CHAPTER - I

INTRODUCTION
To enhance the Reliability of a system, many ways have been devised. Redundancy has been the foremost tool in this direction. Taking into consideration the situation and the requirements, it has been utilized in different ways as 2-unit standby redundant system, a multiple-unit standby redundant system [58], also by allocation of redundant components to various locations as shown by Moore and Shannon [52]. Reliability of the system is considerably improved by having the provisions of repair, and repair coupled with preventive and/or corrective maintenance. Barlow and Proschan [9,10] have dealt the problems of P.M. (Preventive Maintenance) as replacement problems. Morse [59] considered 1-unit system with repair and P.M., 2-unit standby redundant system with repair and P.M. have been discussed in many papers like [61]. Regarding undergoing P.M., studies have been carried out when an operative unit operates for a certain specified time [43], or for a time whose distribution is general [61], without failing, the operation of the working unit stops for P.M. In this work, Corrective Maintenance (CM) as well as P.M. are being performed besides repair of the failed unit — on the operative unit by scheduling with the help of random checks. Here, CM means the repair of hidden faults (non-self reporting) of the system that could only be detected while the unit is under operation and P.M. carries the usual meaning as general maintenance of the system to ensure faultfree, smooth functioning of the system. According
to the situation \( H_i/C_i \) could be performed on the operative unit as well as by stopping the functioning of the unit. The provision of random checks has been also inducted by Schneeweiss [82] for finding hidden faults of the system. But he assumes that the time to repair the hidden faults detected at a random check, is negligible. Whereas in this work, it is assumed that the repair time of this hidden fault is a random variable whose distribution is general.

**Historical Background**

In the past, although the term 'reliability' has not been used specifically, yet from time immemorial, person is striving hard to achieve directly or indirectly maximum reliability in every sphere of life with the least efforts. Due to complexity, sophistication and automation inherent in present-day technology, the word 'reliability' has become the concern of operation management in industry, government, business and defence. As a matter of fact, after the first world war, the urgent need of reducing the average number of failures of aircrafts, led to the concept of reliability.

An early as 1932, the area of machine maintenance had received the attention of Khintchine and of C.Palm in 1947. The experiences of A.K.Erlang, C.Palm and others in handling the problems of telephone trunking were instrumental in evolving the techniques. Erlang and Palm introduced poisson distribution as an input distribution of calls in telephone trunking. But
Khintchine (1960) and Osokov provided the final rigorous proofs. The fatigue life in materials was studied by Weibull in the late 1930's. The related topics of extreme value theory were studied by Gumbel (1935) and Epstein (1948) among others. The Weibull distribution, to represent the life length of material, was given by Weibull in 1939. The normal and gamma distributions were given by H.E. Daniels in 1945 and Z.W. Birnbaum and S.C. Saunders in 1958.

The applications of renewal theory of replacement problems were studied by A.J. Lotha in 1935 and N.R. Campbell in 1941. But Feller (1941, 1949) developed renewal theory as a mathematical discipline. The summary of the known mathematical results in renewal theory was presented by Walter Smith in 1958. The area of quality control received greater attention by statisticians in 1940's.

Since the mid-1950 much work has been done on reliability analysis. Since 1950 Aeronautical Radio Inc. (ARINC) has concentrated on military reliability problems in addition to analysis of defective tubes. To improve the reliability of the electronic equipments, in 1952, Defence Department in America formed an Advisory Groups on Reliability of Electronic Equipment (AGREE) and its first report came out in June 1957.

The basic definition of reliability was presented by Robert Lusser of R and D division, Red Stone Arsenal at a Symposium in San Diego in 1952. The assumption of an exponential failure distribution in life testing is the works of
D.J. Davis (1952), Epstein Sabel (1953) and Agree Report (1957). The Weibull Distribution received greater importance in many life testing procedures, with the works of J.H. Kao (1956, 1958) and Zelens Dannemiller papers (1959). G.P. Steck (1957), J.R. Rosenblatt (1963), and A. Madausky (1958) worked on the missile reliability problems. The papers of Moore and Shannon were related to relay network reliability. In 1956, the report of G. Weiss introduced the semi-Markov process to problems of system maintainability. In 1958 Walter Smith presented many mathematical known results in renewal theory. The study of Tate on the minimum unbiased estimate for the probability that a system would survive a specified time based on n observed times of failure, is important. Repairman problems received the attention of Feller (1957), Cox and Smith (1961), Tuckacs (1962) and Barlow (1962). In 1960 Epstein B and Hosford J.E. discussed two unit redundant systems. In 1967 Jewell W.S. analysed the general repair time in case of two-element redundant system.

Besides finding the reliability of the system, studies have been carried out to evaluate other measures as system availability and mean time to system failure (MTSF) etc., using different techniques to suit the requirements. Branson and Shah [12] applied Semi-Markov Process (SMP) method when repair time distribution was general but time to failure remained exponential. Srinivasan and Gopalan [92] applied regenerative point technique in their model. Thus studies like
irregular short supervision, delayed repair, preventive maintenance, n-spare system with a single repair facility, imperfect switching, k out of -n F system etc. were made by different authors in the 1970's. Scheduling of maintenance by periodic checks is considered in many studies [9, 10, 61]. W.G. Schneeweiss [82] inducted the provision of random checks.

Recently K. Murari and C. Maruthachalam apply supplementary variable technique to analyse the models with alternating periods in many papers [56]. Murari and Vibha Goyal introduce different types of repair facilities in the models of reliability theory [31]. The concept of random shocks and Preventive Maintenance has been inducted in some work by Murari and A.A.H. Al-Al [1].

Reliability

Reliability means the ability of a system to preserve the properties necessary to serve its purpose under the normal conditions of operation in the course of the stipulated period of time. In the literature, reliability has been defined in many ways, but the definition which has been mostly accepted by the reliability authorities is given by the Electronic Industries Association (EIA) U.S.A. which states:

'Reliability is the probability of an item performing its intended function over a given period of time under the operating conditions encountered'

This definition stresses four significant elements, namely:

1. Probability
2. Intended function
3. Time
4. Operating conditions
These four terms play an important role in the theory of reliability. Let us discuss each of them one by one.

**Probability** : Prolonged observations of the occurrence or nonoccurrence of an event \( A \) for a large number of repeated trials that occur under an invariable set of conditions show that for a broad range of phenomena the number of occurrences or nonoccurrences of \( A \) obeys stable laws. Namely, if we denote by \( m \) the number of occurrences of an event \( A \) in \( n \) independent trials, it is found that the ratio \( m/n \) for sufficiently large \( n \) in the majority of such series of observations is almost constant. Large deviations being progressively rarer as the number of trials is increased.

So, the limit \( \lim_{n \to \infty} m/n \) is called the probability of the occurrence of the event \( A \).

**Intended function** : In all cases, criteria has to be established specifying or defining clearly what is considered to be the intended function of the item and this intended function should be performed satisfactorily. For an example, if the intended function is to travel from Dehradun to Delhi in a Car in a stipulated period and while performing the task (journey) some minor problem (say one of the four spark-plugs becomes defective) is being created, but the journey is completed in a stipulated period, then it is said that the intended function is performed satisfactorily. Whereas if some thing goes wrong with the engine - a major breakdown - resulting which the journey may not be performed in a specified period and the intended function is not performed satisfactorily.
Time: A statement about the duration of functioning or the intended life of an item is a prerequisite for discussing the reliability of the item. Since reliability stated as a probability of performing a system function for the stipulated period of time.

Operating conditions: If the operating conditioning of a system are changed, its performance may be changed to the extent that the system becomes totally inoperative. Operating conditions as acceleration, acoustics, corrosive atmosphere, gravity, humidity, pressure, vibration, voltage, torque, etc. if changed, the performance of the system is bound to be effective. Thus defining reliability, the operating conditions under which a system functions, should be included.

With the help of reliability only probabilities or averages are predicted, it can not be used to predict discrete events. Reliability does not predict whether a given device will function for an abnormally long period of time or it will fail immediately after being put into operation.

Quantitative definition: If \( T \) is the time to failure of a device (\( T \) is itself a random variable), then the probability that the device will work without failure in the given operating conditions upto time \( t \), is defined as the reliability \( R(t) \) of the device,

\[
R(t) = P \{ T > t \} \tag{1.1}
\]

The device or mechanism is assumed to be working properly in the beginning i.e. at \( t = 0 \) and the equipment can not function
for ever without failure. In terms of reliability, this statement can be expressed as \( R(0) = 1 \) and \( R(\infty) = 0 \), \( R(t) \) is a non-increasing function between these two limits. Generally it is decreasing but during certain periods such as periods of rest without failure it remains constant. If unreliability to time \( t \) is \( Q(t) \), then

\[
Q(t) = 1 - R(t) 
\]  

(1.2)

By knowing the nature of failure of a unit, its reliability could be easily found out. To find it out let us suppose

\[
r(t)\Delta t = P \{ \text{failure in interval } (t, t+\Delta t) | \text{given that the component (unit) has operated upto time } t \} 
\]

\[
= P \{ t < T \leq t + \Delta t | T > t \} 
\]  

(1.3)

Applying 1.1 and the theorem of joint probability

\[
R(t+\Delta t) = R(t)[1-r(t)\Delta t] 
\]

\[
\frac{1}{R(t)} \cdot \frac{R(t+\Delta t) - R(t)}{\Delta t} = -r(t) 
\]

when \( \Delta t \to 0 \)

\[
\frac{1}{R(t)} \cdot \frac{dR(t)}{dt} = -r(t) 
\]

on integrating

\[
\int \frac{dR(t)}{R(t)} = -\int_{0}^{t} r(t) dt 
\]

\[
\frac{R(t)}{R(0)} = -\int_{0}^{t} r(t) dt 
\]

\[
\log e R(t) = -\int_{0}^{t} r(t) dt 
\]

By using \( R(0) = 1 \)

\[
R(t) = \exp (-\int_{0}^{t} r(t) dt) 
\]  

(1.4)
\( r(t) \) is called instantaneous failure rate or hazard rate. When \( r(t) \) is independent of time, 
\[
R(t) = e^{-r \cdot t}
\]
and \( r \) is known as constant exponential failure rate.

**Systems**: Systems (engineering systems) are composed of subsystems, units and components. The units of the system are connected in different ways to give us different system configurations as series, parallel, a combination of series, parallel, a combination of series parallel configurations. In addition we have also standby and \((k \cdot n)\) system configurations.

**Series configuration**: This configuration is depicted in Fig. 1.1. All the units of the system are connected in a chain form. That is why sometimes it is also called chain system. In this case the system fails if any of the unit of the system fails.

If \( R_i(t) \) is the probability of functioning the \( i \)th unit without failure in time \( t \), the reliability of the system with \( n \) such units, connected in series, is given by
\[
R(t) = R_1(t), R_2(t), \ldots, R_n(t)
\]
\[
= \prod_{i=1}^{n} R_i(t)
\]
Thus the reliability of a system whose units are connected in a series is the product of the reliabilities of its units. The simplest example is of a lead in which the glow bulbs are connected in a series and if any of the bulbs fails, the complete lead fails.
FIG. 1.1 SERIES CONFIGURATION

FIG. 1.2 PARALLEL CONFIGURATION

FIG. 1.3 SLIDING STANDBY
Parallel Configuration: In this configuration, if some of the components of the system fail, the whole system does not fail. Parallel configuration is often referred to as redundancy. A Block diagram for parallel system is shown in Fig. 1.2. If all the components of the system fail, only then the system fails.

If $R_i(t)$ is the probability of functioning the $i$th unit without failure in time $t$, the reliability of the system with $n$ such units, connected in parallel, is given by

$$R(t) = 1 - \text{probability of the system failure in time } t$$
$$= 1 - \text{probability that all the } n \text{ components fail during time}$$
$$= 1 - [1 - R_1(t)] \cdot [1-R_2(t)] \cdot ... \cdot [1-R_n(t)]$$
$$= 1 - \prod_{r=1}^{n} [1-R_r(t)] \quad (1.6)$$

Standby System: The simplest example of standby redundant system is a motor vehicle loaded with a spare wheel. When any of the working wheel fails, immediately it is replaced by the spare wheel. Redundancy improves the reliability of any system many fold. In this system, only one unit is in operation at a time. When the operative unit fails, a standby unit starts operating and the failed unit is off the line. Depending upon the requirement, mostly two types of standby (cold and warm) units are utilized:

(i) If the off-line unit can fail, it is called warm standby unit.

(ii) If the off-line unit cannot fail, it is called cold standby unit.
FIG. 1.4 SLIDING STANDBY WITH AFL

FIG 1.5 STANDBY SYSTEM

FIG 1.6 SERIES PARALLEL CONFIGURATION
Sometimes we require sliding standby. If \( n \) components are connected in a series, a sliding standby component as shown in Fig. 1.3, is attached to function when any of the \( n \) components fails. There may be more than one component for acting as sliding standby components depending upon the reliability requirement. We can also have sliding standby with an Automatic Fault Locator (AFL) to locate the faulty component, to disconnect it and connecting the standby component. AFL's are generally provided in automatic and highly complex system. Standby redundancy configuration having AFL is depicted in Fig. 1.4.

If \( L_i \) is the life time of the \( i \)th unit of a system consisting of \( n \) cold standby independent units, the reliability of the system

\[
R(t) = P\{L_1 + L_2 + \ldots + L_n \geq t\}. \quad (1.7)
\]

And if \( R_i(t) = e^{-\lambda t} \) \( \forall \ i \)

the system reliability \( R(t) \), becomes

\[
R(t) = e^{-\lambda t}[1 + \lambda t + \ldots + \frac{(\lambda t)^{n-1}}{(n-1)!}] \\
\]

In a warm standby system at any instant of time, all units or some of them may be in working condition and some of them may not. So the reliability of such system can not be easily found out.

The reliability \( R(t) \) of a two-unit cold standby system consisting of components \( A \) and \( B \) is given by

\[
R(t) = \int_t^\infty f_1(t)dt + \int_0^t [f_1(t_1) \int_{t-t_1}^\infty f_2(t)dt]dt_1, \quad (1.8)
\]
FIG 1.7 PARALLEL SERIES CONFIGURATION

FIG 1.8 MIXED PARALLEL CONFIGURATION

FIG 1.9 A BRIDGE DIAGRAM
where \( f_1(t) \) and \( f_2(t) \) are the time to failure density functions of components A and B respectively.

Here component A is successfully operating up to time \( t \) or component A fails at any time \( t'_1 < t \) and component B operates for the remaining period \( (t-t'_1) \).

The configuration of this system is shown in Fig. 1.5. On failure of the main current, the automatic current supply from the standby generator is the common example of this system.

**r out of n configuration \((r,n)\):** This type of system contains \( n \) components, out of which \( r \) components must be good for operating the system. A common example is a battery having \( n \) cells and for having the requisite voltage out of \( n \) cells a minimum of \( r \) cells should be in operation.

If the components are identical and independent and the failure rate \((\lambda)\) is constant, the reliability for a \((r,n)\) configuration is expressed by

\[
R(t) = \sum_{k=r}^{n} \binom{n}{k} e^{-\lambda t} [1-e^{-\lambda t}]^{n-k} \tag{1.9}
\]

There are many other configurations as series parallel, parallel series, mixed parallel and in the form of bridge, etc. These are depicted in Fig. 1.6 - 1.9.

**Mean Time to System Failure (MTSF)**

It is inevitable that how much good a system may be, it cannot operate for an infinitely long time. Due to aging to
components or some other reasons (shocks due to fluctuation in voltage or physical handling etc.) the system is to fail. To find the failure pattern of the system, we calculate the mean time to system failure. It is defined as the expected time the system is in operation before it completely fails. If \( f(t) \) is the probability density function of time to failure and \( F(t) \) its distribution function then M.T.S.F.

\[
\text{M.T.S.F.} = \int_0^\infty t f(t) \, dt
\]

\[
= -\int_0^\infty t \frac{dR(t)}{dt} \, dt
\]

\[
= -\left\{ tR(t) \right\}_0^\infty + \int_0^{\infty} R(t) \, dt
\]

\[
= \int_0^{\infty} R(t) \, dt \tag{1.10}
\]

since \( t \left. R(t) \right|_0^\infty \to 0 \) as \( t \to \infty \), for a realizable system must fail after a finite operating time.

**Availability**

The term reliability is concerned with the system's capability of survival and maintainability is related to the system capability of repair. By combining these two we obtain the system availability, a measure of system effectiveness as a whole.

According to the requirement, 'availability' is used in different forms.
Intrinsic (Inherent) Availability ($A_i$):

Intrinsic Availability is expressed as

$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

where

* MTBF = Mean time between failure
* MTTR = Mean time to repair

$A_i$ can be used to determine the extent to which a designer has achieved his maintainability and reliability objectives.

System Availability (Instantaneous Availability): It is the probability that the system will be available for use at a given instant of time $t$ and is denoted by $A(t)$.

Mission Time (Average) Availability: Average Availability $A(T)$ is defined as the expected fraction of a given interval of time $(0,T)$ for which the system is in operational state.

$$A(T) = \frac{1}{T} \int_0^T A(t) \, dt$$

Steady state availability: For an operating system, the expected fraction of time in the long run for which the system operates satisfactorily is defined as steady state availability $A_{\infty}$,

$$A_{\infty} = A(\infty) = \lim_{T \to \infty} A(T).$$
Maintainability: In the evaluation of any system, maintainability plays a very important part. How often the system fails (reliability) and how long it is down (maintainability) are vital considerations in determining its worth. British specification BS 3811: 1974 [7] states that 'maintenance is a combination of any actions carried out to retain an item in, or restore it to, an acceptable standard'. The Department of Defence, USA, defines it thus: 'Maintainability is a quality of the combined features and characteristics of equipment design which permits or enhances the accomplishment of maintenance by personnel of average skill under natural and environmental conditions under which it will operate'. Quantitatively it may be defined as the probability that a system will be restored to its operational effectiveness within a given period of time, when the maintenance action is performed in accordance with prescribed procedures and resources.

Broadly, maintenance is divided into two sub-heads -

(i) Preventive Maintenance (PM)

(ii) Corrective Maintenance (CM)

PM carries the usual meaning as general maintenance of the system as lubrication, refuelling, cleaning, adjustment, alignment etc. Generally CM involves minor repairs, that may crop up between inspections. Normally both the maintenance are carried out at pre-determined intervals but in this work M/CM is scheduled
by checkings at random intervals as sometimes periodic inspections are not practical. Also in this work, CM means the repair of hidden faults (non-self reporting) that could only be detected while the unit is under operation. PM and/or CM may be performed while the unit is under operation. PM and/or CM may be performed while the unit is operative or the unit stops functionings, according to the need of the circumstances.

**Markov Process**: A Markov process is a process in which only the last state occupied by the process is taken into account for determining the future behaviour. With this assumption the probability of making a transition in any state of the process depends only on the state presently occupied i.e.

If \( t > t_n > \ldots > t_2 > t_1 \)

\[
\Pr \{ a < X(t) < b \mid X(t_1) = x_1, \ldots, X(t_n) = x_n \} = \Pr \{ a < X(t) < b \mid X(t_n) = x_n \}
\]

where \((X(t))\) is a set of possible values of the random variable \(X_n\) of the stochastic process \(X_n, n \geq 1\).

**Semi-Markov Process**: It is a process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state, before a transition occurs, is a random variable depending upon the last transition made. Thus at transition instants the semi-Markov process behaves just like a Markov process. If system has \(N\)
states 1, 2, ... i, ... j, ... N, such that transition from one state i to any other state j takes place with transition probability $p_{ij}$, such that

$$\sum_{j=1}^{N} p_{ij} = 1,$$

and that the time spent in state i before moving to state j is a random variable whose distribution depends on both these states, then we have a semi-Markov process.

**Regenerative Process**: A time point at which the system history prior to the time point, is irrelevant to the system conditions is called a regeneration point. A process in which each point is a regeneration point is called a regenerative process. Let $\{X(t)\}$ be a stochastic process having a countably infinite number of states 0, 1, 2, ... k, ... . Assuming that there are epochs $t_1, t_2, ...$ at which the process probabilistically restarts from scratch, i.e., with probability 1 there exists an epoch $t_1$ such that the process beyond $t_1$ is a probabilistic replica of the whole process starting at $t = 0$; then the same property holds for $t_2, t_3, ...$. Such a process is known as a regenerative process.

**First Passage Time**: It is the time taken to reach state j for the first time if the system is in state i at time zero. Thus $\Theta_{ij}$ is the first passage time of the system from state i to
state \( j \) and \( f_{ij}(t) \) is the probability that \( Q_{ij} \) is less than or equal to \( t \) i.e.,

\[
f_{ij}(t) = P\{Q_{ij} \leq t\}
\]

**Transforms**

**Laplace Transforms (L.T.)** : If \( f(t) \) be a function of a positive real variable \( t \). Then the Laplace transform (L.T.), \( f^*(s) \) of the function \( f(t) \) is defined as

\[
f^*(s) = \int_0^\infty e^{-st} f(t) dt
\]

for the values of \( s \) for which the integral exists.

**Laplace Stieltjes Transform (L.S.T.)** : If \( t \) be a non-negative random variable with distribution function \( F(t) \) which is piecewise continuous, then L.S.T. of \( F(t) \), \( F^{**}(s) \) is defined as

\[
F^{**}(s) = \int_0^\infty e^{-st} dF(t)
\]

Also

\[
F^{**}(s) = s \int_0^\infty e^{-st} F(t) dt = sF^*(s)
\]

and

\[
F^{**}(s) = \int_0^\infty e^{-st} f(t) dt = f^*(s)
\]

as \( f(t) = \frac{dF(t)}{dt} \) .
Convolution: If $X$ and $Y$ are two independent, positive, integral-valued random variables with probability distributions given by

$$P\{X = k\} = a_k, \quad P\{Y = j\} = b_j$$

The sum $Z = X + Y$ is another random variable whose distribution is given by

$$P\{Z = r\} = C_r$$

$$C_r = a_0 b_r + a_1 b_{r-1} + \ldots + a_r b_0 = \sum_{s=0}^{r} a_s b_{r-s}$$

This new sequence $[C_r]$ is known as convolution of $[a_k]$ and $[b_k]$.

Laplace Convolution: If $f(x)$ and $g(x)$ are two functions of $x$ such that $f(x) \equiv 0$ and $g(x) \equiv 0$ for $x < 0$, the Laplace convolution of $f(x)$ and $g(x)$ is given by

$$f(x) \text{ (LC)} g(x) = \int_{0}^{x} f(u)g(x-u)du$$

Stieltjes Convolution: If $F(x)$ and $G(x)$ are two functions of $x$ such that $F(x) \equiv 0$ and $G(x) \equiv 0$ for $x < 0$, the Stieltjes convolution of $F(x)$ and $G(x)$ is given by

$$F(x) \text{ (SC)} G(x) = \int_{0}^{x} F(x-u) dG(u)$$

In this work these convolutions are widely utilized. Since their L.T. and L.S.T. reduce to a very simplified form.
\[
[f(x)(LC)g(x)]^\infty = f^\infty(x)g^\infty(x)
\]

\[
[F(x)(SC)G(x)]^\infty = F^\infty(x)G^\infty(x)
\]

**Distributions**

If X is a random variable, the distribution function F(x) is defined as

\[
F(x) = P\{X \leq x\} = \int_0^x f(t) dt, \text{ with the assumption that } x \geq 0
\]

Here f(t) is called as probability density function.

**Exponential distribution**

In the present work this distribution has been used for many variables as time to random check, and time to failure of an operative unit. The pdf of the random variable whose distribution is exponential is given by

\[
f(t) = \begin{cases} 
\lambda e^{-\lambda t}, & t \geq 0 \\
0, & t < 0
\end{cases}
\]

The parameter \( \lambda \) takes only positive values. Here

\[
R(t) = P\{X > t\} = e^{-\lambda t}
\]

and

\[
E[X] = \int_0^\infty t \lambda e^{-\lambda t} dt = \int_0^\infty R(t) dt = \frac{1}{\lambda}
\]

This is the only distribution which obeys Markovian property.

**Erlang distribution**

The pdf of this distribution is given by

\[
f(t) = \begin{cases} 
\frac{(\lambda t)^{r-1} e^{-\lambda t}}{(r-1)!}, & t \geq 0 \\
0, & t < 0
\end{cases}, \lambda > 0, r \geq 1
\]
The MTSF = \frac{r}{r \lambda} = \frac{1}{\lambda}

when r = 1, this distribution reduces to exponential distribution. If we have r independent exponential random variables each with parameter r \lambda, the sum of these random variables is a random variable whose distribution would be Erlang distribution with parameters r and \lambda. When r \to \infty, this distribution becomes deterministic with constant time equal to 1/\lambda.

Its Laplace transform is given by

\[ f^*(s) = \left( \frac{r \lambda}{s + r \lambda} \right)^r. \]

It is a very versatile distribution and hence it is used in various models.

**Summary**

The whole study is covered in Eight Chapters. The work of succeeding chapters is summarized below.

**CHAPTER - 2**

Three models are considered in this chapter. In all the models the system is single-unit and the unit is operative under CM. In each model, at the random checks, the operative unit undergoes CM with different probabilities. For undergoing CM, in model 1, the probabilities are 1/2 and p; in model 2, the probabilities are 1/2, 3/4 and 7/8; in model 3 the probabilities are 1/2, 3/4, 7/8 and 1 (i.e. the unit has to undergo CM).
CHAPTER - 3

In this chapter also three models each with single-unit system, and unit being operative while under CM, are considered. At random checks on the operative unit in model 1, the probabilities for undergoing CM are \((1-p), (1-p^2), (1-p^3), \ldots\). Whereas in models 2 and 3, the probabilities for undergoing CM are restricted to \((1-p), (1-p^2), \ldots (1-p^k)\). In model 2, at the kth, (k+1)th, (k+2)th, \ldots random checks, the probability of undergoing CM remains \((1-p^k)\). Whereas in model 3 at the kth random check, the unit has to undergo CM. As a matter of fact, the need of kth random check only arises if in all the preceding [(k-1)th, (k-2)th, \ldots 2nd and 1st] random checks CM was not needed.

CHAPTER - 4

Two models, each with a two-unit system are considered in this chapter. In model 1, when the unit undergoes CM at a random check it remains operative and thus may fail also. But the rates of failure when the unit is operative and under CM and when it is operative and without CM are different. In model 2, when the unit is under CM, it does not remain operative but may fail. Here also failure rates are different when the unit is not operative but under CM and when it is operative but without CM.
CHAPTER - 5

In this chapter a two-unit system is considered. At random checks on the operative unit when the unit undergoes CM, neither it works nor it fails. The probabilities for undergoing CM of the operative unit at the random checks are $p_1$ and $p_2$.

CHAPTER - 6

In this chapter two models, each with a two-unit system, are considered. In model 1, the operative unit at random checks undergoes Preventive Maintenance (PM) with probability $p$ and remains without PM with probability $q (= 1-p)$. While the unit is under PM, neither it works nor it fails. In model 2, the operative unit at random checks undergoes CM and PM with probabilities $p$ and $r$ respectively and remains without PM and CM with probability $q (= 1-p-r)$. While the unit is under PM, it works and thus may fail but the failure rate is higher than the failure rate when the unit is operative but not under PM. While the unit is under CM, neither it works nor it fails.

CHAPTER - 7

A two-unit system is considered in this chapter. While the unit undergoes CM with probability $p$ at the random checks on the operative unit, neither it works nor it fails. While the unit is operative, with probability $a$ there is a repairable type of failure of the unit and with probability $b (= 1-a)$, the failure is of non-repairable type. When both the units of the
In this chapter, a two-unit system with a waiting for the arrival of the service for CM is considered. The random checking is performed by a different agency which is automatic and always available at the site. For repair of the failed unit, service is available without any loss of time. While the unit is under CM, neither it works nor it fails.