CHAPTER-6
COMPARATIVE RELIABILITY ANALYSIS OF TWO SIMILAR HOT STANDBY
PLC SYSTEM WITH PREVENTIVE/CORRECTIVE MAINTENANCE
ALONG WITH AND WITHOUT REPAIR FACILITY

Introduction: After going through many number of papers, it has been observed that considerations are depicted regarding abstract possibilities. In this paper we have considered purely industrial case. The data required for the study of this correspondence is being collected from Parle Biscuits Ltd. Bahadurgarh. In case-I we have provided a repair facility in which two repairmen are available, with the help of which repair of simultaneously failed PLC's can be carried out at the same time. In case-II, only one PLC is being taken care of at a time for repair second has to wait for repair if failed simultaneously.

To go deep into the matter two types of maintenances can be consulted and those are preventive and corrective maintenances. As plant is being closed on Sundays and two hours daily at lunch time, preventive maintenance is conducted by regular appointed engineer. Whenever fault arises, the hot standby is being repaired by the repairman in due course of time such an idea of practical case has not been reported so far in the literature of reliability while studying two unit Hot standby systems. Our aim is to fill in such existing gaps.

The model is analysed by making use of Markov process and regenerative point technique. Various measures of system effectiveness such as mean time to system failure, steady state availability, total fraction of busy time of regular repairman per unit time for operative unit as well as hot standby, expected time for which the regular engineer is busy in repairing operative & hot standby units are obtained.

Introducing the above ideas, through this paper we are able to find out that which case is profitable for industry. Also we have found out that whether it is beneficial in keeping regular engineer or every time calling
expert from the concerned industry when PLC fails. Profit in both the cases is compared.

Graphs pertaining to particular case are plotted and interpretations are made accordingly.

Case-I: HOT STANDBY SYSTEM WITH REPAIR FACILITY

Case-II: HOT STANDBY SYSTEM WITHOUT REPAIR FACILITY.

Fig. 6.1

NOTATIONS :

O : operative unit.

Hs : Hot standby in operating state.

fr : operative unit under repair of regular repairman.

fr₁ : Hot standby failed suddenly while in operating state and is
under repair of regular repairman.

\( f_{wr} \): Failed unit waiting for the repair of regular repairman.

\( f_d \): Both the units are in down state. System is not in use and they are under preventive maintenance.

\( \lambda \): Failure rate of operative unit.

\( \alpha \): Rate at which unit enters down state. System is not in use. Both are under preventive maintenance.

\( \beta \): Rate at which both unit comes out of down state.

\( \beta_1 \): Rate at which hot stand by unit is repaired by regular repairman.

\( \beta_2 \): Rate at which operative unit is repaired by regular repairman.

\[ o \]: operative unit.

\[ r \]: down state.

\[ s \]: Regenerative state

\[ \square \]: Failed state.

**Transition Probabilities:**

\[
\begin{align*}
q_{01} &= \alpha e^{-(\alpha+\delta+\lambda)t} & q_{02} &= \lambda e^{-(\alpha+\gamma+\lambda)t} & q_{03} &= \delta e^{-(\alpha+\delta+\lambda)t}.
\end{align*}
\]

\[
\begin{align*}
q_{10} &= \beta e^{-\beta t} & q_{20} &= \beta_2 e^{-(\beta_2+\lambda)t} & q_{25} &= \lambda e^{-(\beta_2+\lambda)t} & q_{42} &= \beta_1 e^{-(\beta_1+\beta_2)t} & q_{43} &= \beta_2 e^{-(\beta_1+\beta_2)t} & q_{52} &= 2\beta_2 e^{-2\beta_2 t}.
\end{align*}
\]

The non-zero elements \( p_{ij} \) are given below:

\[
\begin{align*}
p_{01} &= \frac{\alpha}{\delta + \delta + \lambda}, & p_{02} &= \frac{\lambda}{\delta + \alpha + \lambda}, & p_{03} &= \frac{\delta}{\alpha + \delta + \lambda}, & p_{10} &= 1, \\
p_{20} &= \frac{\beta_2}{\beta_2 + \lambda}, & p_{25} &= \frac{\lambda}{\lambda + \beta_2}, & p_{30} &= \frac{\beta_1}{\beta_1 + \lambda}, & p_{34} &= \frac{\lambda}{\beta_1 + \lambda}.
\end{align*}
\]
Mean Sojourn Times : \( m_i \)

\[
\begin{align*}
\mu_0(t) &= \frac{1}{\alpha + \delta + \lambda}, \\
\mu_1(t) &= \frac{1}{\beta}, \\
\mu_2(t) &= \frac{1}{\lambda + \beta_2}, \\
\mu_3(t) &= \frac{1}{\lambda + \beta_1} \\
\mu_4(t) &= \frac{1}{\beta_1 + \beta_2}, \\
\mu_5(t) &= \frac{1}{2\beta_2}
\end{align*}
\]

MTSF (Mean time to system failure)

Let \( \phi_i(t) \) be the c.d.f. of first passage time from regenerative state \( i \) to a failed state. To determine the mean time to system failure of system we regard failed states of the system absorbing. By probabilistic arguments, we obtain following recursive relations.

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \phi_1(t) + Q_{02}(t) \phi_2(t) + Q_{03}(t) \phi_3(t) \\
\phi_1(t) &= Q_{10}(t) \phi_0(t) \\
\phi_2(t) &= Q_{20}(t) \phi_0(t) + Q_{25} \\
\phi_3(t) &= Q_{30}(t) \phi_0(t) + Q_{34}
\end{align*}
\]

Taking Laplace Stieltjes transform of above equations and solving for \( \phi_0^{**}(s) \) the mean time to system failure, when the system starts from state ‘\( \phi_0 \)’ is given by

\[
\phi_0 = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N}{D}.
\]

where

\[
N = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3
\]

\[
D = 1 - p_{01} - p_{02}p_{20} - p_{03}p_{30}
\]

Availability Analysis:

Let \( A_i(t) \) be the probability that the system is in up state at instant \( t \) given that the system entered regenerative state \( i \) at \( t = 0 \). Using the arguments of theory of regenerative processes, the availability \( A_i(t) \) is seen to satisfy the following recursive relations.
\[ A_0(t) = M_0(t) + q_{01}(t) A_1(t) + q_{02}(t) A_2(t) + q_{03}(t) A_3(t) \]
\[ A_1(t) = q_{10}(t) + A_0(t) \]
\[ A_2(t) = M_2(t) + q_{20}(t) A_0(t) + q_{25}(t) A_5(t) \]
\[ A_3(t) = M_3(t) + q_{30}(t) A_0(t) + q_{34}(t) A_4(t) \]
\[ A_4(t) = q_{42}(t) A_2(t) + q_{43}(t) A_3(t) \]
\[ A_5(t) = q_{52}(t) A_2(t) \]

where
\[ M_0(t) = e^{-(\lambda+\delta+\alpha)t} \quad M_2(t) = e^{-(\lambda+\beta_2)t} \quad M_3(t) = e^{-(\lambda+\beta_1)t}. \]

Taking Laplace transforms of above equations and solving them for
\[ A_0^*(s), \]
we get
\[ A_0^*(s) = \frac{N_1(s)}{D_1(s)} \]

where
\[ N_1(s) = \overline{M}_0^*(s) \{1 - q_{34}^*(s) q_{43}^*(s) - q_{25}^*(s) q_{52}^*(s) + q_{25}^*(s) q_{34}^*(s) q_{43}^*(s) \}
+ \overline{M}_2^*(s) \{q_{02}^*(s) - q_{02}^*(s) q_{34}^*(s) q_{43}^*(s) + q_{03}^*(s) q_{34}^*(s) q_{42}^*(s)\}
+ \overline{M}_3^*(s) \{q_{03}^*(s) - q_{03}^*(s) q_{25}^*(s) q_{52}^*(s)\} \quad (6.41) \]

and
\[ D_1(s) = (1 - q_{01}^*(s) q_{10}^*(s)) \{1 - q_{52}^*(s) q_{25}^*(s) + q_{25}^*(s) q_{34}^*(s) q_{43}^*(s) q_{52}^*(s) - q_{34}^*(s) q_{43}^*(s)\}
- q_{02}^*(s) \{q_{20}^*(s) - q_{20}^*(s) q_{34}^*(s) q_{43}^*(s)\} - q_{03}^*(s) \{q_{20}^*(s) q_{42}^*(s) + q_{30}^*(s) - q_{25}^*(s) q_{30}^*(s) q_{52}^*(s)\} \quad (6.42) \]

the steady state availability of the system is given by
\[ A_0 = \lim_{s \to 0} s A_0^*(s) = \frac{N_1}{D_1} \]

where
\[ N_1 = \mu_0 (p_{02} + p_{20} (1 - p_{34} p_{43})) - \mu_2 (p_{02} p_{34} p_{43} - p_{03} p_{34} p_{42}) + \mu_3 p_{03} p_{20} \quad (6.43) \]
\[ D_1 = \mu_0 p_{02} (1 - p_{34} p_{43}) + \mu_1 p_{02} p_{20} (1 - p_{34} p_{43}) + \mu_2 (p_{02} - p_{02} p_{34} p_{43} + p_{03} p_{34} p_{42}) + \mu_3 p_{03} p_{20} + \mu_4 p_{02} p_{20} p_{34} + \mu_5 (p_{02} p_{25} p_{34} p_{43} - p_{02} p_{25} - p_{03} p_{25} p_{34} p_{42}). \quad (6.44) \]
Busy period analysis of regular repairman for corrective maintenance.

Let BR_i(t) be the probability that the repairman is at instant t is busy in repair, given that the system entered the regenerative state i at t = 0 for BR_i(t). By probabilistic arguments we have the following recursive relations

\[ BR_0(t) = q_{01}(t) \cdot BR_1(t) + q_{02}(t) \cdot BR_2(t) + q_{03}(t) \cdot BR_3(t) \]
\[ BR_1(t) = q_{10}(t) \cdot BR_0(t) \]
\[ BR_2(t) = G_{21}(t) + q_{20}(t) \cdot BR_0(t) + q_{25}(t) \cdot BR_5(t) \]
\[ BR_3(t) = q_{30}(t) \cdot BR_0(t) + q_{34}(t) \cdot BR_4(t) \]
\[ BR_4(t) = G_{22}(t) + q_{42}(t) \cdot BR_2(t) + q_{43}(t) \cdot BR_3(t) \]
\[ BR_5(t) = G_{23}(t) + q_{52}(t) \cdot BR_2(t) \]

(6.45-6.50)

where

\[ G_{21}(t) = e^{-(\lambda+\beta_2)t}, \quad G_{22}(t) = e^{-(\beta_1+\beta_2)t}, \quad G_{23}(t) = e^{-2\beta_2t} \]

Taking Laplace transform of above equations and solving them for \( BR_0^*(s) \), we get

\[ BR_0^*(s) = \frac{N_2(s)}{D_1(s)} \]

Where

\[ N_2^*(s) = G_{21}^*(s) \{q_{02}^*(s) - q_{02}^*(s) q_{34}^*(s) q_{43}^*(s) + q_{03}^*(s) q_{34}^*(s) q_{42}^*(s)\} + G_{23}^*(s) \{q_{02}^*(s) q_{25}^*(s) - q_{02}^*(s) q_{25}^*(s) q_{34}^*(s) q_{43}^*(s) + q_{03}^*(s) q_{25}^*(s) q_{34}^*(s) q_{42}^*(s)\} + G_{22}^*(s) \{q_{03}^*(s) q_{34}^*(s) - q_{03}^*(s) q_{25}^*(s) q_{23}^*(s)\} \]

(6.51)

and \( D_1(s) \) is specified.

The total fraction of time for which the system is under of regular repairman.
where
\[
N_2 = \mu_2(p_{02} - p_{02}p_{34}p_{43} - p_{03}p_{34}p_{42}) + \mu_4p_{03}p_{20}p_{34} + \mu_5p_{25}(p_{02} - p_{02}p_{34}p_{42} + p_{03}p_{34}p_{42}).
\] (6.52)
and \(D_1\) is already specified.

**Busy period analysis for the Hot stand by unit under the repair of regular engineer.**

Let \(BH_i(t)\) be the probability that the repairman is busy at instant \(t\) for repair of hot standby, given that the system entered the regenerative state \(i\) at \(t = 0\) for \(BH_i(t)\). By probabilistic arguments, we have the following recursive relations.

\[
\begin{align*}
BH_0(t) &= q_{01}(t)BH_1(t) + q_{02}(t)BH_2(t) + q_{03}(t)BH_3(t) \\
BH_1(t) &= q_{10}(t)BH_0(t) \\
BH_2(t) &= q_{20}(t)BH_0(t) + q_{25}(t)BH_5(t) \\
BH_3(t) &= G_{11}(t) + q_{30}(t)BH_0(t) + q_{34}(t)BH_4(t). \\
BH_4(t) &= G_{12}(t) + q_{42}(t)BH_2(t) + q_{43}(t)BH_3(t) \\
BH_5(t) &= q_{52}(t)BH_2(t).
\end{align*}
\] (6.53-6.58)

where
\[
G_{11}(t) = e^{-(\lambda + \beta_1)t}, \quad G_{12}(t) = e^{-(\beta_1 + \beta_2)t}
\]

Taking laplace transform of above equations and solve them for \(BH_0^*(s)\), we get
\[
BH_0^*(s) = \frac{N_3(s)}{D_1(s)}
\]

Where
\[
N_3(s) = G_{11}^*(s)\{q_{03}^*(s) - q_{03}^*(s)q_{25}^*(s)q_{52}^*(s)\} + G_{12}^*(s)\{q_{03}^*(s)q_{34}^*(s) - q_{03}^*(s)q_{25}^*(s)q_{34}^*(s)q_{52}^*(s)\}
\] (6.59)
and \(D_1(s)\) is already specified.
The total fraction of time for which the system (hot stand by unit) is under repair of regular repairman, in steady state is given by

\[ BH_0 = \lim_{s \to 0} sBH_0^* (s) = \frac{N_3}{D_1} \]

where \( N_3 = p_{03}p_{20}\{\mu_3p_{03} + \mu_4p_{34}\} \).

(6.60)

and \( D_1 \) is already specified.

**Busy period analysis for preventive maintenances by regular repairman.**

Let \( BP_i(t) \) be the probability that regular engineer is busy at instant \( t \), given that the system entered regenerative state \( i \) at \( t = 0 \) for \( BP_i(t) \). By probabilistic arguments we have following recursive relations.

\[\begin{align*}
BP_0(t) &= q_{01}(t)BP_1(t) + q_{02}(t)BP_2(t) + q_{03}(t)BP_3(t) \\
BP_1(t) &= \bar{G}(t) + q_{10}(t)BP_0(t) \\
BP_2(t) &= q_{20}(t)BP_0(t) + q_{25}(t)BP_5(t) \\
BP_3(t) &= q_{30}(t)BP_0(t) + q_{34}(t)BP_4(t) \\
BP_4(t) &= q_{42}(t)BP_2(t) + q_{43}(t)BP_3(t) \\
BP_5(t) &= q_{52}(t)BP_2(t) \
\end{align*}\]

(6.61-6.66)

where

\[ \bar{G}(t) = e^{-\beta t} \]

Now taking Laplace transform of the above equations and solving them for \( BP_0^*(s) \), we get

\[ BP_0(s) = \frac{N_4(s)}{D_1(s)} \]

where \( N_4(s) = \bar{G}^*(s)q_{01}^*(s)\{1-q_{34}^*(s)q_{43}^*(s) - q_{25}^*(s)q_{52}^*(s) + q_{25}^*(s)q_{34}^*(s)q_{43}^*(s)q_{52}^*(s)\} \)

(6.67)

and \( D_1(s) \) is already specified.

The total fraction of time for which the system is under preventive maintenance of regular repairman, in steady state is given by
\[ \lim_{s \to 0} sBP_0^* (s) = \frac{N_4}{D_1}. \]

\[ N_4 = 1 p_{01} p_{20} [1 - p_{34} p_{43}]. \]  

and \( D_1 \) is already specified.

**Expected time for the regular engineer repairing the PLC system.**

Let \( S_i(t) \) be the probability that the unit is under the repair of regular repairman at time \( t \). Using probabilistic arguments we get following recursive relations.

\[
\begin{align*}
R_0(t) &= Q_{01}(t) s R_1(t) + (1 + R_2(t)) s Q_{02}(t) + (1 + R_3(t)) s Q_{03}(t) \\
R_1(t) &= Q_{10}(t) s R_0(t) \\
R_2(t) &= (1 + R_5(t)) s Q_{25}(t) + Q_{20}(t) s R_0(t) \\
R_3(t) &= (1 + R_4(t)) s Q_{34}(t) + Q_{30}(t) s R_0(t) \\
R_4(t) &= Q_{42}(t) s R_2(t) + Q_{43}(t) s R_3(t). \\
R_5(t) &= Q_{52}(t) s R_2(t)
\end{align*}
\]

Taking Laplace Steiltjes transform for above relations and solve them for \( R_0^*(s) \), we get

\[ R_0^*(s) = \frac{N_5(s)}{D_1(s)} \]

Where

\[
N_5(s) = (1 - Q_{34}^*(s)) Q_{43}^*(s) - Q_{25}^*(s) Q_{43}^*(s) - Q_{25}^*(s) Q_{34}^*(s) Q_{43}^*(s) Q_{52}^*(s) Q_{02}^*(s) + Q_{03}^*(s)) + Q_{03}^*(s) Q_{25}^*(s) + Q_{02}^*(s) Q_{25}^*(s) Q_{34}^*(s) Q_{43}^*(s) Q_{52}^*(s) Q_{03}^*(s) Q_{34}^*(s) Q_{42}^*(s) - Q_{03}^*(s) Q_{25}^*(s) Q_{34}^*(s) Q_{52}^*(s)
\]

and \( D_1(s) \) is already specified.

The Expected Time for which hot standby and operating unit is under the repair of regular repairman is given by,

\[ R_0 = \frac{N_5}{D_1} \]
where
\[ N_5 = (p_{02} + p_{03}) (1 - p_{34} p_{43} - p_{25} - p_{25} p_{34} p_{43}) + p_{02} p_{25} + p_{02} p_{25} p_{34} p_{43} + p_{03} p_{34} + p_{03} p_{25} p_{34} p_{42} - p_{03} p_{25} p_{34} \]
and \( D_1 \) is already specified.

**CASE-II**

**State Transition Probabilities \( (q_{ij}) \)**

\[
q_{01} = \alpha e^{-\left(\alpha + \delta + \lambda\right)t} \\
q_{10} = \beta e^{-\beta t} \\
q_{30} = \beta_1 e^{-\left(\beta_1 + \lambda\right)t} \\
q_{52} = \beta_2 e^{-\beta_2 t}
\]

Non-zero elements \( p_{ij} \) are given as.

\[
p_{ij} = \lim_{s \to 0} \int_0^\infty q_{ij}(t) \, dt
\]

\[
p_{01} = \frac{\alpha}{\alpha + \delta + \lambda} \\
p_{02} = \frac{\lambda}{\alpha + \delta + \lambda} \\
p_{03} = \frac{\delta}{\alpha + \lambda + \delta}
\]

\[
p_{10} = 1 \\
p_{20} = \frac{\beta_2}{\beta_2 + \lambda} \\
p_{25} = \frac{\lambda}{\beta_2 + \lambda}
\]

\[
p_{30} = \frac{\beta_1}{\beta_1 + \lambda} \\
p_{34} = \frac{\lambda}{\beta_1 + \lambda} \\
p_{42} = 1
\]

\[
p_{52} = 1.
\]

**Mean Sojourn Times \( (m_{ij}) \)**

\[
m_0(t) = \frac{1}{\alpha + \delta + \lambda} \\
m_1(t) = \frac{1}{\beta} \\
m_2(t) = \frac{1}{\lambda + \beta_2} \\
m_3 = \frac{1}{\lambda + \beta_1}
\]

\[
m_4 = \frac{1}{\beta_1} \\
m_5 = \frac{1}{\beta_2}
\]
For MTSF we receive same equations as in case-I.

**Availability**

For case-II we receive following recursive relations by probabilistic arguments. Taking laplace transform of above equations and solving them for \( A_0^*(s) \), the steady state availability of the system is given by

\[
A_0(t) = M_0(t) + q_0(t) A_1(t) + q_{02}(t) A_2(t) + q_{03}(t) A_3(t).
\]

\[
A_1(t) = q_{10}(t) A_0(t)
\]

\[
A_2(t) = M_2(t) + q_{20}(t) A_0(t) + q_{25}(t) A_5(t).
\]

\[
A_3(t) = M_3(t) + q_{30}(t) A_0(t) + q_{34}(t) A_4(t)
\]

\[
A_4(t) = q_{42}(t) A_2(t)
\]

\[
A_5(t) = q_{52}(t) A_2(t).
\]

where

\[
M_0(t) = e^{-(\lambda+\delta+\eta)t} \quad M_2(t) = e^{-(\lambda+\beta_2)t} \quad M_3(t) = e^{-(\lambda+\beta_1)t}
\]

Taking laplace transforms of above equations and solving them for \( A_0^*(s) \), we get

\[
A_0^*(s) = \frac{N_1(s)}{D_1(s)}
\]

where

\[
N_1(s) = \{1-q_{34}^*(s)q_{25}^*(s)q_{52}^*(s)+q_{25}^*(s)q_{34}^*(s)q_{52}^*(s)\} + \bar{M}_2^*(s)
\]

\[
\{q_{02}^*(s) - q_{02}^*(s)q_{34}^*(s) + q_{03}^*(s)q_{34}^*(s)q_{42}^*(s)\} + \bar{M}_3^*(s)\{q_{03}^*(s) - q_{03}^*(s)q_{52}^*(s)q_{52}^*(s)\}
\]

\[
q_{25}^*(s)q_{52}^*(s)
\]

and

\[
D_1(s) = (1-q_{01}^*(s)q_{10}^*(s))\{1-q_{25}^*(s)q_{25}^*(s) + q_{25}^*(s)q_{34}^*(s)q_{52}^*(s) - q_{34}^*(s)q_{43}^*(s)\} - q_{02}^*(s)\{q_{20}^*(s) - q_{20}^*(s)q_{34}^*(s)\} - q_{03}^*(s)\{q_{20}^*(s)q_{34}^*(s)q_{42}^*(s) + q_{30}^*(s) - q_{25}^*(s)q_{30}^*(s)q_{52}^*(s)\}
\]

The steady state availability of the system is given by
\[
A_0(t) = \lim_{s \to 0} s A^*(s) = \frac{N_1}{D_1}
\]

where
\[
N_1 = \mu_0(p_{02} + p_{20}) + \mu_2 p_{03}p_{34} + \mu_3 p_{03}p_{20}
\]
\[
D_1 = \mu_0 p_{02} + \mu_1 p_{01}p_{20} + \mu_2(p_{02} + p_{03}p_{34}) + \mu_3 p_{03}p_{20} + \mu_4 p_{02}p_{20}p_{34} + \\
\mu_5[-p_{02}p_{25} - p_{03}p_{25}p_{34}].
\]

**Busy period analysis of regular repairman for corrective maintenance.**

Following recursive relations will be obtained by probabilistic arguments for \( BR_i(t) \). Taking laplace transform of above relations and solving them for \( BR_0^*(s) \), the total fraction of time for which the system is under repair of regular repairman.

\[
BR_0(t) = q_{01}(t)BR_1(t) + q_{02}(t)BR_2(t) + q_{03}(t)BR_3(t)
\]
\[
BR_1(t) = q_{10}(t)BR_0(t)
\]
\[
BR_2(t) = \overline{G}_{21}(t) + q_{20}(t)BR_0(t) + q_{25}(t)BR_5(t)
\]
\[
BR_3(t) = q_{30}(t)BR_0(t) + q_{34}(t)BR_4(t).
\]
\[
BR_4(t) = q_{42}(t)BR_2(t)
\]
\[
BR_5(t) = \overline{G}_{22}(t) + q_{52}(t)BR_2(t).
\]

where
\[
\overline{G}_{21}(t) = e^{-(\lambda + \beta_2)t} \quad \overline{G}_{22}(t) = e^{-\beta_2t}
\]

Taking Laplace transform of above equations and solving them for \( BR_0^*(s) \), we get
\[
BR_0^*(s) = \frac{N_2(s)}{D_1(s)}
\]

Where
\[ N_2^*(s) = \tilde{G}_{21}^*(s) \{q_{02}^*(s) - q_{02}^*(s) q_{34}^*(s) + q_{03}^*(s) q_{34}^*(s) q_{42}^*(s)\} + \tilde{G}_{23}^*(s) \{q_{25}^*(s) - q_{25}^*(s) q_{34}^*(s) + q_{34}^*(s) q_{42}^*(s)\} \]
\[ + \tilde{G}_{22}^*(s) \{q_{02}^*(s) q_{25}^*(s) - q_{02}^*(s) q_{25}^*(s) q_{M}^*(s) + q_{03}^*(s) q_{25}^*(s) q_{42}^*(s)\} \]
\[ \tilde{G}_{22}^*(s) \{q_{02}^*(s) - q_{02}^*(s) q_{34}^*(s) + q_{03}^*(s) q_{34}^*(s) q_{42}^*(s)\} \]
(6.119)

and \( D_1(s) \) is specified.

The total fraction of time for which the system is under repairman.

\[ BR_0 = \lim_{s \to 0} s BR_0^*(s) = \frac{N_2}{D_1} \]

where

\[ N_2 = \mu_2(p_{02}p_{03}p_{34}) + \mu_4p_{03}p_{20}p_{34} + \mu_2p_{25}(p_{02}p_{02}p_{34} + p_{03}p_{34}) \]
(6.120)

and \( D_1 \) is already specified.

**Busy period analysis for Hot standby under the repair of regular repairman.**

Following recursive relations are obtained by using probabilistic arguments for \( BH_i(t) \). Taking laplace of above relations and solving them for \( BH_0(t) \), the fraction of time for which unit (hot stand by) is under repair of regular repairman, in steady state is given by

\[ BH_0(t) = q_{01}(t) @ BH_1(t) + q_{02}(t) @ BH_2(t) + q_{03}(t) @ BH_3(t) \]
\[ BH_1(t) = q_{10}(t) @ BH_0(t) \]
\[ BH_2(t) = q_{20}(t) @ BH_0(t) + q_{25}(t) @ BH_5(t) \]
\[ BH_3(t) = \tilde{G}_{11}(t) + q_{30}(t) @ BH_0(t) + q_{34}(t) @ BH_4(t). \]
\[ BH_4(t) = \tilde{G}_{12}(t) + q_{42}(t) @ BH_2(t) \]
\[ BH_5(t) = q_{52}(t) @ BH_2(t). \]
(6.121-6.126)

\[ \tilde{G}_{11}(t) = e^{-(\lambda_1+\beta_1)t}, \quad \tilde{G}_{12}(t) = e^{-(\beta_1+\beta_2)t} \]

Taking laplace transform of above equations and solve them for \( BH_0^*(s) \), we get
\[ BH_0^*(s) = \frac{N_3(s)}{D_1(s)} \]

Where
\[ N_3(s) = \bar{G}_{11}(s) \{q_{03}(s) - q_{03}(s) q_{25}(s) q_{52}(s)} + \bar{G}_{12}(s) \{q_{03}(s) q_{34}(s) - q_{03}(s) q_{25}(s) q_{52}(s)} \]

(6.127)

and \( D_1(s) \) is already specified.

The total fraction of time for which the system (hot stand by unit) is under repair of regular repairman, in steady state is given by

\[ BH_0 = \lim_{s \to 0} s BH_0^*(s) = \frac{N_3}{D_1} \]

where
\[ N_3 = p_{03}p_{20} [\mu_3p_{03} + \mu_4p_{34}] \]

(6.128)

and \( D_1 \) is already specified.

**Busy period analysis for preventive maintenance by regular repair.**

Following recursive relations are obtained by probabilistic arguments for \( BP_i(t) \). Taking laplace transform of above relations and solving them for \( BP_0^*(s) \), the total fraction of time for which the system is under preventive maintenance of regular repairman, in steady state is given by

\[ BP_0(t) = q_{01}(t)BP_1(t) + q_{02}(t)BP_2(t) + q_{03}(t)BP_3(t) \]
\[ BP_1(t) = q_{10}(t)BP_0(t) \]
\[ BP_2(t) = q_{20}(t)BP_0(t) + q_{25}(t)BP_5(t) \]
\[ BP_3(t) = q_{30}(t)BP_0(t) + q_{34}(t)BP_4(t) \]
\[ BP_4(t) = q_{42}(t)BP_2(t) \]
\[ BP_5(t) = q_{52}(t)BP_2(t) \]

(6.129-6.134)

where
\[ \bar{G}(t) = e^{-\beta t} \]

Now taking Laplace transform of the above equations and solving them for \( BP_0^*(s) \), we get
\[ BP_0(s) = \frac{N_4(s)}{D_1(s)} \]

Where \( N_4(s) = G^*(s)q_{01}^*(s) \{1-q_{34}^*(s) - q_{25}^*(s) q_{52}^*(s) + q_{25}^*(s) q_{34}^*(s) q_{52}^*(s)\} \) \hspace{1cm} (6.135)

and \( D_1(s) \) is already specified.

The total fraction of time for which the system is under preventive maintenance of regular repairman, is steady state is given by \( BP_0 \)

\[ = \lim_{s \to 0} s BP_0^*(s) = \frac{N_4}{D_1} \]

\[ N_4 = \mu_1 p_{01} p_{20}. \] \hspace{1cm} (6.136)

and \( D_1 \) is already specified.

**Expected time for the regular engineer repairing the PLC.**

\[ R_0(t) = Q_{01}(t) + R_1(t) + (1+R_2(t)) + Q_{02}(t) + (1+R_3(t)) + Q_{03}(t) \]
\[ R_1(t) = Q_{10}(t) + R_0(t) \]
\[ R_2(t) = Q_{20}(t) + R_0(t) + Q_{25}(t) + (1+R_5(t)) \]
\[ R_3(t) = Q_{30}(t) + R_0(t) + Q_{34}(t) + R_4(t) \]
\[ R_4(t) = Q_{42}(t) + (1+R_2(t)) \]
\[ R_5(t) = Q_{52}(t) + R_2(t) \] \hspace{1cm} (6.137-6.142)

Taking Laplace Steiltjes transform for above relations and solve them for \( R_0^{**}(s) \), we get

\[ R_0^{**}(s) = \frac{N_5(s)}{D_1(s)} \]

Where
\[ N_5(s) = (1-Q_{34}^{**}(s) - Q_{25}^{**}(s) Q_{52}^{**}(s) - Q_{25}^{**}(s) Q_{34}^{**}(s) Q_{52}^{**}(s)) (Q_{02}^{**}(s) + Q_{03}^{**}(s)) + Q_{02}^{**}(s) Q_{25}^{**}(s) + Q_{02}^{**}(s) Q_{25}^{**}(s) Q_{34}^{**}(s) + Q_{03}^{**}(s) Q_{34}^{**}(s) + Q_{03}^{**}(s) Q_{25}^{**}(s) Q_{34}^{**}(s) - Q_{03}^{**}(s) Q_{25}^{**}(s) Q_{34}^{**}(s) Q_{52}^{**}(s) \] \hspace{1cm} (6.143)

and \( D_1(s) \) is already specified.
The Expected Time for which hot standby and operating unit is under the repair of regular repairman is given by,

\[ R_0 = \frac{N_5}{D_1} \]

where

\[ N_5 = (p_{02} + p_{03}) \{1 - p_{25}\} + p_{02}p_{25} - p_{03}p_{25} - p_{03}p_{34} + p_{03}p_{25}p_{34} \] (6.144)

and \( D_1 \) is already specified.

**Profit Analysis**

In steady state the expected total profit (for both the cases) is given by

\[ P = C_0 A_0 - C_1 B R_0 - C_2 B H_0 - C_3 B P_0 - C_4 R_0 \] (6.145)

where

- \( C_0 \) = Revenue per unit up time.
- \( C_1 \) = cost per unit up time for which regular repairman is busy in repair of operative unit.
- \( C_2 \) = cost per unit up time for which regular repairman is busy for repair of hot standby.
- \( C_3 \) = cost per unit up time for which unit is in down state.
- \( C_4 \) = cost for the regular engineer in case-A and cost for the expert repairman to be called upon whenever unit fails in case-B.

**20. Graphical Interpretation:**

(i) Fig. (6.2) & (6.3) shows the behaviour of M.T.S.F. w.r.t. failure rate with \( \beta_2 \) repair by regular engg) and \( \beta \) (down state rate) as parameters respectively. Here MTSF decreases with increase in failure rate. However it attains greater values with the increase in \( \beta_2 \) and decrease in \( \beta \). Whereas MTSF remains same for both case-I & case-II.
(2) In fig (6.4) We have plotted graph between profit and revenue per unit up time for with and without repair facility for regular repairman. Here we get more profit with without repair facility.

(3) In fig. (6.5) We have plotted graph between profit and revenue per unit up time for visiting and regular repairman for with repair facility. Here visiting repairman provides us with greater profit.

(4) Graph is being plotted between profit and failure rate for with and without repair facility for regular repairman in figure (6.6). Here without repair facility gives us more profit.

(5) Figure (6.7) plots the graph between availability and failure rate for regular and visiting repairman with repair facility. Availability is more with visiting repairman.

(6) Availability is plotted with failure rate in figure (6.8) for with and without repair facility for regular repairman. Availability is more with repair facility.

21 Conclusion

(i) Here we can simply o with permanent engineer without repair facility as far as revenue and PLC at Parle Biscuits Ltd. are concerned.

(ii) Expert repairman bears less cost but regular engineer is involved in down state repairs & maintenances which reduces failure rate of PLC. Therefore we will again prefer regular engineer.

(iii) As we have seen that in case-II. We have got slightly more profit. But considering all the factors we can avoid repair facility. Because PLC is a very versatile device and we are not coming across simultaneous big failures. Small failures are cured by regular appointed engineers with less of effort and revenue.

(iv) Since the engineers of PARLE BISCUIT Ltd. Bahadurgarh are sent for regular training sessions which improves the quality of professionals and reduce the requirement of experts. Also regular
engineers increases the life of PLC by constant inspections and maintenances whereas

(v) Mean Time to system failure remains unchanged in all the four cases.
MTSF V/S Failure Rate ($\lambda$) for different values of repair rate by regular repairman ($\beta_r$)

$\alpha = .05, \beta = .2, \beta_1 = .04, \delta = .0002$

$\beta_2 = 3$

$\beta_2 = 1.5$

Fig. 6.2
MTSF V/S failure Rate($\lambda$) for different values of down state rate ($\beta$)

$\alpha = .05$, $\beta_2 = 1.5$, $\beta_1 = .04$, $\delta = .0002$

Fig. 6.3
Profit vs Revenue per unit up time for with repair facility (W.R) and without repair facility (N.R) for regular repairman.

Fig. 6.4

Profit vs Revenue per unit up time for with repair facility (W.R) and without repair facility (N.R) for regular repairman.

\[ \alpha = .05, \beta_1 = 1, \beta_2 = 1.5, \delta = .0002, \lambda = .00035 \]

Fig. 6.4
Profit v/s Revenue per unit up time for regular and visiting reprises for with repair facility (W.R.)

\[ \alpha = 0.05, \beta_1 = 1, \beta_2 = 1.5, \delta = 0.0002, \lambda = 0.0035 \]

- Reg.rep
- Vis.rep

Fig. 6.5
Fig. 6.6
Failure Rate ($X$)

Profit vs. Failure rate with and without repair facility for regular engineer.

$\alpha = 0.05, \beta_1 = 1, \beta_2 = 1.5, \delta = 0.0002$

$W.R$  $N.R$

Profit

$-50000$ $0$ $50000$

Failure Rate ($X$)
Fig. 6.7

Availability vs. Failure Rate $ \alpha$ for Regular & Visiting Repairment with Repair Facility

$\alpha = 0.05, \beta_1 = 1, \beta_2 = 1.5, \delta = 0.0002$
Availability vs. Failure Rate for with and without repair facility for Regular Repairment

α = 0.5, β₁ = 1, β₂ = 1.5, δ = 0.0002

Fig. 6.8