PREFACE

The theory of stochastic process is concerned with the structure of families of random variables $X_t$ where $t$ is a parameter running over a suitable index set $T$. The main elements distinguishing stochastic process are in the nature of the state space, the index parameter and the dependence relations among the random variables.

Martingale concepts and methodology have provided a far-reaching apparatus vital to the analysis of all kinds of functionals of stochastic processes. The martingale property expresses a relation that occurs in numerous contexts and has become a basic tool in both theoretical and applied probability. Martingale theory requires an extensive use of conditional expectations.

The first chapter is devoted to a study of characterization of stochastic process by means of stochastic integrals. The notion of infinitely divisible and stable distribution is discussed. A review of the recent work on the characterization of Wiener and stable process and connected results through stochastic integrals is observed. Also the characterization of symmetric stable laws through regression properties is discussed. The system of random variables defined by system of Walsh functions is proved as a Wiener process. The end of this chapter is dealt about the characterization of stable, symmetric and Wiener process under the assumption that two stochastic integrals are identically distributed.

The parallel behaviour of harmonic functions with martingales are analysed in
the second chapter. The extrapolation method suggested by Burkholder is given analogous results, by carried over to harmonic functions. A few elementary propositions from standard martingale theory are proved and some of the inequalities for continuous function which satisfies the growth condition are derived. A fruitful analogy in harmonic analysis, based on the idea of subordination is given. A number of equivalent characterizations of conformal martingale are proved. The remarkable property that the stratonovich solution coincides with the Ito solution is discussed. The basic inequalities for real martingales and elementary properties of bounded mean oscillation martingales are derived.

A detailed study of stochastic approach to variational principles are discussed in the third chapter. The problem of stochastic calculus of variations using dynamic programming is derived. The structure of jump process though conditional distribution and a local description is presented. The representation of all local martingales on the sigma fields generated by such a process as integrals with respect to certain fundamental family of martingales is explained. An optimal control problem on queues, using the calculus of martingales is suggested.

A passage from a Markov chain to a Martingale is developed in the fourth chapter. The limiting probabilities of a Markov chain is identified as points on the torus, using the major principles on differential geometry. The theory of algebra-geometric and analytic phenomena into very simple statements about the combinatorics of cones in affine spaces over the reals is discussed.
A simpler proof for the threshold theorems due to Williams and Whittle using Rajarashi technique is given in the fifth chapter. Some simple applications of martingales to epidemic models are discussed. The relation between Downston's model and the general epidemic is derived and also a generalisation of Daniel's classical results is presented. Better approximation results are given by taking the parameters as functions of infectives. The threshold results for the deterministic and stochastic versions of the homogeneous SI model with recruitment death due to the disease are compared. Finally, a model is developed for the spread of an epidemic in a closed, homogeneously mixing population in which new infections occur at rate differs from the rate of standard general epidemic.

An algorithm for analyzing finite security markets possess a duality structure is given in the sixth chapter, in contrast to functional analytic derivation. The central result about the notion of arbitrage opportunities is explained. A probabilistic method for establishing the basic equivalence between the absence of arbitrage opportunities and the existence of a certain martingale measure is obtained.

At the end of this dissertation, the geometrical aspects of the theory of martingale is identified. A geometrical proof for "Doob decomposition theorem" is given. Also an isomorphism is identified between the stochastic matrices under multiplication and the semigroup of linear endomorphism. It is also pointed out that the vertices of a simplex are nothing but the rows of a stochastic matrix. Using the embedding technique, the special elements such as regular, idempotent of the semigroup from the simplex are identified at the end of this last chapter.