CORTISOL SECRETION OF PARKINSONIOn SUBJECTS USING BIRTH-DEATH PROCESS
CHAPTER VIII

CORTISOL SECRETION OF PARKINSONIAN SUBJECTS USING BIRTH-DEATH PROCESS

8.1 INTRODUCTION

Finally Chapter eight, is devoted to the study of “Stochastic Model for Cortisol Secretion of Parkinsonian subjects using Birth-Death Processes”[106]. The current study was to evaluate Cortisol responses to an experimental psychologic stressor. The plasma Cortisol concentration has been considered as on index of CRF-ACTH secretion. No consistent change in both plasma Cortisol levels and Urinary steroid excretion has been observed in patients with Cushing’s disease after either rapid administration of L-dopa or prolonged therapy with L-dopa. In patients with Parkinson’s disease receiving L-dopa for 3 or 4 weeks the baseline Urinary steroids excretion is not significantly modified [13][118]. However, the Chronic treatment of parkinsonian subjects[49][134] as well as the acute administration of L-dopa in normal subjects, blunts or completely inhibits the plasma Cortisol response to insulin induced hypoglycemia and other stressful conditions.

In experiments evidence has been obtained the dopamine, rather than norepinephrine is the mediator which inhibits ACTH response to hypoglycaemic stress after L-dopa administration. The blockade of dopamine β hydroxylase with disulfiram did not overcome the L-dopa induced inhibition of the plasma Cortisol response to hypoglycaemia.
In this paper a birth-death process \([25][37][68][75]\), \(X \equiv \{x(t), t \geq 0\}\), will always be a process taking values in \(N = \{0, 1, 2\}\) with birth rates \(\{\lambda_n, n \in N\}\) and death rates \(\{\mu_n, n \in N\}\). Two such processes with transition functions \(\{p_{ij}(t)\}\) and \(\{\bar{p}_{ij}(t)\}\). They are similar if, for all \(i, j \in N\), there are constants \(c_{ij}\) such that

\[
\{\bar{p}_{ij}(t)\} = c_{ij}p_{ij}(t) \text{ for all } t \geq 0.
\]

8.2 NOTATIONS

\(\lambda\) is the mean value of first four points in the medical graph.

\(\mu\) is the mean value of last points of the medical graph.

\(\lambda_n(x)\) - the birth rates. \(n \in N\).

\(\mu_n(x)\) - the death rates \(n \in N\).

8.3 EXAMPLE

Plasma Cortisol and blood glucose responses to insulin-induced hypoglycaemia in basal conditions after 100mg of L-dopa one hour before insulin administration and after the same dose of L-dopa in healthy disulfiram-primed subjects. This finding is consistent with the report that the increase found in plasma Cortisol in the early morning was completely suppressed in 20 subjects. Every 30 mts. collect the saliva sample.
8.4 FAMILIES OF SIMILAR BIRTH–DEATH PROCESS

In this context a birth-death process $\mathcal{X} = \{x(t), t \geq 0\}$, will always be a process taking values in $\mathbb{N} = \{0, 1, 2, \ldots\}$ with birth rates $\{\lambda_n, n \in \mathbb{N}\}$ and death rates $\{\mu_n, n \in \mathbb{N}\}$.

Two such processes with transition functions $\{p_{ij}(t)\}$ and $\{\tilde{p}_{ij}(t)\}$ are said to be similar if

For all $i, j \in \mathbb{N}$, there are constants $c_{ij}$ such that

$$\{\tilde{p}_{ij}(t)\} = c_{ij}p_{ij}(t) \text{ for all } t \geq 0.$$
We will assume that the transition functions \( \{ p_{ij}(t), i, j \in \mathbb{N} \} \) of the birth–death process \( \mathcal{X} = \{X(t), t \geq 0\} \) with birth rates \( \{\lambda_n, n \in \mathbb{N}\} \) and death rates \( \{\mu_n, n \in \mathbb{N}\} \) satisfying \( \sum_{n=0}^{\infty} (\pi_n + (\lambda_n \pi_n)^{-1}) = \infty \) and if \( \mu_0 > 0 \), we let.

\[
\alpha_n = \lambda_n + \mu_n, \quad \beta_{n+1} = \lambda_n \mu_{n+1}, \quad n \in \mathbb{N}
\]

The question of which birth-death processes are similar to \( \mathcal{X} \) may now be phrased as follows. Can we identify, besides \( \{\lambda_n, n \in \mathbb{N}\} \) and \( \{\mu_n, n \in \mathbb{N}\} \) all other sets of birth rates \( \{\bar{\lambda}_n, n \in \mathbb{N}\} \) and death rates \( \{\bar{\mu}_n, n \in \mathbb{N}\} \) such that.

\[
\bar{\alpha}_n + \bar{\mu}_n = \alpha_n, \quad \bar{\lambda}_n \bar{\mu}_{n+1} = \beta_{n+1}, \quad n \in \mathbb{N}
\]

This problem can be transformed into a problem involving chain sequences.

A sequence \( \{a_n\}_{n=1}^{\infty} \) is a chain sequence if there exists a second sequence \( \{g_n\}_{n=0}^{\infty} \) such that

\begin{enumerate}
  \item \( 0 \leq g_0 < 1, \quad 0 < g_n < 1, \quad n = 1,2, \ldots \)
  \item \( a_n = (1-g_{n-1}) g_n, \quad n = 1,2, \ldots \)
\end{enumerate}

The sequence \( \{g_n\} \) is called a parameter sequence for \( \{a_n\} \).

If both \( \{g_n\} \) and \( \{h_n\} \) are parameter sequence for \( \{a_n\} \) then.

\[
g_n < h_n, \quad n = 1,2, \ldots \quad \text{if } g_0 < h_0.
\]
Every chain sequence \( \{a_n\} \) has a minimal parameter sequence \( \{m_n\} \) uniquely determined by the condition \( m_0 = 0 \), and it has a maximal parameter sequence \( \{M_n\} \) characterized by the fact that \( M_0 > g_o \) for any other parameter sequence \( \{g_n\} \). For every \( x \), \( 0 \leq x \leq M_0 \) here is a unique parameter sequence \( \{g_n\} \) for \( \{a_n\} \) such that \( g_0 = x \).

Returning to the context of the birth-death process \( \mathcal{X} \), we let

\[
\gamma_n = \frac{\beta_n}{\alpha_{n-1}\alpha_n} \quad n = 1, 2, \ldots
\]

And observe that \( \{\gamma_n\}_{n=0}^\infty \) is a chain sequence. Since we can write

\[
\gamma_n = \left(1 - \frac{\mu_{n-1}}{\lambda_{n-1} + \mu_{n-1}}\right) \frac{\mu_n}{\lambda_n + \mu_n}
\]

So that \( \{\mu_n / (\lambda_n + \mu_n)\} \) constitutes a parameter sequence for the chain sequence \( \{\gamma_n\} \). Our task is now to find all parameter sequences for the chain sequence \( \{\gamma_n\} \). Since there is a one-to-one correspondence between parameter sequences for \( \{\gamma_n\} \) and sets of birth and death rates satisfying indeed, for every parameter sequence \( g = \{g_n\} \) we can construct the corresponding birth rates \( \{\lambda_n^{(g)}\} \) and death rates \( \{\mu_n^{(g)}\} \) by letting,

\[
\lambda_n^{(g)} = \alpha_n (1 - g_n), \quad \mu_n^{(g)} = \alpha_n g_n \quad n \in \mathbb{N}
\]

The problem of identifying all parameter sequences for a chain sequence for which one parameter sequence is known. In our setting the solution may be formulated as follows.
Case i. \((\mu_0 = 0)\) Let

\[ S_{-1} = 0, \quad S_n = \lambda_0 \sum_{k=0}^{n} (\lambda_k \pi_k)^{-1}, \quad n \in \mathbb{N}, \]

and \( S = \lim_{n \to \infty} S_n \)

(Possibly \( S = \infty \)). Then all parameter sequences for \( \{\gamma_n\} \) are given by

\[ \{g_n(x)\}, \quad 0 \leq x \leq \frac{1}{S}. \]

Where

\[ g_0(x) = x, \quad g_n(x) = \frac{\mu_n}{\lambda_n + \mu_n} \frac{1 - xS_{n-2}}{1 - xS_{n-1}} \quad n \geq 1. \]

It follows in particular that \( \left\{ \frac{\mu_n}{\lambda_n + \mu_n} \right\} \) is the only parameter sequence for \( \{\gamma_n\} \) if \( S = \infty \).

Case ii \((\mu_0 > 0)\). Let

\[ T_{-1} = 0, \quad T_n = \mu_0 \sum_{k=0}^{n} (\mu_k \pi_k)^{-1}, \quad n \in \mathbb{N}, \]

and \( T = \lim_{n \to \infty} T_n \)

Possibly \( T = \infty \). Then all parameter sequence for \( \{\gamma_n\} \) are given by

\[ \{g_n(x)\}, \quad -\infty \leq x \leq \frac{1}{T} \]

where
$$g_n(x) = \frac{\mu_n}{\lambda_n + \mu_n} \frac{1 - xT_{n-1}}{1 - xT_n} \quad n \in \mathbb{N},$$

It is interesting to observe that the maximal parameter sequence is obtained for $x=1/T$. So the sequence $\left\{ \frac{\mu_n}{\lambda_n + \mu_n} \right\}$ is the maximal parameter sequence for $\gamma_j$ if $T = \infty$.

Translating these results in terms of birth and death rates we obtain the following theorem.

**Theorem 8.1**

A birth-death process $\mathcal{A}$ with birth rates $\{\lambda_n, n \in N\}$ and death rates $\{\mu_n, n \in N\}$ is not similar to any other birth-death process if and only if $\mu_0 = 0$ and $\sum_{n=0}^{\infty} (\lambda_n \pi_n)^{-1} = \infty$.

In the opposite case the process is similar to any member of an infinite, one parameter family of birth-death processes $\{\mathcal{A}^{(x)} \mid 0 \leq x \leq 1/S\}$ if $\mu_0 = 0$,

and $\{\mathcal{A}^{(x)} \mid \infty \leq x \leq 1/T\}$ if $\mu_0 > 0$. The birth rates $\lambda_n(x)$, $n \in N$ and death rates $\mu_n(x)$, $n \in N$, of $\mathcal{A}^{(x)}$ are given by

$$\mu_0(x) \equiv \lambda_0(x),$$

$$\lambda_n(x) \equiv \lambda_n \frac{1 - xS_n}{1 - xS_{n-1}},$$

$$\mu_{n+1}(x) \equiv \mu_n \frac{1 - xS_{n-1}}{1 - xS_n}, n \geq 0.$$
\[ \lambda_n(x) \equiv \lambda_n \frac{1 - xT_{n+1}}{1 - xT_n} \]

\[ \mu_n(x) \equiv \mu_n \frac{1 - xT_{n-1}}{1 - xT_n}, \ n \geq 0 \]

If \( \mu_0 > 0 \), the quantities \( S_n \equiv \lambda_0 \sum_{k=0}^{n-1} (\lambda_k \pi_k)^{-1} \ n \in \mathbb{N} \).

and \( S \equiv \lim_{n \to \infty} S_n \)

If \( \mu_0 > 0 \), the quantities \( T_n \equiv \mu_0 \sum_{k=0}^{n-1} (\mu_k \pi_k)^{-1} \ n \in \mathbb{N} \).

and \( T \equiv \lim_{n \to \infty} T_n \)

We first want to make some additional remarks on the example already discussed by [37]. So let \( \{x(t), t \geq 0\} \) be the birth-death process with constant birth rates \( \lambda_n = \lambda \) and constant death rates \( \mu_n = \mu \). Since \( \mu_0 > 0 \) this process is transient. The first to that its transition functions can be represented by

\[ p_{ij}(t) = \left( \frac{\lambda}{\mu} \right)^{j-1} \frac{1}{\sqrt{\mu}} e^{-(\lambda+\mu)t} \{ I_{j-1}(2t\sqrt{\lambda\mu}) - I_{j+1}(2t\sqrt{\lambda\mu}) \}. \ t \geq 0. \]

Where \( I_n(.) \) is the nth modified Bessel function, It is readily seen that the quantities \( T_n \),

\[ T_n = \frac{\lambda^{n+1} - \mu^{n+1}}{(\lambda - \mu)\lambda^n} \ n \in \mathbb{N}. \]

\[ T = \begin{cases} \frac{\lambda}{(\lambda - \mu)} & \text{if } \lambda > \mu \\ \infty & \text{if } \lambda \leq \mu \end{cases} \]

Hence we conclude from the theorem that for each \( x \) in the interval

\(-\infty \leq x \leq 1/T.\)
The process $\mathcal{X}^{(x)}$ with rates $\lambda_n(x) = \lambda + \mu - \mu_n(x)$

$$
\mu_n(x) = \lambda \mu \frac{(\lambda - \mu) \lambda^{n-1} - x(\lambda^n - \mu^n)}{(\lambda - \mu) \lambda^n - x(\lambda^{n+1} - \mu^{n+1})} \quad n \in \mathbb{N}.
$$

is similar to $x$, in accordance with [37].

In Fig. 8.2. We show graphs of $\mu_n(x)$ for $n \in \mathbb{N}$. $\lambda$ is the mean value of first four points in the medical graph using birth process and $\mu$ is the mean value of last four points of the medical graph using death process.
8.1 TABLE

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<th>$\mu_0(x)$</th>
<th>$\mu_1(x)$</th>
<th>$\mu_2(x)$</th>
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Using medical value we get the Calculation result of $\mu_n(x)$.

y axis: $\mu_n(x)$.  Series 5: $\mu_0(x)$,  Series 4: $\mu_1(x)$,  Series 3: $\mu_2(x)$,  Series 2: $\mu_3(x)$,  Series 1: $\mu_4(x)$
8.5 CONCLUSION

If we consider a birth-death process \( \mathcal{X} \equiv \{x(t), t \geq 0\} \) taking values in the finite set \( \mathbb{N} \equiv \{0,1,2,\ldots,n\} \) with birth rates \( \lambda_n, n \in \mathbb{N} \) and death rates \( \mu_n, n \in \mathbb{N} \), all strictly positive except \( \mu_0 \) and \( \lambda_N \), which may be equal to 0. When \( \mu_0 > 0 \) the process may escape from \( \mathbb{N} \), via 0, to an absorbing state -1, and when \( \lambda_N > 0 \) the process may escape from \( \mathbb{N} \) via \( N+1 \) to an absorbing state \( N+1 \). The model is fitted with the birth-death processes taking the values in \( \mathbb{N} = \{0,1,2,\ldots\} \) but allow the death rate in state 0 to be a positive so that escape from \( \mathbb{N} \) is possible. The model is fitted with birth and death process and gives good results than the medical report.