Preface
In the evolution of any scientific discipline there is a period in which attempts are made to develop mathematical theories in order to account for and explain the observations generated by the phenomena with which the discipline is concerned. The theoretical and experimental observations can then be compared in order to see if the model is a reasonable one i.e. if it is capable of accounting for the experimental observations obtained under the conditions stipulated by the model.

A physical quantity is defined by a series of operations and calculations, of which it is a result. New definitions are needed to explain any advanced study of mechanical behaviour. The methods of evaluating the efficiency of materials and of investigating mechanics of their fracture suggested fracture Mechanics quite important in material science. Cracks in materials and composites usually appear during a stage of manufacture or in a very early stage of use.

Fatigue is an important consideration in structural design and for continued operation of safety critical structures and components. The problem of fatigue offers immense scope for advanced work during different stages of operation and maintenance of structures. Due to uncertainty in the fracture, probability distributions play an important role in failure modeling. In chapter 1 we deal with different aspects of the random fatigue problem. In the first section we give basic definitions and discuss the distributions used
in failure modeling. Different approaches of random fatigue problem is dealt in section 2. Characteristics of some basic damage models are given in section 3. Section 4 deals with general shock models.

It has been recognised that fatigue failure in materials result from the nucleation and propagation of cracks. However during their course of propagation the cracks encounter various types of metallurgical structures and imperfections so that the rate of propagation is in general varying in time. On the other hand even in well controlled laboratory condition results obtained from crack growth experiments usually show a considerable statistical dispersion. The experiments show also that the fatigue life of real mechanical components is characteristically random. Randomness of a fatigue process is completely evident if a structure is subjected to time varying random loading.

Various researchers in this field have considered the basic equation representing fatigue crack growth under varied set up. In chapter 2, we give the various models for random fatigue crack growth. We give an elaborate account of the work by Sobczyk [76] and others in this chapter. Crack growth model under random loading is dealt at length in section 1. Crack growth is given as a differential equation model in section 2 and as a Markov chain model in section 3. Crack propagation is illustrated as a diffusion model in section 4. We have found that stochastic modeling given by Keith Ortiz [38] is best suited for fatigue crack growth.
Traffic growth has always been much greater than predicted and with the development of new materials whose properties cannot be obtained using empirical methods, the need to predict the remaining lives of pavements and the design of new pavements to withstand heavier traffic loadings with new axle and suspension configurations require the use of an analytical method as the traditional empirical ones cannot cope. Following the work of John Maddison Read [36] on 'Fatigue cracking of Bituminous paving mixtures', we give an elaborate account of fatigue cracking of bituminous paving mixtures in chapter 3. Starting with basic definitions in section 1, we have given modes of failure of bituminous mixtures in section 2. A general background to fatigue of bituminous mixtures is explained in section 3. Different stages of cracking and varied fatigue test methods and limitations are given in section 4 of this chapter.

Study of design, construction and maintenance of pavements have become inevitable on the pavements connecting various places in the entire extent of any developing country. Fatigue cracking of pavements and structures is dealt in detail in chapter 4. Section 1 deals with common distress in pavements. Reflection cracking in pavements with and without stress absorbing membrane interfaces has been analysed with supportive data obtained from Bangalore University in section 2. The glory of earlier kingdom is reflected in the architecture and other legacy of the rulers. In almost all countries the basic taste of the rulers have been one of erecting monuments which are based on the culture and intellect of the designers in those periods. As days pass by, new rulers have to take up the preservation
and maintenance of such structures. These structures when exposed to natural calamities are affected by cracks which grow and damage the structures. The causes are presented in detail in section 3 of this chapter, with the help of “Hand book of Indian Standards Institution” [31].

Composites are exciting materials which are finding increasing applications in aerospace, defence, transportation, communication, power, electronics, recreation, sporting and numerous other commercial and consumer products. The potential pay off for composite materials is so high that they have become one of the fastest developing research and development areas in materials science. Rapid advancement in the science of fibres, matrix materials, processing, interface structure, bonding and their characteristics on the final properties of composites have taken place in recent years. The technological developments in composite materials are responsible for partially meeting the global industrial demand for materials with improved performance capabilities.

Chapter 5 presents Fracture Mechanics for Composites. The entire classification of composites is presented in the first section. Solutions for failure processes in composites have been taken up for study in section 2. A particular composite namely carbon fibre composite is a material for our study and we have presented the cumulative damage model for fracture in composites in section 3. Inference problem and the estimating procedures for parameters in a stationary process is dealt in section 4. A statistical analysis
of short fatigue crack growth leading to a Markov chain model is dealt with in the last section 5.

Almost all artificial materials used in Engineering practice fall into the class of composites. Cracks in composites usually appear during a stage of their manufacture or in a very early stage of use. Here we give some of the most important effects responsible for the growth and inhibition of cracks in composite materials which determine the mechanism of failure of these materials. Chapter 6 deals with specific types of fracture growth in composites. Based on the studies of Cherepanov [14] parameters influencing crack growth in composites are studied in section 1. Unidirectional fibre composites are important structural elements of many modern composite materials. The fracture of these fibre composites and the field of stresses and strains using stress intensity factors is dealt with in section 2. Growth of transverse cracks in unidirectional fiber composites considering the statistical properties of individual fibres has been studied in section 3. Fatigue failure of composite materials under multiaxial cyclic loading is an important concern in the application of composite structures. A fatigue failure theory and model for the composite laminate under multiaxial loading is given in Section 4.

Chapter 7 gives an idea of the Mathematical techniques that can be used in Fracture Mechanics. To deal with the interplay between more than two basic mechanical behaviour we need advanced tools like tensor calculus. The continuum mechanics is well studied through tensor calculus by
mathematicians such as Eddington [22] Sokol-Nikoff [77] and others. As the path traced by the crack growth in any surface is governed by generalized Hook's law and other considerations, we have given a detailed derivation of this, and with particular relevance to the study of fracture mechanics. Covariant derivative and its application to the solution of mass energy equation and generalized continuum is given in section 1. The subject of topology is of interest in its own right, and it also serves to lay the foundations for future study in analysis, in geometry and in algebraic topological explanations of Engineering problems. Homotopy forms and the fundamental concepts in the study of Algebraic topology, helps us in identifying contractible surfaces. This finds its utility in identifying the possible path for crack growth on a surface. We give specific concepts of homotopy due to S.T. Hu [84] used in our study in section 2.

One of the simplest of all the types of stochastic processes is a Markov chain. A Markov chain is completely specified by its Transition probability matrix and 'initial distribution'. We have developed a Markov chain to explain the crack growth in "F 1140 steel, a ferrite perlite steel with carbon", with a background of doubly stochastic materials, in section 3.

We have presented a mathematical description of some of the most important effects responsible for the growth and effect of cracks in structural, building materials and composite materials which determine the mechanism of the failure of these materials in the thesis. We have adopted modern
applicable mathematical tools in dealing with almost related and unrelated mechanical structures.

The concern of this thesis is with the basic foundation of probability theory and advanced mathematical structures such as stochastic process in the formulation development and solution of crack growth problem.

In this thesis we have unified the different approaches of failure modeling in the study of mechanical structures in the design and manufacture. We have presented expository account of deterministic and stochastic modeling procedures used in the study of crack propagation. Crack problems visualised in traffic and related pavements with the use of bituminous and other structural materials are studied at length with supportive secondary data and the conclusions drawn are found to be in agreement with the theory.

The use of modern algebraic, topological and tensor tools have come in handy for illustration and we hope these techniques would be of immense use in placing the fracture Mechanics study in the most general abstract set up.