CHAPTER 1

Optical Turbulence and Adaptive Optics

1.1 Introduction

The image obtained in the presence of atmospheric turbulence is highly aberrated to such an extent that the image may not be often recognizable. This problem of atmospheric distortion is of interest to a broad range of disciplines that encompass the astronomical community; designers of high-energy laser beam control systems for communications as well as ground based imaging systems. For example, the twinkling of nighttime stars has frustrated the astronomers since the invention of telescope. The random intensity fluctuations known as optical turbulence, occurs since the radiation passes through the thick blanket of air surrounding the earth. This optical turbulence relates to the fluctuations of refractive index of the transmitting medium due to factors such as pressure, temperature and so on [1-2]. Even a small temperature fluctuation of the order of a tenth of a degree might generate strong wavefront perturbations over a propagation path of few hundred meters. An ideal image should be free from such aberrations for signal or target recognition. Hence, the imaging system has to be incorporated with the feature of correcting the image distortion.

A key point in the development of new concepts and methods for adaptive optics system is a basic understanding of the effects and the influence of turbulence on the optical image formation. Wavefront errors, which change slowly, i.e. longer time scales, are usually compensated by active optics implemented on most modern telescopes.

1.1.1. Seeing and angular resolution

Ground based telescopes suffer even under the best seeing conditions from the effects of atmospheric turbulence that cause wavefront degradation. We can
observe the effect of distortions even with the naked eye at night. The stars twinkle due to intensity fluctuations. This phenomenon is known as Scintillation.

When the plane wavefront passes through the thick blanket of atmospheric region it encounters refractive index fluctuations, and as a result the beam is distorted when it reaches the telescope system. Here the transmitted wavefront conditions have to be explored qualitatively and qualitatively with respect to the influence of randomly distributed refractive index fluctuations. First, there is generally an over all tilt of the wavefront that is different from the initial one. Thus the centroid of the image pattern does not lie at the same spot in the focal plane at which the light was originally aimed. This tilt will of course change with time, so the centroid of the focal plane image will seem to dance around. Secondly, there are many sub-wavefront parts with different tilts, spread over telescope mirror in projection. These will lead to distinct spots of essentially telescope-diffraction limited size, called speckles that are randomly placed in the focal plane. As the refractive index fluctuations do, speckles move around, and come and go, with time. Finally, there are small scale changes due to changing curvature of the wavefront. If the curvature is high enough, the corresponding rays are significantly defocused, compared to the nominal focal plane.

In space, where the light does not have to travel though the atmosphere, the image of a point source is limited only by the optics of the telescope and the diffraction of light. A perfect telescope would have a theoretical diffraction limited angular resolution of

\[
\alpha = \frac{1.22 \lambda}{D}
\]  

(1.1)

Where

- \(\alpha\) – Angular resolution of the imaging system
- \(\lambda\) - Wavelength of light
- \(D\) - Diameter of the telescope unit

Atmospheric turbulence degrades the image primarily by phase fluctuation over the aperture of the telescope. For short exposures these fluctuating aberrations
will cause the light to scatter and form a speckle pattern in the image plane, where the individual speckles are diffraction-limited. Due to the turbulence, the angular resolution is described as:

\[ \Delta \alpha' = \frac{1.22 \lambda}{r_0(\lambda)} \]  

Where \( \Delta \alpha' \) is the seeing angle and \( r_0 \) is the Fried parameter, which is the phase coherence length across the turbulent wavefront, i.e. the distance over which the wavefront is not significantly perturbed. The larger the \( r_0 \), better is the seeing resolution limit [2-4].

For example, under diffraction limited imaging (i.e., free of aberration) the angular resolution of image with respect to a 10m Keck Telescope and Human eye are estimated as follows:

- D= 10m Keck Telescope \( \alpha = 0.013 \) arc sec.
- D= 7-8mm (dilated Human eye) \( \alpha = 0.3 \) arc sec.

Due to the influence of turbulence, the seeing resolution for the both system would be

- Keck Telescope \( \alpha = 0.5 \) arc sec
- Human eye \( \alpha = 1 \) arc minute

The Fried parameter is an important parameter for modeling the turbulent wavefront and will be explained in more detail in the next section.

1.1.2 Imaging through atmospheric turbulence and the Fried parameter

Three parameters are often used to characterize seeing: the Fried Parameter ‘\( r_0 \)’, the time scale ‘\( t_0 \)’ on which image changes significantly while seeing, and the
isoplanatic angle \( \theta_{ip} \). The resolution of seeing-limited images obtained through an atmosphere with turbulence characterized by a Fried Parameter \( r_0 \) is the same as the resolution of diffraction-limited images taken with a telescope of diameter \( r_0 \). The Fried parameter is the typical measure of refractive index fluctuations. In terms of the Fried parameter, the Full Width at Half Maximum (FWHM) of the seeing disk turns out to about

\[
\Delta \theta_{\text{seeing}} = 1.22 \frac{\lambda}{r_0} \quad (1.3)
\]

The Fried parameter is seen to vary with site and weather conditions roughly in the range 3-30cm at visible wavelengths. Also Fried parameter gives estimation for the number of speckles in the focal plane, as the ratio of telescope area to that of typical index fluctuation,

\[
N = \frac{D^2}{r_0^4} \quad (1.4)
\]

It is predicted from the Kolmogorov theory [2] of turbulence, and the known dispersion of air, that Fried parameter varies with wavelength as \( r_0 \propto \lambda^{6/5} \). Thus the FWHM diameter of the seeing disk should get smaller gradually as wavelength increases, as \( \Delta \theta_{\text{seeing}} \propto \lambda^{-1/5} \) [2-5].

The time scale for a significant change in the arrangement of index fluctuations is most often determined by the speed of the wind. Therefore,

\[
t_0 = \frac{r_0}{v} \quad (1.5)
\]

Where,

- \( r_0 \) - beam diameter or Fried parameter
- \( v \) - wind velocity.
Thus to make an image of the pattern sharp in details, or to make an optical correction toward the best seeing, one must act quickly, with exposure or reaction times in a few to few tens of milliseconds.

Plane waves originating in two point objects very close together on the sky can be considered to traverse essentially the same pattern of refractive index fluctuations on their way into the optics; those from more distant objects pass through a different set of cells. The reference angle by which the object points are close and far is desired by the isoplanatic angle ‘$\theta_{ip}$’.

$$\theta_{ip} = r_0/3h$$  \hspace{1cm} (1.6)

Where,

$r_0$ - Fried parameter

$h$ – Elevation above the telescope

There are many models describing atmospheric turbulence, but the most commonly used one is the structure function introduced by Kolmogorov in 1961 as can be found in the book by Tyson [2]. These structure functions describe the random functions used in turbulence theory.

Fig.1.1 shows the effect of atmospheric turbulence on the propagating light beam. Light from a point source or star traveling to the earth, due to the large distance, preserves a plane wavefront. In the earth’s atmosphere these wavefronts are distorted randomly while moving through different layers or cells of air with differences in the refractive index. These variations in the refractive index arise from variations in density, which are caused by temperature fluctuations. The structure function $D_n(r)$, which describes the properties of the atmospheric turbulence, gives the variation in refractive index ($N$) between two points on the wavefront separated by a distance ($r_1-r_2$) and can be written as follows:

$$D_n(r) = \langle |n(r_1) - n(r_2)|^2 \rangle = C_n^2 \cdot r^{2/3},$$  \hspace{1cm} (1.7)
Scintillation contributes much less to the image degradation than distortions of the phase of the wavefront. The phase structure function $C_n^2$ and $r_0$ across the diameter of the telescope is given by

$$D_\phi(r) = 6.88 \left( \frac{r}{r_0} \right)^{5/3} \text{rad}^2,$$

(1.8)

In the Kolmogorov model, this equation uses the Fried parameter [2-8]. The Fried parameter $r_0$ is given by,

$$r_0(\lambda, \xi) = 0.185\lambda^{6/5} \cos^{3/5} \xi \left( \int C_n^2(h) dh \right)^{-3/5}$$

(1.9)

Where $\lambda$ is the wavelength of light, $\xi$ the zenith angle (angle of observation) and $C_n^2$ the structure constant for the refractive index variations which is integrated through the optical path of turbulence. Equation 1.9 describes the spatial distribution of the turbulence. The refractive index is also a function of altitude or height above the ground and the extensive studies performed by Fried has been summarized by Robert Tyson [2].

Since $r_0$ increases with $\lambda$ as in eqn.1.9, the image quality is better at longer wavelengths. For a further analysis one also needs to know how fast these fluctuations evolve with time. The speed of the turbulence varies with height; the temporal variations are often described by an average wind velocity $V_a$, which is typically around 10 m per second. The temporal variations are given by

$$\tau_0 = 0.314 \left( \frac{r_0}{V_a} \right)$$

(1.10)

which are on the order of a few milliseconds. These temporal variations are also called the coherence.
Point Source or Star

Angular resolution is
\[ \Delta \alpha = 1.22 \frac{\lambda}{D} \]
or if \( r_0 < D \)
\[ \Delta \alpha = 1.22 \frac{\lambda}{r_0} \]

Atmospheric Turbulence

Perfect Image

2.44 \( \frac{\lambda}{D} \)

Telescope Aperture

Imaging in Fourier Transform of Pupil plane

Image Plane

Intensity Profile

Long Exposure

Short Exposure

2.44 \( \frac{\lambda}{D} \)

2.44 \( \frac{\lambda}{r_0} \)

2.44 \( \frac{\lambda}{D} \)

Fig. 1.1 Atmospheric turbulence and Image resolution
1.1.3 Strehl Ratio

To quantify the performance of an AO-system, the Strehl ratio of the point spread function (PSF) is used. The PSF is interpreted as the image plane intensity distribution that results from imaging a point source. The point spread function is a very useful performance measure, since the resolution of an adaptive optics system is determined directly from the width of the PSF. The Strehl ratio is defined as the ratio of the central intensities of the aberrated PSF and the diffraction-limited PSF of the instrument.

For a circular aperture with an aberration function $\psi(\rho, \theta)$, which describes the wavefront distortion (in units of $\mu$m or nm) as a function of the spherical coordinates ($\rho, \theta$), the Strehl ratio is given by

$$S = \frac{1}{\pi} \left| \int_0^{2\pi} \int_0^{\infty} \rho \partial_{\rho} \partial_{\theta} e^{ik\psi(\rho,\theta)} d\rho d\theta \right|^2$$

(1.11)

From this equation it is immediately clear that $0 < S \leq 1$. Also $S=1$ when $\psi$ is constant and for a varying $\psi$, $S \leq 1$. The Strehl ratio tends to be larger for longer wavelengths (smaller $k$). In the case of atmospheric turbulence, only the statistical properties of $\psi$ are known. If the rms phase error $\sigma_\psi \lesssim k \sigma_\psi$ is smaller than about 2 rad, $S$ can be approximated by the so-called extended Marechal approximation:

$$S = e^{-\sigma_\psi^2}$$

(1.12)

The Wavefront variance related to Fried parameter is defined as,

$$\sigma_\psi^2 = 1.03 (D/r_0)^{5/3}$$

(1.13)

The above two equations show that the Strehl ratio for images obtained with a telescope of diameter $D = r_0$ is $S = 0.36$. 
For \( D > r_0 \), the Strehl ratio decreases precipitously with telescope diameter. Equivalently, \( S \) decreases sharply with decreasing wavelength, since \( r_0 \) is proportional to \( \lambda^{6/5} \).

Adaptive optics systems are used to enhance the capability of optical Imaging systems by actively compensating for aberrations in a real time closed loop fashion. These aberrations, such as atmospheric turbulence, optical fabrication errors, thermally induced distortions or laser device aberrations etc., produce adverse effects on the wavefront. The effects of aberration can be studied in terms of spot size of the beam, beam divergence, beam quality, jitter, brightness variation and seeing condition. The twinkling of stars or distorted images across a paved road on a hot summer day is caused by turbulence in the atmosphere. Distortions like these can be corrected by adaptive optics [8].

1.2 Adaptive optics

Adaptive optics is a field concerned with the correction of optical beams that have been blurred in this fashion and it allows us to achieve near diffraction-limited imaging. The principal uses of adaptive optics are to improve the image quality in optical and infrared telescopes, to image and track the rapidly moving space objects, and to compensate for laser beam distortion through the atmosphere and high-energy laser beam guidance. Basically, any imaging system that involves propagation through a turbulent medium can benefit from the use of adaptive optics.

Adaptive optics systems generally consist of three main components or subsystems: a wavefront sensor, a control computer, tip-tilt compensator and a deformable mirror. The wavefront sensor determines the aberrations present in the wavefront, the control computer calculates the corrections that need to be made and the deformable mirror implements these corrections. A generalized adaptive optics system for astronomical imaging is shown in figure 1.2 below.
Adaptive Optics System

In this figure, light from a point source or star traveling to the earth, due to the large distance, is considered to be a plane wavefront. In the earth's atmosphere these wavefronts are distorted randomly while moving through different layers or cells of air with differences in the refractive index. This propagating light beam has to be properly launched to the telescope arrangement coupled with adaptive optics system. The incoming light from a telescope is incident on a tilt mirror and a deformable mirror. The tilt mirror is used to correct global tilt aberrations, as these are difficult to correct with a deformable mirror by itself. The beam splitter reflects off a part of the incoming wavefront to a wavefront sensor (WFS). The WFS estimates the phase aberrations and the control computer sends correction signals to the tip-tilt as well as the deformable mirror in a closed loop fashion.
The objective of the adaptive system is to cancel out the atmospheric effect on the beam to obtain flat wavefront in the signal detector of the science camera.

The principle behind adaptive optics is the phenomenon of phase conjugation, the process of which is shown in figure 1.3 below.

![Wavefronts and Mirrors](image)

**Fig. 1.3 Phase Conjugation**

It is helpful to think of a wavefront as a surface rather than a continuous beam and realize that the phase of a wavefront is nothing more than its shape. When we adjust our thinking in this fashion, we see that phase conjugation allows us to correct a distorted optical wavefront by generating an aberration that directly opposes the shape of the wavefront.

In the Fig.1.3, we see an aberrated input beam incident on both a conventional (flat) mirror (Fig 1.3 a) and a deformable mirror (Fig. 1.3 b). After being reflected
by the conventional mirror, the aberration is still present. In fig 1.3(b) however, we have changed the shape of the deformable mirror so that it directly opposes the shape of the incident beam, and hence the reflected beam is unaberrated.

1.2.1 Wavefront Sensors

Determining the phase or shape of an incident beam of light is difficult because the wavelength of light is so small. The optical path difference across a wavefront is typically on the order of the wavelength and the beam itself travels faster than any detector can track it. To this end, several methods have been developed, in particular shearing interferometry, curvature sensing, and Shack-Hartmann sensing [2]. In recent times point-diffraction interferometer, pyramid sensor and phase retrieval sensors are also attempted for adaptive optics application [3, 7]. Currently, the Shack-Hartmann sensor is the most commonly used, for its accuracy, vibration insensitive and ease of implementation.

1.2.2 Control Computer

The important constituent of an AO system is the control algorithm, which must be capable of converting wavefront data taken by the sensor into corrector control signals in a period of time short enough in order that the turbulence has not decorrelated significantly between sensing and correction.

1.2.3 Wavefront Reconstruction

By wavefront reconstruction, we mean that we are linearly combining the slope measurements into a surface that describes the wavefront. Once we know what the shape of the wavefront is, we can generate actuator commands for the deformable mirror to correct it. We can also think of wavefront reconstruction as the state estimation problem encountered in controlling dynamic systems.
Following Block diagram (Fig. 1.4) shows the wavefront reconstruction path that signals take the information from the wavefront measurement for the correction system. In most cases, only a few of these paths are used. In many cases, however, parallel paths are employed to control the lower-order aberration modes, such as focus and tilt via one path, and simultaneously control the higher-order wavefront errors with a deformable mirror. This method has been used in many adaptive optics systems.

![Diagram](image)

**Fig. 1.4** The various paths of wavefront control in conventional adaptive optics system

Wavefront reconstruction can be separated into two approaches, modal and zonal. A modal reconstruction scheme treats the wavefront as the sum of a series of basis functions and is the method used in this thesis. The zonal approach divides the aperture into an array of independent sub apertures or zones. Many practical adaptive optics systems use the zonal approach, as it allows us to generate actuator commands directly from the measured gradients, without actually reconstructing the wavefront in the process. When designing the
wavefront reconstruction, it is helpful to realize that low order aberrations are most easily corrected using modal reconstruction, while high order aberrations are most easily corrected using zonal reconstruction.

Basically any practical implementation of wavefront reconstruction chooses less processing time; fewer control loops and less software dependence. Adaptive optics control systems often analyze and direct many parallel channels of information, based on single-channel linear processing algorithms. However, in practice, optical control system under control may not be linear throughout the entire range and methods such as multivariate signal analysis are effectively implemented based on control theory concepts.

In the zonal reconstruction approach, there are several different geometries we can use, the main three being the Fried [09], Hudgin [10] and Southwell [11] geometries. The difference between these geometries is in how the gradient measurements are related to the points at which the wavefront is evaluated. The differences in the geometries and the resulting reconstruction calculations can be found in references [02] and [07]. Zonal wavefront reconstruction is a well-developed method for optimization currently available.

The two main problems inherent with wavefront reconstruction are uniqueness and noise. The problem of uniqueness arises because we have to discretize the wavefront spatially. We can only measure the wavefront at a finite set of sample points, so we do not have a continuous map of the wavefront. This means that there is an infinite number of wavefronts with different shapes that can satisfy the conditions required between any two points. The second problem is the addition of noise. The gradient measurements from the wavefront sensor are corrupted by random noise from ambient light, the quantization of energy of the photons incident on the detector, and the electrons added by the detection process. We can deal with both of the aforementioned problems by using statistical solutions and some tricks in matrix algebra for image processing.
Most of the zonal wavefront reconstruction methods utilize some sort of least-square estimation to determine the phase values from the slope measurements. These estimations can be optimized in minimizing the mean square error of the estimate and the actual value. Some of the classical methods of optimal wavefront reconstruction can be found in [09,12-18]. These methods either assume a priori knowledge or use measurements of atmospheric statistics to optimize the wavefront estimation. Some of these classical techniques that deserve special attention are in ref [11] and [16].

The main problem encountered with the classical technique is that atmospheric conditions may vary. These changes take place over a matter of a few minutes, so the wavefront must be recalculated periodically. Modern adaptive techniques are utilized to deal with this problem [18-20] and also, a Kalman filter approach is outlined in [21]. Unfortunately, these techniques provide computational difficulty, so it may not be possible to realize them in real-time. Here we have performed a simple self-adaptive control algorithm for global tilt correction for closed loop AO operation and Direct Slope method for Deformable correction to achieve wavefront compensation in real time.

1.2.4 Deformable Mirror

A deformable mirror is typically a thin membrane with several actuators attached to it that allow us to adjust the shape of the mirror. As we saw in figure 1.3, if the deformable mirror takes the conjugate shape of an aberrated wavefront, it can correct the wavefront with little error. Fig.1.5 shows the deformable mirror compensating an aberrated wavefront.

Deformable mirrors are complex components and are characterized by their actuator spacing, stroke length, dynamic range, uniformity and hysteresis. Because the physical distance through which the mirror surface is required to
move is often less than a micron, these deformable mirrors must also be high precision components.

Deformable Mirror

Fig. 1.5 Deformable mirror in action

There are several different types of deformable mirrors, which generally vary in how they are machined and the type of actuators used. These actuators are typically made of some form of piezoelectric material (PZT). One of the biggest difference is in the face sheet, whether it is continuous or segmented. A segmented mirror have low coupling between adjacent actuators. This helps in correcting high order aberrations where high spatial frequencies are required but it poses a problem in correcting lower order aberrations where smooth surface or low spatial frequencies are required. A full treatment of deformable mirrors found in [03, 07].
1.3 Various types of Adaptive schemes

The realization of adaptive optics by hardware systems can be classified into four generic types, as shown in Fig. 1.6(a), 1.6(b), 1.6(c) and 1.6(d). This divides the systems into either beam propagation or imaging systems and into received or outgoing wave systems. The transmitted wavefront systems use a laser to irradiate a target and sense the return from a glint on the target to interrogate the intervening atmosphere for phase aberration information.

In the Phase Conjugation approach, the return signal is combined with the outgoing signal on the wavefront modifier, a deformable mirror for example, and sent to the wavefront sensor, which determines the differences between the two waves. The processor then extracts the beacon phase and applies the conjugate to the outgoing wave, which then arrives at the target with the atmospheric distortion ideally exactly compensated. The aperture tagging system applies spatial wavefront modifications at distinct temporal frequencies. These are deconvolved from the return signal and are modified on a trial basis so as to drive a hill-climbing servomechanism, to maximize the power density in the target glint.
Fig. 1.6(a) Phase conjugation

Fig. 1.6(b) Aperture Tagging
The Wavefront compensation and Image Sharpening systems on the above mentioned figures 1.6 (c) and 1.6 (d) are appropriate for imaging systems, such
as astronomical observatories or ground based satellite reconnaissance systems. The wavefront compensation system is similar in spirit to the phase conjugate approach for the active system. The object is the source of return signal, which is passed from the telescope primary to the wavefront corrector, which in the case of a deformable mirror, is commanded to produce a null in the incoming and processing wavefronts. A portion of the collected signal is passed to the wavefront sensor and processor, which produces a local map of the wavefront deviation from the ideal. The conjugate of this map is applied to the mirror so that the main incoming signal will have the atmospheric distortion effectively removed before it is viewed by the imaging system, e.g., the eyeball of the astronomer.

The final system portrayed in figure is 1.6(d), the image sharpening method. Here the image is perturbed on a trial basis and the effect is measured by means of a criterion such as flux in a specified area at the image plane. Hill climbing techniques are then used to alter the trials to maximize this selected criterion.

1.4 History of Adaptive Optics

The U.S Department of Defense started developing techniques to overcome the effects of atmospheric seeing in the 1970s when they began to use surveillance satellites. The declassification of military accomplishments in the 1990s has combined with significant private sector and educational institutional developments to allow observatories to begin installing adaptive optics (AO) in many locations. At present time, there are many adaptive optics programs in the design and construction phase and the developments in adaptive optics are categorized in terms of hardware or components, innovative centroid estimation, post processing, system developments, control systems and Laser guided star imaging around the world [2,3,22-43]. To quote a few, adaptive optics system are either built or being in use at U.S. Air Force Starfire Optical Range (SOR) With Xinetics deformable mirrors, Canda-France-Hawaii Telescope at Mount kea,
University of Hawaii, European Southern Observatory at La Silla (ADONIS), Telescopio Nazionale Galileo (TNG) at Asiago Astrophysical Observatory, Max-Planck Institute in Germany, Lick Observatory, Mt. Palomar adaptive Optics system, University of Illinois in USA, National Solar Observatory, W.M.Keck Telescope on Mount kea, University of Chicago, Alpha LAMP Integration in USA, Gemini Telescope, an astronomy consortium of the United states, Canada, U.K, Chile, Brazil, and Argentina, and Subaru-NAO in Japan. Many pioneer researchers such as Babcock, Fried, and Roddier have contributed a great deal to the technology of AO.

Many recent astronomical discoveries can be directly attributed to new optical observation capabilities. With the new generation of Very Large Telescopes entering into operation, the role of AO systems is extremely important. With this capability, their huge light gathering along with the ability to resolve small details, both spatially and spectrally, has the potential to bring major advances in ground-based astronomy in the new decade. Adaptive optics seeks to overcome the degrading effects upon image quality of some intervening medium, the optical characteristics of which are evolving rapidly with time.

To appreciate the daunting task faced by designers of adaptive optics systems, one should understand that an initially plane wavefront traveling 20 km through the turbulent atmosphere accumulates, across the diameter of a large telescope, phase errors of a few micrometers. These have to be sensed with a minimum number of photons and corrected to about 1/50 of a micrometer every millisecond or so. Another complication is that, for short integration times, the field of view over which the atmospheric wavefront distortions and hence the images are correlated, the isoplanatic angle, is very small (only a few arc second for visible wavelengths).