CHAPTER 4

EXPECTED TIME FOR RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM ASSOCIATED WITH CORRELATED INTER-DECISION TIMES WHEN THRESHOLD DISTRIBUTION HAS SCBZ PROPERTY.

4.1 INTRODUCTION

For a single graded system, Sathiyamoorthy and Elangovan (1998(a)) have obtained the mean and variance of the time to recruitment (i) when the number of exits forms a sequence of independent and identically distributed random variables, (ii) the random threshold is geometric and (iii) the inter-decision times are independent and identically distributed random variables. Later, for the same manpower system, Sathiyamoorthy and Parthasarathy (2003) have obtained the mean and variance of the time to recruitment when (i) the loss of manhours process is a sequence of independent and identically distributed random variables and (ii) the random threshold has Setting the Clock Back to Zero (SCBZ) property.
Mariappan and Srinivasan (2001(a)) have also obtained the mean time for recruitment in a single graded system using shock model approach when the inter-decision times are correlated exchangeable and exponential random variables also they have obtained the mean time for recruitment in a single graded system using shock model approach when the bivariate process formed by loss of manpower and inter-decision times forms a correlated renewal sequence.

In this chapter, an organization with two grades subjected to loss of manpower due to staff depletions caused by policy decisions taken by the organization is considered. Assuming that each grade has its own random threshold whose distribution has SCBZ property and the inter-decision times are exchangeable and constantly correlated exponential random variables, two mathematical models are constructed based upon an appropriate univariate policy of recruitment. The objective of this chapter is to find the mean time for recruitment in the organization for both the models.

The rest of this chapter is organised as follows: In section 4.2, description of Model 1 is given, analytical expression for meantime to recruitment is obtained and the special cases are discussed.
In section 4.3, Model 2 is described and a similar computation is carried out using a different univariate policy of recruitment. In section 4.4, both the models are numerically illustrated and relevant conclusions are made.

4.2 MODEL 1

DESCRIPTION AND ANALYSIS OF THE MODEL

Assumptions:

1. An organization having two grades (grades A and B) takes policy decisions at random epochs in $[0, \infty)$, and at every decision making epoch, a random number of persons quit the organization.

2. There is an associated loss of manhours to the organization if a person quits and it is linear and cumulative.

3. Each grade has its own threshold level and the threshold distribution has SCBZ property. Recruitment is made whenever the total number of exits exceeds the threshold level in both the grades.

4. The inter – decision times are exchangeable constantly correlated exponential random variables.

5. The process which generates the number of exits and the threshold are mutually independent.

6. Mobility of man power from one grade to the other is permitted.
**Notations:**

- \( U_i \): time between the \((i-1)\)th and \(i\)th decision epoch. \( U_i \)’s are exchangeable and constantly correlated exponential random variables.
- \( X_i \): discrete random variable denoting the total number of persons who leave the organization from the two grades at the \(i\)th decision epoch. \( i = 1, 2, ... \) \( X_i \)'s independent and identically distributed random variables.
- \( Y_A, Y_B \): continuous random variables denoting the threshold of levels for the grades A and B respectively and the distributions \( Y_A \) and \( Y_B \) follows SCBZ property.
- \( Y \): \( Y = \max(Y_A, Y_B) \)
- \( V_k(t) \): probability that there are \( k \) decisions in \((0, t]\).
- \( g(.) \): probability density function of \( X_i, i = 1, 2, ... \)
- \( g^*(.) \): Laplace transform of \( g(.) \)
- \( g_k(.) \): \( k \)-fold convolution of \( g(.) \)
- \( f(.) \): probability density function of inter-decision times
- \( f_k(.) \): \( k \)-fold convolution of \( f(.) \)
- \( F_k(.) \): \( k \)-fold convolution of \( F(.) \)
- \( H_A(.) \): distribution function of \( Y_A \) with parameters \( \theta_1, \theta_2 \text{ and } \mu_1 \)
- \( H_B(.) \): distribution function of \( Y_B \) with parameters \( \theta_3, \theta_4 \text{ and } \mu_2 \)
- \( H(.) \): distribution function of \( Y \).
- \( T \): time for recruitment in the organization.
$L(t) : \text{distribution function of } T$

$L^*(s) : \text{Laplace-Stieltje's transform of } L(t)$.

$R : \text{correlation between any } U_i \text{ and } U_j, \ i \neq j$

$\phi(n,x) : \int_0^x e^{-\tau} \tau^{n-1} d\tau$

$a : \text{mean of } U_i, i = 1, 2, \ldots$

$b : a(1-R)$

$m : m = m(s) = \frac{1}{(1+bs)}$

$E(T) : \text{mean time for recruitment}$

**MAIN RESULTS**

In this subsection an analytical expression for the mean time to recruitment is obtained.

Since $Y = \max(Y_A, Y_B)$, using (2.2.2) to (2.2.5) in chapter 2, the distribution of $Y$ is given by,

$$H(x) = H_A(x) H_B(x)$$

$$\therefore 1 - H(x) = 1 - H_A(x) H_B(x)$$

The probability distribution of the thresholds $Y_A$ and $Y_B$ for the two grades respectively are given by,

$$1 - H_A(x) = p_1 e^{-(\alpha_1 + \mu)x} + q_1 e^{-\beta_1 x}$$

and

$$1 - H_B(x) = p_2 e^{-(\alpha_2 + \mu)x} + q_2 e^{-\beta_2 x}$$
where
\[ p_i = \frac{\theta_1 - \theta_2}{\mu_1 + \theta_1 - \theta_2}; q_i = \frac{\mu_1}{\mu_1 + \theta_1 - \theta_2}; p_2 = \frac{\theta_3 - \theta_4}{\mu_2 + \theta_3 - \theta_4}; q_2 = \frac{\mu_2}{\mu_2 + \theta_3 - \theta_4} \]

where \( \mu_1 \) and \( \mu_2 \) are the parameters of the exponentially distributed truncated random variables.

Since \( Y = \max (Y_1, Y_2) \),
\[
1 - H(x) = p_1 e^{-(\theta_1 + \mu_1)x} + p_2 e^{-(\theta_3 + \mu_2)x} + q_1 e^{-(\theta_1)x} + q_2 e^{-(\theta_3)x} - q_1 p_2 e^{-(\theta_1 + \theta_3 + \mu_1)x} - p_1 q_2 e^{-(\theta_3 + \mu_2)x}
\]
\[ -p_1 q_2 e^{-(\theta_3 + \theta_4 + \mu_1)x} - q_1 p_2 e^{-(\theta_1 + \theta_3 + \mu_2)x} - q_1 q_2 e^{-(\theta_4 + \mu_1)x} \] (4.2.1)

The probability that the threshold level is not reached till 't' is,
\[
P[T > t] = \sum_{k=0}^{\infty} P \left( \text{k instants of exits in (0,t]} \right) \text{ and the cumulative loss of manpower in these k decisions does not reach the threshold level}\]
\[ = \sum_{k=0}^{\infty} V_k (t) P \left( \sum_{i=1}^{k} X_i < Y \right) \] (4.2.2)

We now calculate \( P \left( \sum_{i=1}^{k} X_i < Y \right) \).

Using the law of total probability and (4.2.1),
\[
P \left( \sum_{k=0}^{\infty} X_i < Y \right) = \sum_{r=0}^{\infty} P \left[ Y > \sum_{k=0}^{\infty} X_i, \sum_{k=0}^{\infty} X_i = r \right] P \left[ \sum_{k=0}^{\infty} X_i = r \right]
\]
\[ = \sum_{r=0}^{\infty} P[Y > r] P \left( \sum_{k=0}^{\infty} X_i = r \right) \]

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\[
= \sum_{r=0}^{\infty} p_r e^{-(\theta_1 + \mu_1) r} P \left( \sum_{k=0}^{\infty} X_k = r \right) + \sum_{r=0}^{\infty} p_2 e^{-(\theta_1 + \mu_2) r} P \left( \sum_{k=0}^{\infty} X_k = r \right) \\
+ \sum_{r=0}^{\infty} q_1 e^{-(\theta_1) r} P \left( \sum_{k=0}^{\infty} X_k = r \right) + \sum_{r=0}^{\infty} q_2 e^{-(\theta_2) r} P \left( \sum_{k=0}^{\infty} X_k = r \right) \\
- \sum_{r=0}^{\infty} p_1 p_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2) r} P \left[ \sum_{k=0}^{\infty} X_k = r \right] - \sum_{r=0}^{\infty} p_1 q_2 e^{-(\theta_1 + \theta_2 + \mu_1) r} P \left[ \sum_{k=0}^{\infty} X_k = r \right] \\
- \sum_{r=0}^{\infty} p_2 g_1 e^{-(\theta_2 + \theta_1 + \mu_1) r} P \left[ \sum_{k=0}^{\infty} X_k = r \right] - \sum_{r=0}^{\infty} q_1 q_2 e^{-(\theta_1 + \theta_2) r} P \left[ \sum_{k=0}^{\infty} X_k = r \right] \quad (4.2.3)
\]

Now write

\[
a_1 = \sum_{r=0}^{\infty} e^{-(\theta_1 + \mu_1) r} P(X_i = r); a_2 = \sum_{r=0}^{\infty} e^{-(\theta_1 + \mu_2) r} P(X_i = r); a_3 = \sum_{r=0}^{\infty} e^{-(\theta_2) r} P(X_i = r); \\
a_4 = \sum_{r=0}^{\infty} e^{-(\theta_1) r} P(X_i = r); a_5 = \sum_{r=0}^{\infty} e^{-(\theta_1 + \theta_2 + \mu_1) r} P(X_i = r); \\
a_6 = \sum_{r=0}^{\infty} e^{-(\theta_1 + \mu_1) r} P(X_i = r); a_7 = \sum_{r=0}^{\infty} e^{-(\theta_2 + \mu_2) r} P(X_i = r); \\
a_8 = \sum_{r=0}^{\infty} e^{-(\theta_2 + \theta_1) r} P(X_i = r) \quad (4.2.4)
\]

Now

\[
\sum_{r=0}^{\infty} e^{-\theta r} P(X_i = r) = E(e^{-\theta X_i}), i = 1, 2, ...
\]

and

\[
\sum_{r=0}^{\infty} e^{-\theta r} P(\sum_{i=1}^{k} X_i = r) = \prod_{i=1}^{k} E(e^{-\theta X_i})
\]
\[
\sum_{r=0}^{\infty} p_1 e^{-(\theta + \mu)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = p_1 E\left(\frac{e^{-(\theta + \mu)\sum_{k=0}^{\infty} X_i}}{r!}\right)
\]
\[
\sum_{r=0}^{\infty} p_1 e^{-(\theta + \mu)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = p_1 \prod_{r=1}^{\infty} E\left(e^{-(\theta + \mu)\sum_{k=0}^{\infty} X_i}\right)
\]
\[
= p_1 \alpha_1^k, \quad (4.2.5)
\]

Similarly for other terms of (4.2.3) we can show that
\[
\sum_{r=0}^{\infty} p_2 e^{-(\theta_1 + \mu_2)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = p_2 \alpha_2^k \quad (4.2.6)
\]
\[
\sum_{r=0}^{\infty} q_1 e^{-(\theta_1)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = q_1 \alpha_3^k \quad (4.2.7)
\]
\[
\sum_{r=0}^{\infty} q_2 e^{-(\theta_2)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = q_2 \alpha_4^k \quad (4.2.8)
\]
\[
\sum_{r=0}^{\infty} p_1 p_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = p_1 p_2 \alpha_5^k \quad (4.2.9)
\]
\[
\sum_{r=0}^{\infty} p_2 q_2 e^{-(\theta_2 + \mu_2)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = p_2 q_2 \alpha_6^k \quad (4.2.10)
\]
\[
\sum_{r=0}^{\infty} p_1 q_2 e^{-(\theta_1 + \mu_1 + \mu_2)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) = p_1 q_2 \alpha_7^k \quad (4.2.11)
\]
\[
\sum_{r=0}^{\infty} q_1 q_2 e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)r} P\left(\sum_{k=0}^{\infty} X_i = r\right) q_1 q_2 \alpha_5^k = q_1 q_2 \alpha_8^k \quad (4.2.12)
\]

Using the results (4.2.5) to (4.2.12) in (4.2.3)
\[
P(T > t) = \sum_{r=0}^{\infty} \alpha_k(t) \left\{ \begin{array}{l}
\alpha_1^k + p_2 \alpha_2^k + q_1 \alpha_3^k + q_2 \alpha_4^k - p_1 p_2 \alpha_5^k \\
- p_1 q_2 \alpha_6^k - p_2 q_1 \alpha_7^k - q_1 q_2 \alpha_8^k 
\end{array} \right\}
\]

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As in chapter 2, \( L^*(s) \) can be obtained as follows

\[
P(T > t) = \sum_{k=0}^{\infty} \left[ F_k(t) - F_{k+1}(t) \right] \left\{ \begin{array}{l}
\left. p_1 a_1^k + p_2 a_2^k + q_1 a_3^k + q_2 a_4^k - p_1 p_2 a_5^k \right. \\
\left. -p_1 q_3 a_6^k - p_2 q_4 a_7^k - q_1 q_2 a_8^k \right.
\end{array} \right.
\]

(4.2.13)

Since \( L(t) = P(T \leq t) = 1 - P(T > t) \)

\[
L(t) = \sum_{k=0}^{\infty} F_k(t) \left\{ \begin{array}{l}
\left. p_1 a_1^k + p_2 a_2^k + q_1 a_3^k + q_2 a_4^k - p_1 p_2 a_5^k \right. \\
\left. -p_1 q_3 a_6^k - p_2 q_4 a_7^k - q_1 q_2 a_8^k \right.
\end{array} \right.
\]

(4.2.14)

From Gurland, J (1955), Laplace-Stieltje's transform of \( F_k(t) \) is given by

\[
F_k^*(s) = \frac{1}{(1 + bs)^k} \left[ 1 + \left( \frac{kRbs}{(1-R)(1+bs)} \right) \right]^{1-k}
\]

Taking Laplace-Stieltjes transform on both sides of (4.2.14), we get

\[
L^*(s) = \sum_{k=0}^{\infty} \left[ \begin{array}{l}
(1 + bs)^{-k} \\
\left[ 1 + \left( \frac{kRbs}{(1-R)(1+bs)} \right) \right]^{1-k}
\end{array} \right] \times
\left\{ \begin{array}{l}
p_1 a_1^k + p_2 a_2^k + q_1 a_3^k + q_2 a_4^k - p_1 p_2 a_5^k \\
-p_1 q_3 a_6^k - p_2 q_4 a_7^k - q_1 q_2 a_8^k
\end{array} \right.
\]

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\[ L^*(s) = \sum_{m=0}^{\infty} \frac{m^k}{kR(1-m)1-R} \left\{ p_1 a_1 a_1^k + p_2 a_2 a_2^k + q_1 a_3 a_3^k + q_2 a_4 a_4^k - p_1 p_2 a_5 a_5^k \right\} \]

\[ \text{Since} \quad E(T) = -\frac{d}{ds}[L^*(s)]_{s=0} \]  

(4.2.15)

As in Chapter 3, using (4.2.15) in (4.2.16) the mean time for recruitment is found to be

\[ E(T) = \left( \frac{b}{1-R} \right) \left[ \frac{p_1}{a_1} + \frac{p_2}{a_2} + \frac{q_1}{a_3} + \frac{q_2}{a_4} - \frac{p_1 p_2}{a_5} - \frac{p_1 q_2}{a_6} - \frac{p_2 q_1}{a_7} - \frac{q_1 q_2}{a_8} \right] \]  

(4.2.17)

(i.e) \[ E(T) = a \left[ \frac{p_1}{a_1} + \frac{p_2}{a_2} + \frac{q_1}{a_3} + \frac{q_2}{a_4} - \frac{p_1 p_2}{a_5} - \frac{p_1 q_2}{a_6} - \frac{p_2 q_1}{a_7} - \frac{q_1 q_2}{a_8} \right] \]  

(4.2.17) gives the mean time for recruitment.

**SPECIAL CASE**

Suppose \( X_i, i = 1, 2, \ldots \) follows Poisson distribution with parameter \( \lambda \)

\[ a_i = \sum_{r=0}^{\infty} e^{-(\beta + \mu)r} p(X_i = r) \]

\[ = \sum_{r=0}^{\infty} e^{-(\beta + \mu)r} \frac{e^{-\lambda} \lambda^r}{r!} \]
\[
E(T) = a_1 = e^{-(\lambda_1 + \mu)}
\]

\[
= e^{-\lambda_1} e^{-\lambda_2} e^{-\lambda_3} e^{-\lambda_4} e^{-\lambda_5}
\]

Similarly,

\[
a_2 = e^{-(\lambda_1 + \mu)} ; a_3 = e^{-(\lambda_2 + \mu)} ; a_4 = e^{-(\lambda_3 + \mu)} ; a_5 = e^{-(\lambda_4 + \lambda_5 + \mu + \rho)}
\]

\[
a_6 = e^{-(\lambda_1 + \rho)} ; a_7 = e^{-(\lambda_2 + \rho)} ; a_8 = e^{-(\lambda_3 + \rho)}
\]

Using (4.2.18) and (4.2.19) in (4.2.17) we get

\[
E(T) = a_1 = \left[ \frac{p_1}{1-e^{-(\lambda_1 + \mu)}} + \frac{p_2}{1-e^{-(\lambda_2 + \mu)}} + \frac{q_1}{1-e^{-(\lambda_3 + \mu)}} \right]
\]

\[
+ \left[ \frac{q_2}{1-e^{-(\lambda_1 + \rho)}} - \frac{p_1 p_2}{1-e^{-(\lambda_1 + \mu + \rho)}} - \frac{p_2 q_1}{1-e^{-(\lambda_1 + \mu)}} \right]
\]

\[
+ \left[ \frac{p_2 q_1}{1-e^{-(\lambda_2 + \rho)}} - \frac{q_2}{1-e^{-(\lambda_2 + \rho)}} \right]
\]

Equation (4.2.20) gives the meantime for recruitment for the special case.

**NUMERICAL ILLUSTRATION:**

In this section model 1 is numerically illustrated and relevant conclusions are made.
Fixing $\mu_1 = 2; \mu_2 = 0.8; \theta_1 = 0.5; \theta_2 = 0.4; \theta_3 = 0.3; \theta_4 = 0.6; b = 0.2$ and varying $R$ and $\lambda$, the values of $E(T)$ are computed and tabulated in Table 4.2.1

<table>
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<th>$\lambda$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<td>0.5307</td>
<td>0.3748</td>
<td>0.2967</td>
<td>0.2503</td>
</tr>
<tr>
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<td>0.4372</td>
<td>0.3462</td>
<td>0.2920</td>
</tr>
<tr>
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<td>0.9288</td>
<td>0.6558</td>
<td>0.5192</td>
<td>0.4380</td>
</tr>
<tr>
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<td>1.2384</td>
<td>0.8744</td>
<td>0.6923</td>
<td>0.5840</td>
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<tr>
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<td>3.7151</td>
<td>2.6233</td>
<td>2.0769</td>
<td>1.7521</td>
</tr>
</tbody>
</table>

**CONCLUSION:**

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda$ decreases, keeping other parameters fixed. In other words, when the number of exits decreases on the average, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as $R$ increases, keeping other parameters fixed.
(iii) The meantime to the recruitment decreases for negative correlation and increases for positive correlation when \( R \) and \( \lambda \) varies simultaneously.

4.3 MODEL 2

DESCRIPTION AND ANALYSIS OF THE MODEL

Assumptions:

1. An organization having two grades A and B takes policy decisions at random epochs in \([0, \infty)\), and at every decision making epoch, a random number of persons quit the organization.

2. There is an associated loss of manhours to the organization if a person quits and it is linear and cumulative.

3. Each grade has its own threshold level and the threshold distribution has SCBZ property. Recruitment is made whenever the total number of exits crosses in any one of the two thresholds.

4. The inter-decision times are exchangeable and constantly correlated.

5. The process which generates the number of exits and the threshold are mutually independent.

6. The mobility of manpower from one grade to the other grade is not permitted.
Notations:

\( U_i \) : time between the \((i-1)^{th}\) and \(i^{th}\) decision epoch. \( U_i \)'s are exchangeable and constantly correlated exponential random variables.

\( X_i \) : discrete random variable denoting the total number of persons who leave the organization from the two grades at the \(i^{th}\) decision epoch. \( i = 1, 2, \ldots \). \( X_i \)'s are independent and identically distributed random variables.

\( Y_A, Y_B \) : continuous random variables denoting the threshold of levels for the grades A and B respectively and the distribution \( Y_A \) and \( Y_B \) follows SCBZ property. \( Y = \min (Y_A, Y_B) \)

\( V_k(t) \) : probability that there are \( k \) decisions in \([0, t]\).

\( g(.) \) : probability density function of \( X \)

\( g^*(.) \) : Laplace transform of \( g(.) \)

\( g_k(.) \) : \( k \)-fold convolution of \( g(.) \)

\( f(.) \) : probability density function of inter-decision times

\( f_k(.) \) : \( k \)-fold convolution of \( f(.) \)

\( F_k(.) \) : \( k \)-fold convolution of \( F(.) \)

\( H_A(.) \) : distribution function of \( Y_A \) with parameters \( \theta_1, \theta_2 \) and \( \mu_1 \)

\( H_B(.) \) : distribution function of \( Y_B \) with parameters \( \theta_3, \theta_4 \) and \( \mu_2 \)

\( H(.) \) : distribution function of \( Y \).

\( T \) : time for recruitment in the organization
L(t) : distribution function of T
L*(s) : Laplace-Stieltje’s transform of L(t).
R : correlation between any U_i and U_j, i \neq j

\phi(n,x) : \int_0^x e^{-\tau} \tau^{n-1} d\tau

a : mean of U_i, i = 1, 2,\ldots
b : a(1-R)
m : m = m(s) = \frac{1}{1+bs}

E(T) : mean time for recruitment

**MAIN RESULT**

In this subsection an analytical expression for the mean time to recruitment is obtained.

As in chapter 2, the distribution of Y is given by

\[ 1 - H(x) = p_1p_2e^{-(\theta_1+\lambda_1+\mu_2)x} + p_1q_2e^{-(\theta_1+\lambda_1+\mu_2)x} + q_1p_2e^{-(\theta_2+\lambda_1+\mu_1)x} + q_1q_2e^{-(\theta_1+\theta_2)x} \]

(4.3.1)

As in Model 1, it can be shown that

\[ L(t) = \sum_{k=1}^{\infty} F_k(t) \{ p_1p_2 a_5^k + p_1q_2 a_6^k + p_2q_1 a_7^k + q_1q_2 a_8^k \} \]

(4.3.2)

and

\[ L^*(s) = \sum_{k=1}^{\infty} \left[ \left( 1+bs \right)^{-k} / 1 + \left( kR bs / (1-R)(1+bs) \right) \right] \left\{ p_1p_2 a_5^k + p_1q_2 a_6^k + p_2q_1 a_7^k + q_1q_2 a_8^k \right\} \]
\[
E(T) = \frac{d}{ds} \left[ L^*(s) \right]_{s=0}
\]

from (4.3.4) one can find that

\[
E(T) = b \left[ \frac{p_1 p_2 a_5}{a_5} + p_2 q_1 a_6 a_7 + q_1 q_2 a_8 \right]
\]

(i.e) \( E(T) = a \left[ \frac{p_1 p_2}{a_5} + \frac{p_2 q_1}{a_6} + \frac{q_1 q_2}{a_8} \right] \)  (4.3.4)

where \( a, i = 5, 6, 7, 8 \) and \( \bar{a} = 1 - a \) are given by (4.2.4)

Equation (4.3.4) gives the mean time for recruitment.

**SPECIAL CASE**

Suppose \( X_i, i = 1, 2, \ldots \) follows Poisson distribution with parameter \( \lambda \).

\[
a_5 = \sum_{r=0}^{\infty} e^{-(a_1 + \theta_1 + \mu_1 + \mu_2) r} P(X_i = r)
\]
\[ 
E(T) = \sum_{r=0}^{\infty} e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2) r} \frac{\lambda^r e^{-\lambda}}{r!} \\
= e^{-\lambda} \sum_{r=0}^{\infty} \frac{(\lambda e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)})^r}{r!} \\
= e^{-\lambda} e^{\lambda e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)}} \\
(\text{i.e.}) \quad a_5 = e^{\lambda e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)}} 
\]

Similarly,
\[ 
a_6 = e^{\lambda e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)}}; \quad a_7 = e^{\lambda e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)}}; \quad a_8 = e^{\lambda e^{-(\theta_1 + \theta_2 + \mu_1 + \mu_2)}} \] 

(4.3.6)

Now
\[ 
E(T) = a \left[ \frac{P_1 P_2}{1-e^{-(\theta_1 + \mu_1 + \theta_2 + \mu_2)}} + \frac{P_1 q_2}{1-e^{-(\theta_1 + \mu_1 + \theta_2 + \mu_2)}} + \frac{P_2 q_1}{1-e^{-(\theta_1 + \theta_2 + \mu_2)}} + \frac{q_1 q_2}{1-e^{-(\theta_1 + \theta_2 + \mu_2)}} \right] 
\]

(4.3.7)

Equation (4.3.7) is an analytical expression to the mean time for recruitment.

**NUMERICAL ILLUSTRATION:**

In this section model 2 is numerically illustrated and relevant conclusions are made.

Fixing \( \mu_1 = 2; \mu_2 = 0.8; \theta_1 = 0.5; \theta_2 = 0.4; \theta_3 = 0.3; \theta_4 = 0.6; b = 0.2 \) and varying \( R \) and \( \lambda \), the values of \( E(T) \) are computed and tabulated in Table 4.2.1
Table 4.2.1

<table>
<thead>
<tr>
<th>R</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4</td>
<td>0.3246</td>
<td>0.2079</td>
<td>0.1729</td>
<td>0.1581</td>
<td>0.1508</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.3787</td>
<td>0.2425</td>
<td>0.2018</td>
<td>0.184</td>
<td>0.1760</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5681</td>
<td>0.3638</td>
<td>0.3026</td>
<td>0.2766</td>
<td>0.2640</td>
</tr>
<tr>
<td>0.4</td>
<td>0.7574</td>
<td>0.4850</td>
<td>0.4035</td>
<td>0.3688</td>
<td>0.3520</td>
</tr>
<tr>
<td>0.8</td>
<td>2.2723</td>
<td>1.4551</td>
<td>1.2106</td>
<td>1.1065</td>
<td>1.0559</td>
</tr>
</tbody>
</table>

CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda$ decreases, keeping other parameters fixed. In other words, when the number of exits decreases on the average, the mean time for recruitment increases.

(ii) The mean time to recruitment increases as R increases, keeping other parameters fixed.

(iii) The meantime to the recruitment decreases for negative correlation and increases for positive correlation when R and $\lambda$ varies simultaneously.