CHAPTER - IV

TWO-UNIT COLD-STANDBY REDUNDANT SYSTEM SUBJECT TO RANDOM CHECKING AND CORRECTIVE MAINTENANCE WHILE REMAINING OPERATIVE OR BECOMING INOPERATIVE
Here two models, each with a two-similar unit system are considered. In both models, initially in state 1, one unit operates and the other remains as a cold-standby. On failing of the operative unit with exponential failure rate $\lambda$, the unit undergoes repair and the standby unit starts operating i.e. the system transits to state 2. Before completion of the repair of the failed unit, if the other operating unit also fails, the system fails.

At random checks on the operating unit, when the system is in state 1, with probability $p$, the unit undergoes CM, i.e. the system transits to state 3 and with probability $q(=1-p)$, the unit does not require CM, i.e. the system remains in state 1. While the unit is under CM, in model 1 it continues to operate but with different failure rate whereas in model 2 the unit under CM does not operate but may fail and $k$ is the exponential failure rate in both cases. Thus pdf and cdf of time to failure when the unit is without CM and under CM are different and equal to $f(t)(=\lambda e^{-\lambda t})$ and $F(t)$; and $f(t)(=ke^{-kt})$ and $L(t)$ respectively. There is only one agency to perform all the jobs i.e. random checking, repair and CM.
MODEL - 1

Notations for the diagram

0 operative standby

cs cold standby

wr waiting for repair, while the other unit is under repair

ur under repair

uc under CM

ur.r under repair continued from state r.

Regenerative states

Regenerative states are those when the system enters state 1 from states 2 and 3; state 2 from states 1, 3 and 4; state 3 from state 1. Thus states 1, 2 and 3 are regenerative states and state 4 is a failed state.
FIG. 4.1 STATE TRANSITION DIAGRAM

- OPERATIVE STATE
- FAILED STATE
- REGENERATION POINT

State 1: (o, cs)
State 2: (o, ur)
State 3: (u, c, cs)
State 4: (u, r, 2, wr)

Transitions: k, q
M. T. S. F. Analysis

\[ \varphi_1(t) = Q_{11}(t)(SC)\varphi_1(t) + Q_{12}(t)(SC)\varphi_2(t) + Q_{13}(t)(SC)\varphi_3(t) \]

\[ \varphi_2(t) = Q_{24}(t) + Q_{21}(t)(SC)\varphi_1(t) \]

\[ \varphi_3(t) = Q_{31}(t)(SC)\varphi_1(t) + Q_{32}(t)(SC)\varphi_2(t) \]

(4.1.1)

transition (p.d.f.)s, are

\[ q_{11}(t) = qe^{-\lambda t} v e^{-vt} \]

\[ q_{12}(t) = \lambda e^{-\lambda t} e^{-vt} \]

\[ q_{13}(t) = e^{-\lambda t} pve^{-vt} \]

\[ q_{21}(t) = e^{-\lambda t} g(t) \]

\[ q_{24}(t) = G(t) e^{-\lambda t} \]

\[ q_{31}(t) = e^{-kt} m(t) \]

\[ q_{32}(t) = ke^{-kt} \Pi(t) \]

\[ q_{22,4}(t) = \{1-e^{-\lambda t}\} c(t) \]

(4.1.2)

Taking L.S.T. of eqs. (4.1.1) and solving for \( \varphi_1^*(s) \) we get

\[ \varphi_1^*(s) = \frac{Q_{24}^*(s) \{ Q_{12}^*(s) + Q_{32}^*(s) Q_{13}^*(s) \}}{1-Q_{11}^*(s)-Q_{12}^*(s)Q_{21}^*(s) - Q_{13}^*(s) \{ Q_{31}^*(s)+Q_{32}^*(s)Q_{21}^*(s) \}} \]

(4.1.3)
It is verified that $Q^{xx}(o) = 1$, which implies that $Q_1(t)$ is a proper distribution. Here we have utilized the following

\[ Q^{xx}(o) + Q^{xx}(o) + Q^{xx}(o) = 1 \]
\[ Q^{xx}(o) + Q^{xx}(o) = 1 \]
\[ Q^{xx}(o) + Q^{xx}(o) = 1 \]

The M.T.S.F. \( T_1 \) = \( \lim_\limits{\imath \rightarrow 0^+} \frac{1 - Q^{xx}(s)}{s} \); given that in the beginning, the system started from state 1.

\[
T_1 = \frac{\lambda k \{1 + g^{-^X}(\lambda)\} + p \nu m^{-^X}(k) \{\lambda + k g^{-^X}(\lambda)\}}{\lambda k \left[\lambda + p \nu m^{-^X}(k)\right] g^{-^X}(\lambda)}. \tag{4.1.4}
\]

**Availability Analysis**

\[
AV_1(t) = e^{-\lambda t} e^{-\nu t} + q e^{-\lambda t} e^{-\nu t} (LC) AV_1(t)
\]
\[ + e^{-\nu t} \lambda e^{-\lambda t} (LC) AV_2(t) + p \nu e^{-\nu t} e^{-\lambda t} (LC) AV_3(t) \]

\[
AV_2(t) = e^{-\lambda t} G(t) + e^{-\lambda t} g(t)(LC) AV_1(t)
\]
\[ + \{1 - e^{-\lambda t}\} g(t)(LC) AV_2(t) \]

\[
AV_3(t) = e^{-kt} \overline{M}(t) + e^{-kt} m(t) \ (LC) AV_1(t) \]
\[ + \overline{M}(t) \ k e^{-kt} \ (LC) AV_2(t) \] \tag{4.1.5}
Taking L.T. of the above eq.s. (4.1.5) and solving for \( A^*V_1(s) \)

\[
A^*V_1(s) = \frac{k B g^{-k}(s+\lambda) + (s+\lambda) \left\{ (s+k)(k-\lambda) + B\Delta \right\}}{(s+\lambda) \left\{ (s+k) \{B\Delta + s(k-\lambda)\} - kBg^{-k}(s+\lambda) \right\}} \tag{4.1.6}
\]

where \( B = \frac{(s+k)\lambda + pvkm(s+k)}{\Delta} \)

and \( \Delta = 1-g^k(s) + g^k(s+\lambda) \)

and steady state availability is given by

\[
\bar{A}^*V_1 = \lim_{t \to \infty} [A^*V_1(t)] = \lim_{s \to 0^+} [s \cdot A^*V_1(s)]
\]

\[
\bar{A}^*V_1 = \frac{k \left( \frac{\Sigma'}{\Omega} \right) g^{-k}(\lambda) + (B)_{\Omega} \left\{ c^k(\lambda) + kc^{-k}(\lambda) \right\} + k \left\{ 1 + pv^{-k}(k) \right\}}{k \left\{ 2k+\lambda \right\} + pv^{-k}(k) \left\{ k+\lambda(k+1) \right\} - (B)_{\Omega} \left\{ c^k(\lambda) + \lambda g^k(\lambda) \right\} - \lambda g^k(\lambda) (B')_{\Omega}} \tag{4.1.7}
\]

where \( (B)_{\Omega} = \frac{k \left\{ \lambda + pv^{-k}(k) \right\}}{g^k(\lambda)} \)

\( (B')_{\Omega} = \lambda + pvkm^{-k}(k) / g^k(\lambda) = \frac{k \left\{ \lambda + pv^{-k}(k) \right\} \left\{ g^k(\lambda) - g^k(\Omega) \right\}}{\left\{ g^k(\lambda) \right\}^2} \).
Expected number of repairs per unit time

\[ NR_1(t) = q_{11}(t)(LC)NR_1(t) + q_{12}(t)(LC)[1+NR_2(t)] + q_{13}(t)(LC)NR_3(t) \]
\[ NR_2(t) = q_{21}(t)(LC)NR_1(t) + q_{22.4}(t)(LC)[1+NR_2(t)] \]
\[ NR_3(t) = q_{31}(t)(LC)NR_1(t) + q_{32}(t)(LC)[1+NR_2(t)] \]

Taking L.T. of the above equations and solving for \( NR_1^*(s) \).

\[ NR_1^*(s) = \frac{A[\lambda + pv\bar{m}^*(s+k)]}{s + \lambda \bar{B} + pv\bar{B} \bar{m}^*(s+k)} \]

where

\[ A = \frac{1}{s\Delta} \]
\[ \Delta = 1 - g^*(s) + g^*(s + \lambda) \]
\[ \bar{B} = 1 - B \]
\[ B = g^*(s + \lambda)/\Delta \]

In the steady state the expected number of repairs per unit time (\( NR_1 \)) is given by

\[ NR_1 = \lim_{t \to \infty} [NR_1(t)] \]
\[ = \lim_{s \to 0^+} [sNR_1^*(s)] \]
\[ = \frac{pv\bar{m}^*(k)}{g^*(\lambda) - g^*(0)[\lambda + pv\bar{m}^*(k)]} \]
Expected number of times the corrective maintenance is carried out in unit time.

\[
\text{NCM}_1(t) = q_{11}(t)(\text{LC})\text{NCM}_1(t) + q_{12}(t)(\text{LC})\text{NCM}_2(t) + q_{15}(t)(\text{LC})[1 + \text{NCM}_5]
\]

\[
\text{NCM}_2(t) = q_{21}(t)(\text{LC})\text{NCM}_1(t) + q_{22,4}(t)(\text{LC})\text{NCM}_2(t)
\]

\[
\text{NCM}_3(t) = q_{31}(t)(\text{LC})\text{NCM}_1(t) + q_{32}(t)(\text{LC})\text{NCM}_2(t)
\]  
(4.1.11)

Taking L.T. of the above equations and solving for \(\text{NCM}_1^*(s)\).

\[
\text{NCM}_1^*(s) = \frac{\Delta p v}{s[\Delta(s + \lambda + p \nu \bar{A} - \lambda g^*(s + \lambda)]}
\]  
(4.1.12)

where

\[
\Delta = 1 - g^*(s) + g^*(s + \lambda)
\]

\[
\bar{A} = \frac{\bar{m}^*(s + k)[(s + k)g^*(s) + sg^*(s + \lambda)]}{\Delta(s + k)}
\]

In steady-state the expected number of times the CM is carried out in unit time \((\text{NCM}_1)\) is given by

\[
\text{NCM}_1 = \lim_{t \to \infty} [\text{NCM}_1(t)] = \lim_{s \to 0} [s\text{NCM}_1^*(s)] = \frac{k_p v[g^*(\lambda) - g^*(0)]}{k[p v g^*(\lambda) - \lambda g^*(0)] + \bar{m}^*(k)[g^*(\lambda) + k g^*(0)]}
\]  
(4.1.13)
\[ g^*(\lambda) = \left(\frac{5K}{1+5K}\right)^K \]  
\[ \lambda \text{(Failure rate)} = 1 \]  
\[ m^*(\lambda) = \left(\frac{n\theta}{1+n\theta}\right)^n \]  
\[ \mu \text{(Repair rate)} = 5 \]
MODEL - 2

Notations for the diagram

Notations of model 1 are also applicable here.

Additional Notations

\( wr \) waiting for repair, while the facility is engaged in CM or the repair of the other unit

\( uc.r \) under CM continued from state \( r \).

Regenerative states

Regenerative states are those when the system enters state 1 from states 1, 2 and 5; state 2 from states 1, 3, 4, 5 and 6; state 3 from state 1; state 6 from state 4. Thus states 1, 2, 3 and 6 are regenerative states and states 4, 5 and 6 are failed states.

\[\begin{align*}
\varphi_1(t) &= Q_{11}(t)(SC)\varphi_1(t) + Q_{12}(t)(SC)\varphi_2(t) + Q_{13}(t)(SC)\varphi_3(t) \\
\varphi_2(t) &= Q_{25}(t) + Q_{21}(t)(SC)\varphi_1(t) \\
\varphi_3(t) &= Q_{34}(t) + Q_{31}(t)(SC)\varphi_1(t) + Q_{32}(t)(SC)\varphi_2(t)
\end{align*}\]

(4.2.1)
FIG. 4.2 STATE TRANSITION DIAGRAM

- OPERATIVE STATE
- FAILED STATE
- REGENERATION POINT
transition (pdf)s

\[
\begin{align*}
q_{11}(t) &= F(t)q \ h(t) = e^{-\lambda t} q \ v \ e^{-vt} \\
q_{12}(t) &= H(t) f(t) = e^{-vt} \lambda e^{-\lambda t} \\
q_{13}(t) &= \bar{F}(t) p \ h(t) = e^{-\lambda t} p \ v \ e^{-vt} \\
q_{21}(t) &= \bar{F}(t) g(t) = e^{-\lambda t} g(t) \\
q_{25}(t) &= \bar{G}(t) f(t) = \bar{G}(t) \lambda e^{-\lambda t} \\
q_{31}(t) &= \bar{F}(t) \bar{L}(t) m(t) = e^{-\lambda t} e^{-kt} m(t) \\
q_{32}(t) &= \bar{M}(t) \bar{F}(t) f(t) = \bar{M}(t) e^{-\lambda t} ke^{-kt} \\
q_{34}(t) &= \bar{M}(t) \bar{L}(t) f(t) = \bar{M}(t) e^{-kt} \lambda e^{-\lambda t} \\
q_{22.5}(t) &= F(t) g(t) = [1-e^{-\lambda t}] g(t) \\
q_{32.4}(t) &= \bar{L}(t) F(t) m(t) = e^{-kt} [1-e^{-\lambda t}] m(t) \\
q_{36.4}(t) &= \bar{M}(t) F(t) f(t) = \bar{M}(t) [1-e^{-\lambda t}] ke^{-kt} \\
q_{6.2}(t) &= g(t) = g(t)
\end{align*}
\]

Taking L.S.T. of eqs.(4.2.1) and solving for \( \phi_1^{**}(s) \)

\[
\phi_1^{**}(s) = \frac{Q_{12}^{**}(s) Q_{25}^{**}(s) + Q_{15}^{**}(s) \ [Q_{34}^{**}(s) + Q_{52}^{**}(s) Q_{25}^{**}(s)]}{[1-Q_{11}^{**}(s) - Q_{12}^{**}(s) Q_{21}^{**}(s) - Q_{15}^{**}(s)(Q_{51}^{**}(s) + Q_{52}^{**}(s) Q_{21}^{**}(s))]} 
\]
It is verified that $\phi^{**}(o) = 1$, which implies that $\phi_1(t)$ is a proper distribution.

Here we make use of the following

$$Q_{11}^{**}(o) + Q_{12}^{**}(o) + Q_{13}^{**}(o) = 1$$
$$Q_{21}^{**}(o) + Q_{25}^{**}(o) = 1$$
$$Q_{51}^{**}(o) + Q_{52}^{**}(o) + Q_{55}^{**}(o) = 1$$

The L.T. of the reliability in time $t$ is

$$[1 - \phi_1^{**}(s)]/s$$

The M.T.S.F., given that the system started at the beginning of state 1 is denoted by $T_1$.

$$T_1 = \lim_{s \to 0^+} [1 - \phi_1^{**}(s)]/s$$

$$= \frac{N_r T_1}{D_r T_1}$$

where

$$N_r T_1 = \lambda(\lambda+k)[(\lambda-v)+p\nu\{1+m^*(\lambda+k)+m^*(\lambda+k)\}]$$
$$+ (v+\lambda)[\lambda(\lambda+k)\lambda^*(\lambda)+p\nu m^*(\lambda+k)\{\lambda+k\lambda^*(\lambda)\}]$$

$$D_r T_1 = \lambda(v+\lambda)[\lambda^*(\lambda)(\lambda+k)+p\nu m^*(\lambda+k)\{\lambda+k\lambda^*(\lambda)\}]$$

**Availability Analysis**

$$AV_1(t) = H(t)F(t)+F(t)q h(t)(LC)AV_1(t)$$
$$+ F(t)p h(t)(LC)AV_3(t)+H(t)f(t)(LC)AV_2(t)$$
\[ AV_2(t) = g(t)F(t) + g(t)F(t)(LC)AV_1(t) + F(t)g(t)(LC)AV_2(t) \]

\[ AV_3(t) = \overline{M}(t)\overline{L}(t)F(t) + \overline{L}(t)\overline{F}(t)\overline{m}(t)(LC)AV_1(t) + \overline{M}(t)\overline{F}(t)f(t)(LC)AV_2(t) + \overline{M}(t)F(t)f(t)(LC)AV_3(t) \]  

(4.2.5)

Taking L.T. of the eqs.(4.2.5) and solving for \( AV^*_1(s) \)

\[ AV^*_1(s) = \frac{(s+\lambda+k) \left[ 1 + Rg^*(s+\lambda) \right] + pv\overline{m}^*(s+\lambda+k)}{pv\overline{m}^*(s+\lambda+k) + (s+\lambda) \left[ 1 - Rg^*(s+\lambda) \right]} \]  

\[ AV^*_1(s) = \frac{pv[(s+k) \{ k\overline{m}^*(s+\lambda+k)g^*(s) + m^*(s+k) - m^*(s+k+\lambda) \} + (s+\lambda+k) \{ k\overline{m}^*(s+k)g^*(s) + \lambda(s+k) \}]}{\Delta(s+\lambda)(s+k)(s+\lambda+k)} \]

where \( R = \frac{\Delta(s+\lambda)(s+k)(s+\lambda+k)}{\lambda - g^*(s+k) + g^*(s+\lambda)} \)

steady state availability \( AV_1 \) is given by

\[ AV_1 = \lim_{s \to 0} s AV^*_1(s) \]

\[ AV_1 = \frac{pv\overline{m}^*(\lambda+k) + (\lambda+k) \left[ 1 + R_0 \overline{g}^*(\lambda) \right]}{1 + pv\overline{m}^*(\lambda+k) - g^*(\lambda) \left[ R_0 + \lambda R_0' \right] - \lambda R_0 g^*(\lambda)} \]  

(4.2.7)

where

\[ R_0 = \frac{pv}{\lambda g^*(\lambda)(\lambda+k)} \left[ m^*(k) - m^*(k+\lambda) + (\lambda+k) \{ \lambda + m^*(k) \} \right] \]
\[
\frac{PV[m^*(k) - m^*(k+\lambda) + k\{m^{*'}(k) - m^{*'}(k+\lambda) + km^{*'}(\lambda+k)} +
+ X(2k+X) + k(X+k)m^{*'}(k) + km^{*'}(k) + km^{*'}(\lambda+k) \}}{\lambda kg^*(\lambda)(\lambda+k)}
\]

Expected number of repairs per unit time

\[
NR_1(t) = q_{11}(t)(LC)NR_1(t) + q_{12}(t)(LC)[1+NR_2(t)] + q_{13}(t)(LC)NR_3(t)
\]

\[
NR_2(t) = q_{21}(t)(LC)NR_1(t) + q_{22.5}(t)(LC)[1+NR_2(t)]
\]

\[
NR_3(t) = q_{31}(t)(LC)NR_1(t) + q_{32}(t)(LC)[1+NR_2(t)]
+ q_{32.4}(t)(LC)[1+NR_2(t)] + q_{36.4}(t)(LC)[1+NR_6(t)]
\]

\[
NR_6(t) = q_{62}(t)(LC)[1+NR_2(t)]
\]

Taking L.T. of the above equations and solving for \(NR_1(s)\).

\[
NR_1(s) = \frac{N_T NR_1(s)}{Dr NR_1(s)}
\]

\[
N_T NR_1(s) = \lambda A + PV[EA+D\{1+Ag^*(s)\}]
\]

\[
Dr NR_1(s) = s+\lambda(1-B) - PV[\overline{m^*}(s+\lambda+k)+B\{E+ADg^*(s)\}]
\]

where

\[
A = \frac{1}{s[1-g^*(s)+g^*(s+\lambda)]}
\]

\[
B = \frac{g^*(s+\lambda)}{1-g^*(s)+g^*(s+\lambda)}
\]
\[
D = k \left( \frac{m^{*}(s+k)}{s+k} - \frac{m^{*}(s+k+\lambda)}{s+k+\lambda} \right)
\]

\[
E = \frac{km^{*}(s+\lambda+k)}{(s+\lambda+k)} + m^{*}(s+k) - m^{*}(s+k+\lambda)
\]

In steady-state expected number of repairs per unit time (\(NR_1\)) is obtained by

\[
NR_1 = \lim_{t \to \infty} [NR_1(t)] = \lim_{s \to 0} [s \cdot NR_1^*(s)]
\]

Expected number of times the corrective maintenance is carried out in unit time.

\[
NCM_1(t) = a_{11}(t)(LC) NCM_1(t) + a_{12}(t)(LC) NCM_2(t) + a_{15}(t)(LC) [1 + NCM_3(t)]
\]

\[
NCM_2(t) = a_{21}(t)(LC) NCM_1(t) + a_{22.5}(t)(LC) NCM_2(t)
\]

\[
NCM_3(t) = a_{31}(t)(LC) NCM_1(t) + a_{32}(t)(LC) NCM_2(t)
\]

\[
+ a_{32.4}(t)(LC) NCM_2(t) + a_{36.4}(t)(LC) NCM_6(t)
\]

\[
NCM_6(t) = a_{62}(t)(LC) NCM_2(t)
\]

(4.2.10)

Taking L.T. of the above equations and solving for \(NCM_1^*(s)\).

\[
NCM_1^*(s) = \frac{\Delta p v}{s[(s+\lambda+p v A)\Delta - \lambda g^{*}(s+\lambda)]}
\]

(4.2.11)
where

\[ \Delta = 1 - g^x(s) + g^x(s + \lambda) \]

\[ \bar{A} = 1 - \Lambda. \]

\[ A = \frac{(s + k)(s + k + \lambda)[m^x(s + k) + m^x(s + k + \lambda)\{g^x(s + \lambda) - g^x(s)\}]}{ \Delta(s + k)(s + k + \lambda) } \]

\[ + k[g^x(s)(s + k + \lambda)m^x(s + k) + (s + k)g^x(s)m^x(s + k + \lambda)] \]

In steady-state the expected number of times the CM is carried out in unit time \((NCM_1)\) is obtained by

\[ NCM_1 = \lim_{t \to \infty} [NCM_1(t)] \]

\[ = \lim_{s \to 0} [sNCM_1^x(s)]. \]