}{\mu_2 + \theta_3 - \theta_4}$$

$$p_3 = \frac{\theta_5 - \theta_6}{\mu_3 + \theta_5 - \theta_6}; q_3 = \frac{\mu_3}{\mu_1 + \theta_5 - \theta_6}$$

(11.3.1)

where $\mu_1, \mu_2, and \mu_3$ are the parameters of the exponentially distributed truncated random variables.
Since \( Y = \min(Y_A, Y_B, Y_C) \)

As in chapter 4 (4.2.1), the distribution function for this model is given by,

\[
1 - H(x) = p_1 p_2 p_3 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2 + \mu_3)} + p_1 p_2 q_2 e^{-(\theta_1 + \theta_3 + \mu_1 + \mu_3)} + p_1 p_2 q_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2 + \mu_3)} + p_1 p_3 q_1 e^{-(\theta_1 + \theta_3 + \mu_1 + \mu_3)} + p_1 p_3 q_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2 + \mu_3)} + q_1 p_2 q_3 e^{-(\theta_2 + \theta_3 + \mu_2 + \mu_3)} + q_1 q_2 p_3 e^{-(\theta_2 + \theta_4 + \mu_2 + \mu_4)} + q_1 q_2 q_3 e^{-(\theta_2 + \theta_3 + \mu_2 + \mu_3)} + q_1 q_2 q_3 e^{-(\theta_2 + \theta_4 + \mu_2 + \mu_4)} \]

(11.3.2)

Now we shall obtain \( L(t) \)

The probability that the threshold level is not reached till \( t \) is,

\[
P[T > t] = \sum_{k=0}^{\infty} P\{ k \text{ instants of exits in } (0, t] \text{ and the cumulative loss of manpower in these decisions does not reach the threshold level} \}
\]

\[
= \sum_{k=0}^{\infty} V_k(t) P\left( \sum_{i=1}^{k} X_i < Y \right) \quad (11.3.3)
\]

\[
= \sum_{k=0}^{\infty} \left[ F_k(t) - F_{k+1}(t) \right] P\left( \sum_{i=1}^{k} X_i < Y \right) \quad (11.3.4)
\]

For \( i = 1, 2, \ldots \), using the law of total probability and (11.3.2),

\[
P\left( \sum_{i=1}^{k} X_i < Y \right)
\]

\[
= \sum_{r=0}^{\infty} P\left[ Y > \sum_{i=1}^{k} X_i \text{ / } \sum_{i=1}^{k} X_i = r \right] P\left[ \sum_{i=1}^{k} X_i = r \right] \]

\[
= \sum_{r=0}^{\infty} P[Y > r] P\left[ \sum_{i=1}^{k} X_i = r \right]
\]

As in Model 2 of chapter 4, one can found that
\[ P \left( \sum_{i=1}^{k} X_i < Y \right) = \sum_{r=0}^{\infty} P_1 P_2 P_3 e^{-\left( \theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3 \right)r} P(X_i = r) + \sum_{r=0}^{\infty} P_1 q_2 q_3 e^{-\left( \mu_1 + \theta_4 + \theta_5 \right)r} P(X_i = r) + \sum_{r=0}^{\infty} P_2 q_1 q_3 e^{-\left( \theta_2 + \theta_4 + \theta_6 \right)r} P(X_i = r) + \sum_{r=0}^{\infty} P_3 q_1 q_2 e^{-\left( \theta_2 + \theta_4 + \theta_5 \right)r} P(X_i = r) + \sum_{r=0}^{\infty} q_1 q_2 q_3 e^{-\left( \theta_4 + \theta_5 + \theta_6 \right)r} P(X_i = r) \]

(11.3.3)

Consider the term in (11.3.3)

\[ \sum_{r=0}^{\infty} P_1 P_2 P_3 e^{-\left( \theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3 \right)r} P(X_i = r) \]

\[ = \sum_{r=0}^{\infty} P_1 P_2 P_3 e^{-\left( \theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3 \right) \sum_{i=1}^{k} X_i} \]

\[ = \sum_{r=0}^{\infty} P_1 P_2 P_3 \prod_{i=1}^{k} E \left[ e^{-\left( \theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3 \right) X_i} \right] \]

\[ = p_1 p_2 p_3 b_1^k \]

Similarly,

\[ \sum_{r=0}^{\infty} P_1 P_2 q_2 e^{-\left( \theta_1 + \theta_2 + \mu_1 + \mu_2 \right)r} P(X_i = r) = p_1 P_2 q_2 b_2^k; \]

\[ \sum_{r=0}^{\infty} P_1 P_3 q_2 e^{-\left( \theta_1 + \theta_2 + \mu_1 + \mu_2 \right)r} P(X_i = r) = p_1 P_3 q_2 b_3^k; \]

\[ \sum_{r=0}^{\infty} P_2 q_2 q_3 e^{-\left( \theta_2 + \theta_3 + \mu_1 \right)r} P(X_i = r) = p_2 q_2 q_3 b_4^k; \]

\[ \sum_{r=0}^{\infty} P_2 q_2 q_5 e^{-\left( \theta_2 + \theta_3 + \mu_1 \right)r} P(X_i = r) = p_2 q_2 q_5 b_5^k. \]
\[
\sum_{r=0}^{\infty} p_2 q_3 e^{-(r_1+q_1+q_2+\mu_1)} P(X_t = r) = p_2 q_3 b_5^k;
\]
\[
\sum_{r=0}^{\infty} p_3 q_1 q_2 e^{-(r_1+q_1+q_2+\mu_1)} P(X_t = r) = p_3 q_1 q_2 b_7^k;
\]
\[
\sum_{r=0}^{\infty} q_1 q_2 q_3 e^{-(r_1+q_1+q_2+\mu_1)} P(X_t = r) = q_1 q_2 q_3 b_8^k.
\]
(11.3.4)

As in Model 1 we write

\[
L(t) = \sum_{k=1}^{\infty} F_k(t) \left[ p_1 p_2 p_3 b_1^k + p_1 p_2 q_3 b_2^k + p_1 p_3 q_2 b_3^k + p_3 q_2 q_3 b_4^k + \right]
\]
\[
\left[ p_2 p_3 q_1 b_5^k + p_2 q_1 q_3 b_6^k + p_2 q_1 q_2 b_7^k + q_1 q_2 q_3 b_8^k \right]
\]
(11.3.5)

Taking Laplace–Stieltje’s transform of (11.3.5) we get

\[
L(s) = \sum_{k=1}^{\infty} \frac{g^k}{[1+\left(\frac{kR(1-g)}{(1-R)}\right)]^x} \left[ p_1 p_2 p_3 \bar{b}_1^{k-1} + p_1 p_2 q_3 \bar{b}_2^{k-1} + p_1 p_3 q_2 \bar{b}_3^{k-1} + p_3 q_2 q_3 \bar{b}_4^{k-1} + \right]
\]
\[
\left[ p_2 p_3 q_1 \bar{b}_5^{k-1} + p_2 q_1 q_3 \bar{b}_6^{k-1} + p_2 q_1 q_2 \bar{b}_7^{k-1} + q_1 q_2 q_3 \bar{b}_8^{k-1} \right]
\]
(11.3.6)

Since

\[
E(T) = -\frac{d}{ds} \left[ L'(s) \right]_{s=0}
\]
(11.3.7)

As in Model 1, using (11.3.6) in (11.3.7) meantime for recruitment is obtained as
\[ E(T) = a \left[ \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} \right] + p_1 \frac{1}{b_4} + p_2 \frac{1}{b_5} + p_2 q_1 \frac{1}{b_6} \]

\[ + p_3 q_1 q_2 \frac{1}{b_7} + q_1 q_2 q_3 \frac{1}{b_8} \]  

\[ (11.3.8) \]

Where \( b_1, b_2, \ldots, b_8 \) are given by

\[ \sum_{r=0}^{\infty} e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_1; \sum_{r=0}^{\infty} e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_2; \]

\[ \sum_{r=0}^{\infty} e^{-\left(\theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_3; \sum_{r=0}^{\infty} e^{-\left(\mu_1 + \theta_2 + \theta_3 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_4; \]

\[ \sum_{r=0}^{\infty} e^{-\left(\theta_3 + \mu_1 + \theta_2 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_5; \sum_{r=0}^{\infty} e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2\right) r} P(X_i = r) = b_6; \]

\[ \sum_{r=0}^{\infty} e^{-\left(\theta_3 + \theta_2 + \theta_1 + \mu_1 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_7; \sum_{r=0}^{\infty} e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right) r} P(X_i = r) = b_8. \]

**SPECIAL CASE:**

Suppose \( X_i, i = 1, 2, \ldots \) follows Poisson distribution with parameter \( \lambda \)

Then,

\[ b_i = \sum_{r=0}^{\infty} e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right) r} P(X_i = r) \]

\[ b_i = \sum_{r=0}^{\infty} e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right) r} \left( e^{-\lambda} \lambda^r / r! \right) \]

\[ - \lambda \left[ 1 - e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right)} \right] \]

\[ = e \left[ 1 - e^{-\left(\theta_1 + \theta_2 + \theta_3 + \mu_1 + \mu_2 + \mu_3\right)} \right] \]
Similarly,

\[ b_2 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}; b_3 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}; \]

\[ b_4 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}; b_5 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}; \]

\[ b_6 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}; b_7 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}; \]

\[ b_8 = e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]} \]

and \( b_i = 1 - b_i, i = 1, 2, \ldots, 8 \) \hspace{1cm} (11.3.9)

Using (11.3.9) in (11.3.8), we get

\[
E(T) = P_1 P_2 P_3 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right) + P_1 P_2 q_3 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right)
\]

\[
+ P_1 P_3 q_2 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right) + P_3 q_2 q_3 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right)
\]

\[
+ P_2 P_3 q_1 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right) + P_2 q_3 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right)
\]

\[
+ P_2 q_1 q_2 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right) + q_1 q_2 q_3 \left( \frac{1}{1 - e^{-\lambda \left[ 1 - e^{-\left( \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \right)} \right]}} \right)
\]

(11.3.10)

Equation (11.3.10) gives the meantime for recruitment.
NUMERICAL ILLUSTRATION:

In this section model 1 is numerically illustrated and relevant conclusions are made.

Fixing \[ \mu_1 = 0.5; \mu_2 = 0.4; \mu_3 = 0.1; \theta_1 = 0.5; \theta_2 = 0.3; \theta_3 = 0.2; \theta_4 = 0.1; \theta_5 = 0.8; \theta_6 = 0.7; b = 2 \]

and varying \( R \) and \( \lambda \), the values of \( E(T) \) are computed and tabulated in Table 11.3.1

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<tr>
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<td>18.9088</td>
<td>12.8612</td>
<td>11.1790</td>
<td>10.5291</td>
<td>10.2467</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as \( \lambda \) decreases, keeping other parameters fixed. In other words, when the number of exits decreases on the average, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as \( R \) increases, keeping other parameters fixed.

(iii) When both \( \lambda \) and \( R \) varies, mean and variance of time to recruitment decreases for negative correlation and increases for positive correlation.