CHAPTER 8

OPTIMAL RESERVE INVENTORY LEVEL BETWEEN TWO GRADES IN MANPOWER PLANNING

8.1 INTRODUCTION

An interesting problem in Inventory Control Theory is to find the optimum size of the buffer between two operating systems. Hanssman (1962), in his paper has considered the problem of optimal reserve inventory between two machines and determined the optimal reserve inventory, assuming constant demand rate. Later this result was extended for stochastic demand.

In this chapter, an organisation having two branches B1 and B2 are considered. Branch B1 has grades G1 and G2. The Branch B2 has Grade G3 which is equivalent to G2 and it serves as an inventory between G1 and G2 in the context of filling up of vacancies that arises in G2. The vacancies in G2 are filled up through a promotion process from G1. In this context, if no suitable person is available in G1 for promotion to G2, then personnel working in G3 are transferred to G2 under certain conditions. G1 is said to be in the upstate, if there is atleast one person in G1 available for promotion to G2. Otherwise, G1 is said
to be in the downstate. The idle time for G2 is the duration in which no personnel is available from G1 and G3 to fill up the existing vacancies in G2. The main objective of this chapter is to find the optimum value of the reserve inventory to be kept in G3, so as to minimize the total cost incurred. The rest of the chapter is organized as follows: In section 8.2, description of the model is given. In section 8.3, analytical expression for the cost equation is obtained. In section 8.4, the optimum reserve inventory level between branches B1 and B2 is obtained for different special cases and the results are illustrated by numerical examples and relevant conclusions are made.

8.2 DESCRIPTION OF THE MODEL

Assumptions:

1. An organisation has two branches B1 and B2.
2. B1 has two Grades G1 and G2 and B2 has Grade G3.
3. The size of G3 is S.
4. Personnel from G1 are promoted to G2 by a definite promotion process to fill up vacancies in G2. If G1 is in the downstate, to fill up vacancies in G2, at most S1 personnel (0 ≤ S1 < S) from G3 are transferred to G2 instantaneously depending upon the need.
Notations:

- \( r_1 \): rate at which the vacancies arises in \( G_2 \).
- \( h_1 \): holding cost per unit of \( S_1 \).
- \( d_1 \): downtime cost for \( G_1 \).
- \( \mu_1 \): average time between two consecutive start of the
downtime duration for \( G_1 \).
- \( g(\tau) \): probability density function of \( \tau \)
- \( G(.) \): distribution of \( \tau \)

We assume that \( G_1 \) is in the downstate and \( G_2 \) is in the
upstate. Till the grade \( G_1 \) is in upstate the process is going on
smoothly. When no one is selected at \( G_1 \) then the Grade 1 comes
to downstate, however \( G_2 \) is in upstate as it gets resource from
\( B_2 \).

In \( B_2 \), among \( S \) persons only \( S_1 \) persons are available for
giving training at \( G_2 \) because there must be a minimum number
of people in \( B_2 \) so that \( B_2 \) may not be closed.

When all \( s_1 \) number of persons from \( B_2 \) are sent for training
at \( G_2 \), \( G_2 \) has no one to give training either from \( G_1 \) or from \( B_2 \)
and idle time for \( G_2 \) starts. Till \( G_1 \) gets the person selected, \( G_2 \) is
in idle state. When the persons are available in \( G_1 \) to give
training in \( G_2 \), again \( G_1 \) comes to the upstate and the process is
going on smoothly, but from \( B_2 \) only one time the persons can be
taken for training so as to postpone the idle time in \( G_2 \).
8.3 MAIN RESULT

Assume that the Grade G1 is in downstate and Grade G2 is in upstate. The cost equation is given as

\[
C(S_i) = h_i S_i + \frac{d_1}{\mu_i} \int_0^\infty \left( \tau - \frac{S_i}{r_i} \right) g(\tau) d\tau
\]  
(8.3.1)

The first term in (8.3.1) refers to the holding cost per unit of S1 and the second term refers to the idle time cost for G2. The optimum value of S1 which minimizes C(S1) can be obtained as follows.

Differentiating (8.3.1) with respect to S1 and equating to zero,

(i.e) \[ \frac{d}{dS_1} (C(S_1)) = 0 \]

\[ h_i + \frac{d_1}{\mu_i} \int_0^\infty \frac{d}{dS_1} \left( \tau - \frac{S_1}{r_i} \right) g(\tau) d\tau = 0 \]

\[ h_i + \frac{d_1}{\mu_i} \int_0^\infty \left( - \frac{1}{r_i} \right) g(\tau) d\tau = 0 \]

\[ h_i - \frac{d_1}{\mu_i r_i} \int_{S_1}^\infty g(\tau) d\tau = 0 \]

\[ h_i - \frac{d_1}{\mu_i r_i} G(S_1, r_i) = 0 \]  
(8.3.2)
Equation (8.3.2) can be solved for $S_1$ when the distribution function $G(.)$ is given.

$C(S_1)$ is minimum for $S_1$ when $\frac{d^2}{dS_1^2}(C(S_1)) > 0$

8.4 SPECIAL CASE

In this section for specific distributions cost analysis are made

Case (i): Suppose $G(.)$ follows uniform distribution over $[0, a]$. In this case, the downtime density of $G_1$ is constant and is independent of time.

The holding cost will arise when $a < \frac{S_i}{r_i}$

When $a > \frac{S_i}{r_i}$, from (8.3.2)

$$h - \frac{d}{\mu r_i} \left( \frac{1}{a} \right) \left( a - \frac{S_i}{r_i} \right) = 0$$

$$S_i = (ar_i) \left( 1 - \frac{h_i r_i}{d_i} \right) \quad (8.4.1)$$

$$S_i = 0 \iff h_i r_i = d_i \quad (8.4.2)$$

$$S_i > 0 \iff d_i > h_i r_i \quad (8.4.3)$$
As \( r_1 \) is real, from (8.4.1), we get

\[
S_1 \leq (a d_1 / 4 h_1 \mu_1) \tag{8.4.4}
\]

**Case (ii)**  Suppose \( G(.) \) follows an exponential distribution with parameter \( \lambda \).

From (8.3.2) it follows that,

\[
h_1 - \left( \frac{d_1}{\mu_1 \eta_1} \right) e^{-\lambda S_1 \eta_1} = 0 \tag{8.4.5}
\]

\[
S_1 = \frac{\eta_1}{[\log(d_1 / h_1 \mu_1 \eta_1)]} \tag{8.4.6}
\]

\[
S_1 > 0 \iff d_1 > h_1 \mu_1 \eta_1 \quad \text{and} \quad S_1 = 0 \iff h_1 \mu_1 \eta_1 = d_1 \tag{8.4.7}
\]

In both the cases, if \( S_1 = 0 \), no inventory is necessary between \( G_1 \) and \( G_2 \) and if \( S_1 > 0 \), then a positive inventory is to be kept between \( G_1 \) and \( G_2 \).
NUMERICAL ILLUSTRATION:

In this section for the cost analysis made in section 8.4, numerical illustrations are given with relevant conclusions.

Case (i): Suppose G(.) is the uniform distribution over [0,a]

The following Table (8.4.1) gives $S_1$ for different combination of the values of $h_1$, $\mu_1$, $r_1$, $d_1$, and $a$.

<table>
<thead>
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<th>$\mu_1$</th>
<th>$r_1$</th>
<th>$h_1$</th>
<th>$d_1$</th>
<th>$a$</th>
<th>$S_1$</th>
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<tbody>
<tr>
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</table>
Case (ii):

G(.) is the exponential distribution with parameter $\lambda$.

The following Table (8.4.2) gives $S_1$ for different combination of the values of $h_1$, $\mu_1$, $r_1$, $d_1$ and $\lambda$.

<table>
<thead>
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<th>$\mu_1$</th>
<th>$r_1$</th>
<th>$h_1$</th>
<th>$d_1$</th>
<th>$\lambda$</th>
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<td>2</td>
<td>0.0</td>
</tr>
</tbody>
</table>
CONCLUSIONS

(i) From the Table (8.4.1) it is observed that, as $|J,I|$ increases, keeping other parameters fixed, $S_1$ decreases and as $|ii|$ increases, keeping other parameters fixed, $S_1$ increases which is realistic.

(ii) From the Table (8.4.2) it is observed that, as $\mu_1$ increases, keeping other parameters fixed, $S_1$ decreases and as $r_1$ increases, keeping other parameters fixed, $S_1$ decreases which is also realistic.