CHAPTER - V

TWO-UNIT COLD-STANDBY REDUNDANT SYSTEM 
WITH DEPENDENT CORRECTIVE MAINTENANCE 
AT SUCCESSIVE RANDOM CHECKINGS
This chapter consists of a two-unit cold-standby system. Initially, the system is in state 1, where one unit is operating and the other is standby. When the operating unit fails, the standby unit starts operating. The failure rate is \( \lambda \) and the pdf and the cdf of time to failure are \( f(t) = \lambda e^{-\lambda t} \) and \( F(t) \) respectively. The failed unit is put to repair and the system transits to state 3.

In state 1, the operating unit is subjected to random checks to ascertain the need of CM. With probability \( p_1 \) the unit undergoes CM and the system transits to state 4. With probability \( q_1 = 1 - p_1 \) the unit does not require CM and the system transits to state 2. In state 2, the operating unit is also subjected to random checks. Here, the operating unit undergoes CM with different probability \( p_2 \) and with probability \( q_2 = 1 - p_2 \) the unit does not need CM and the system remains in state 2. There is only one agency to perform all the jobs, i.e., random checking, repair and CM.

**Notations for the diagram**

- 0: the unit is operating and it has not undergone random checking so far.
- 0-1: the unit is operating and one random checking has been performed and No CM was required at the random checking.
- Cs: Cold standby
FIG. 5. STATE TRANSITION DIAGRAM

- OPERATIVE STATE
- FAILED STATE
- REGENERATION POINT
Regenerative states are those when the system enters
state 1 from states 3 and 4; state 2 from states 1 and 2;
state 3 from states 1, 2, and 5; state 4 from states 1 and 2.

Thus states 1, 2, 3 and 4 are regenerative states and states 5
and 6 are failed states.

\[ M.T.S.E. \]
\[
\varphi_1(t) = Q_{12}(t) \varphi_2(t) + Q_{13}(t) \varphi_3(t) + Q_{14}(t) \varphi_4(t)
\]
\[
\varphi_2(t) = Q_{22}(t) \varphi_2(t) + Q_{23}(t) \varphi_3(t) + Q_{24}(t) \varphi_4(t)
\]
\[
\varphi_3(t) = Q_{36}(t) + Q_{34}(t) \varphi_1(t)
\]
\[
\varphi_4(t) = Q_{45}(t) + Q_{44}(t) \varphi_1(t)
\]

(5.1)

Transition (pdf)s are
\[
q_{12}(t) = F(t) \varphi_{1h}(t) = q_1 e^{-\lambda t} v e^{-\nu t}
\]
\[
q_{13}(t) = H(t) f(t) = e^{-\nu t} \lambda e^{-\lambda t}
\]
\[ q_{14}(t) = \bar{F}(t) p_1 h(t) = e^{-\lambda t} p_1 e^{-\nu t} \]
\[ q_{22}(t) = \bar{F}(t) q_2 h(t) = e^{-\lambda t} q_2 e^{-\nu t} \]
\[ q_{23}(t) = \bar{F}(t) f(t) = e^{-\nu t} \lambda e^{-\lambda t} \]
\[ q_{24}(t) = \bar{F}(t) p_2 h(t) = e^{-\lambda t} p_2 e^{-\nu t} \]
\[ q_{31}(t) = \bar{F}(t) g(t) = e^{-\lambda t} g(t) \]
\[ q_{36}(t) = \bar{G}(t) f(t) = \lambda e^{-\lambda t} \bar{G}(t) \]
\[ q_{41}(t) = \bar{F}(t) m(t) = e^{-\lambda t} m(t) \]
\[ q_{45}(t) = \bar{M}(t) f(t) = \lambda e^{-\lambda t} \bar{M}(t) \]
\[ q_{33.6}(t) = F(t) g(t) = \{1 - e^{-\lambda t}\} g(t) \]
\[ q_{43.5}(t) = F(t) m(t) = \{1 - e^{-\lambda t}\} m(t) \]

Taking L.S.T. of Eqs. (5.1) and solving for \( \phi_1^{**}(s) \)

\[ \phi_1^{**}(s) = \frac{S \phi_{36}^{**}(s) + T \phi_{45}^{**}(s)}{1 - \phi_{22}^{**}(s) - S \phi_{41}^{**}(s) - T \phi_{41}^{**}(s)} \]  \hspace{1cm} (5.5)

where \( S = Q_{12}^{**}(s) Q_{23}^{**}(s) + Q_{13}^{**}(s) \{1 - Q_{22}^{**}(s)\} \)

\[ T = Q_{12}^{**}(s) Q_{24}^{**}(s) + Q_{14}^{**}(s) \{1 - Q_{22}^{**}(s)\} \]

It is seen that \( \phi_1^{**}(0) = 1 \), which implies that \( \phi_1(t) \) is a proper distribution. Here we have utilized the following:

\[ \phi_1^{**}(s) = \frac{S \phi_{36}^{**}(s) + T \phi_{45}^{**}(s)}{1 - \phi_{22}^{**}(s) - S \phi_{41}^{**}(s) - T \phi_{41}^{**}(s)} \]  \hspace{1cm} (5.5)
\[ Q_{12}(0) + Q_{13}(0) + Q_{14}(0) = 1 \]
\[ Q_{22}(0) + Q_{23}(0) + Q_{24}(0) = 1 \]
\[ Q_{31}(0) + Q_{36}(0) = 1 \]
\[ Q_{41}(0) + Q_{45}(0) = 1 \]

MTSF \( T_1 \) = \( \lim_{s \to 0^+} \frac{1 - \varrho(t)}{s} \); given that in the beginning the system is in state 1

\[ T_1 = \frac{\lambda^2 + \nu \{ \lambda(2 - q_2 + q_1) + \nu(p_2 + q_1) \}}{(\nu + \lambda)[\lambda g^{-\lambda}(\lambda) + \nu(p_2 + q_1)]} \] (5.4)

**Availability Analysis**

\[ AV_1(t) = \bar{F}(t)\bar{H}(t) + q_{12}(t)(LC)AV_2(t) + q_{13}(t)(LC)AV_3(t) + q_{14}(t)(LC)AV_4(t) \]
\[ AV_2(t) = \bar{F}(t)\bar{H}(t) + q_{22}(t)(LC)AV_2(t) + q_{23}(t)(LC)AV_3(t) + q_{24}(t)(LC)AV_4(t) \]
\[ AV_3(t) = \bar{F}(t)\bar{H}(t) + q_{31}(t)(LC)AV_1(t) + q_{35}(t)(LC)AV_3(t) \]
\[ AV_4(t) = \bar{F}(t)\bar{H}(t) + q_{41}(t)(LC)AV_1(t) + q_{45}(t)(LC)AV_3(t) \] (5.5)
Taking the L.T. of the eqs. (5.5) and solving for $AVX(s)$, we get

$$AVX(s) = \frac{g^{-X}(s+\lambda) + \Delta \nu m^{-X}(s+\lambda) + \Delta(s+\lambda) \{ s+\lambda+v(p_2+q_1) \}}{\Delta(s+\lambda)[(s+\lambda+v)(s+\lambda+vp_2) - R-vm^{-X}(s+\lambda) \{ vp_2+p_1(s+\lambda) \}]}$$

(5.6)

where

$$\Delta = 1 - g^X(s) + g^X(s+\lambda)$$

$$\Delta_m = m^X(s) - m^X(s+\lambda)$$

$$R = \frac{g^{-X}(s+\lambda)}{\Delta} \left[ \lambda(s+\lambda+v(p_2+q_1)) + \Delta_m \nu \{ vp_2+p_1(s+\lambda) \} \right]$$

and steady-state availability is given by

$$AVX = \lim_{s \to 0} [s AVX(s)]$$

$$AVX = \frac{1+g^X(\lambda) \{ v m^{-X}(\lambda) + \lambda \{ \lambda+v(p_2+q_1) \} - 1 \}}{\lambda g^X(\lambda)[2\lambda+v(1+p_2)-(R')_o - v \{ p_1 m^X(\lambda) + m^{X'}(\lambda)(vp_2 + p_1\lambda) \}]}$$

$$+ \left[ \lambda \{ g^{X'}(\lambda)-g^X(\lambda)+g^X(\lambda) \}((\lambda+v)(\lambda+vp_2) - (R)_o - vm^X(\lambda)\{ vp_2+p_1\lambda \} \right]$$

(5.7)

$$\lambda \{ \lambda+v(1+p_2) \} + \lambda m^{-X}(\lambda) \{ vp_2+p_1\lambda \}$$

$$\lambda [g^X(\lambda)-g^X(\lambda) \{ g^X(\lambda)+g^X(o) \}] +$$

$$\lambda [g^X(\lambda)[\lambda+v \{ m^{X'}(o)-m^X(\lambda) \} (vp_2+p_1\lambda)+vp_1m^{-X}(\lambda)]$$

$$\frac{[g^X(\lambda)]^2}{[g^X(\lambda)]^2}$$
Expected number of repairs per unit time

\[
NR_1(t) = q_{12}(t)(LC)NR_2(t) + q_{13}(t)(LC)[1 + NR_3(t)] + q_{14}(t)(LC)NR_4(t)
\]

\[
NR_2(t) = q_{22}(t)(LC)NR_2(t) + q_{23}(t)(LC)[1 + NR_3(t)] + q_{24}(t)(LC)NR_4(t)
\]

\[
NR_3(t) = q_{31}(t)(LC)NR_1(t) + q_{33.6}(t)(LC)[1 + NR_5(t)]
\]

\[
NR_4(t) = q_{41}(t)(LC)NR_1(t) + q_{43.5}(t)(LC)[1 + NR_5(t)]
\]

(5.8)

Taking L.T. of the above equations and solving for \( NR_1^*(s) \).

\[
NR_1^*(s) = \frac{\lambda + sA(\nu \eta + D + Ap v)}{s[\Delta(s + \lambda + v - Ev\eta - p_1 vB) - \lambda g^*(s + \lambda)]}
\]

(5.9)

where

\[
\Delta = 1 - g^*(s) + g^*(s + \lambda)
\]

\[
A = \frac{m^*(s) - m^*(s + \lambda)}{s \Delta}
\]

\[
B = \frac{m^*(s)g^*(s + \lambda) + m^*(s + \lambda)g^*(s)}{\Delta}
\]

\[
D = \frac{\lambda + Ap_2 v}{\Delta s(s + \lambda + vp_2)}
\]

\[
E = \frac{\lambda g^*(s + \lambda) + \Delta p_2 vB}{\Delta(s + \lambda + vp_2)}
\]

In steady-state expected number of repairs per unit time \( NR_1 \)

is obtained by

\[
NR_1 = \lim_{t \to \infty} [NR_1(t)] = \lim_{s \to 0} [sNR_1^*(s)].
\]
Expected number of times the corrective maintenance is carried out in unit time.

\[ \text{NCM}_1(t) = q_{12}(t)(\text{LC})\text{NCM}_2(t) + q_{15}(t)(\text{LC})\text{NCM}_3(t) + q_{14}(t)(\text{LC})[1 + \text{NCM}_4(t)] \]

\[ \text{NCM}_2(t) = q_{22}(t)(\text{LC})\text{NCM}_2(t) + q_{23}(t)(\text{LC})\text{NCM}_5(t) + q_{24}(t)(\text{LC})[1 + \text{NCM}_4(t)] \]

\[ \text{NCM}_3(t) = q_{31}(t)(\text{LC})\text{NCM}_1(t) + q_{33.6}(t)(\text{LC})\text{NCM}_5(t) \]

\[ \text{NCM}_4(t) = q_{41}(t)(\text{LC})\text{NCM}_1(t) + q_{43.5}(t)(\text{LC})\text{NCM}_5(t) \]  \hspace{1cm} (5.10)

Taking L.T. of the above equations and solving for \( \text{NCM}_1^*(s) \).

\[ \text{NCM}_1^*(s) = \frac{\Delta[p_1v + svq_1B]}{s[(s+\lambda+v-vq_1D-p_1vA)\Delta-\lambda g^*(s+\lambda)]} \]  \hspace{1cm} (5.11)

where

\[ \Lambda = 1 - g^*(s) + g^*(s+\lambda) \]
\[ A = \frac{[g^*(s)m^*(s+\lambda)+m^*(s)g^*(s+\lambda)]}{\Delta} \]
\[ B = p_2v/s(s+\lambda+p_2v) \]
\[ D = \frac{[\lambda g^*(s+\lambda) + \Delta p_2v]}{\Delta(s+\lambda+p_2v)} \]

In steady-state the expected number of times the CM is carried out in unit time (\( \text{NCM}_1 \)) is obtained by

\[ \text{NCM}_1 = \lim_{t \to \infty} [\text{NCM}_1(t)] = \lim_{s \to 0} [s\text{NCM}_1^*(s)]. \]
\[ g^*(\lambda) = (5K/1+5K)^K \]
\[ \lambda (\text{Failure rate}) = 1 \]
\[ m^*(\lambda) = (n\theta/1+n\theta)^n \]
\[ \mu(\text{repair rate}) = 5 \]

**FIG. 5.2**