A charged particle moving through a medium in a straight line with a constant velocity $v$ in time can radiate energy only when the velocity of the source is greater than the phase velocity of light in that medium. This radiation, with its characteristic angle of emission, $\theta_c = \sec^{-1}(\beta)$, is Čerenkov radiation. If the medium is not homogeneous and/or varies with time then in such a medium or near it the situation is different. Under such conditions, a source can emit transition radiation. This new radiation, first noted by Ginzburg and Frank in 1946 [Ginzburg46][Ginzburg80], is defined as the radiation emitted by a charge (or any other source without any intrinsic frequency) moving uniformly along a straight line under inhomogeneous conditions - an inhomogeneous medium, in a medium changing with time or near such a medium.

The simplest problem of this type concerns a charge crossing the boundary between two medium. Far from the boundary in the first medium the particle has certain field's characteristic of its motion and of that medium. Later, when it is deep in the second medium, it has fields appropriate to its motion and that medium. Even if the motion is uniform throughout, the initial and final fields will be different if the two media has different electromagnetic properties. Evidently the fields must reorganise themselves as the particle approaches and passes through the interface. In this process of reorganization some part of the fields are shaken off as transition radiation.

The calculation of transition radiation for the case of two dielectric media was first carried out by Beck [Beck48]. His approach was to use the method of images for finding the field of the particle in the two media and then to introduce the transition radiation field in order to satisfy a boundary condition on the field with time.
The same case was subsequently treated by Garibian [Garibian58] in his search for wave solutions in the radiation zone. Garibian then went on to solve a boundary value problem with the fields expanded in plane incoming and outgoing waves. Other aspects of transition radiation have also been considered in the literature. Pafomov [Pafomov58], Garibian & Chalikian [Garibian59] considered the case of transition radiation in a slab. Garibian’s method was later used by Dooher [Dooher71] to calculate Transition radiation from magnetic monopoles. Recent work by Saveliev [Saveliev02] has led to the development of a theoretical description of transition radiation in the prewave zone, i.e. close to the trajectory of the incident charged particle.

**Figure 3.1:** A schematic diagram of transition radiation emitted from a charge q moving with velocity v along z axis, normally incident upon the boundary between vacuum and a conductor of dielectric constant ε_2. P denotes the point of observation and θ is the angle at which transition radiation is emitted at frequency of ω.
3.1 Theory of transition radiation

We discuss the transition radiation field of a charged particle moving through the interface between two media following Doohar's [Doohar71] approach. The problem is to obtain the solution of Maxwell's equation for the two regions 1 & 2, considering a point charge \( q \) moving with a constant velocity \( v \) along the \( z \) direction, crossing the boundary plane \( z = 0 \) at \( t = 0 \) as shown in Figure 3.1. The first step is to resolve the relevant fields into Fourier Components with respect to time. This approach is adopted from [fermi40]

\[
(\vec{E}, \vec{H}, \vec{D}, \vec{B}) = \int d\omega e^{-i\omega t} (\vec{E}_\omega, \vec{H}_\omega, \vec{D}_\omega, \vec{B}_\omega) \quad \text{---3.1}
\]

Where \( \vec{E} \) and \( \vec{D}, \vec{B} \) are related by the following relations

\[
\begin{align*}
\vec{D}_\omega &= \varepsilon(\omega) \vec{E}_\omega \\
\vec{B}_\omega &= \mu(\omega) \vec{H}_\omega
\end{align*}
\quad \text{---3.2}
\]

By using the particle current density \( \vec{j} \), given by \( \vec{j} = e\vec{v}\delta(\vec{X} - \vec{v}t) \) the Maxwell's equations for region 1,2 as

\[
\nabla \times \vec{H}_{\omega 1,2} = \frac{-i\omega}{c} \varepsilon_{1,2} \vec{E}_{\omega 1,2} + \frac{2en_z}{c} e^{i(\omega υ)z} \delta(\vec{p}) \quad \text{---3.3}
\]

\[
\nabla \times \vec{E}_{\omega 1,2} = \frac{-i\omega}{c} \mu_{1,2} \vec{H}_{\omega 1,2} \quad \text{---3.4}
\]

\[
\nabla \cdot \vec{E}_{\omega 1,2} = \frac{2e}{\nu \varepsilon_{1,2}} \delta(\vec{p}) e^{i(\omega υ)z} \quad \text{---3.5}
\]

\[
\nabla \cdot \vec{H}_{\omega 1,2} = 0 \quad \text{---3.6}
\]

Where \( \vec{X} = \vec{p} + z\vec{n}_z \)

Resolving the field vectors into Fourier components with respect to the transverse displacement vector \( \vec{p} \) we get the following equation;

\[
(\vec{E}_\omega, \vec{H}_\omega, \vec{D}_\omega, \vec{B}_\omega) = \int d^2k e^{-ik\vec{p}} \left[ \vec{E}_\omega (\vec{k}, z), \vec{H}_\omega, \vec{D}_\omega, \vec{B}_\omega \right] \quad \text{---3.7}
\]
Thus equations (3.3-3.6) become
\[
\left[ i \vec{k} + \vec{n}_z \frac{\partial}{\partial z} \right] \times \vec{H}_{\omega,1,2} = -\frac{i\omega}{c} \varepsilon_{1,2} \vec{E}_{\omega,1,2} + \frac{2\varepsilon_0}{2\pi^2 c} e^{i(\omega/v)z} \tag{3.8}
\]
\[
\left[ i \vec{k} + \vec{n}_z \frac{\partial}{\partial z} \right] \cdot \vec{E}_{\omega,1,2} = \frac{i\omega}{c} \mu_{1,2} \vec{H}_{1,2} \tag{3.9}
\]
\[
\left[ i \vec{k} + \vec{n}_z \frac{\partial}{\partial z} \right] \cdot \vec{E}_{\omega,1,2} = \frac{e}{2\pi^2 v\varepsilon_{1,2}} e^{i(\omega/v)z} \tag{3.10}
\]
\[
\left[ i \vec{k} + \vec{n}_z \frac{\partial}{\partial z} \right] \cdot \vec{H}_{\omega,1,2} = 0 \tag{3.11}
\]

The general solution of equations (3.8-3.11) can be written as the sum of a particular solution (particle field), denoted by the superscript $p$, and a homogeneous solution (the radiation field), denoted by a prime. It is clear that the particle fields must be of the form
\[
\begin{pmatrix}
E_p^p \\
H_p^p \\
\vec{D}_{\omega,\omega}^p \\
\vec{B}_{\omega,\omega}^p 
\end{pmatrix} = \begin{pmatrix}
\varepsilon_p^p \\
\varepsilon^p_p \\
\vec{D}_{\omega,\omega}^p \\
\vec{B}_{\omega,\omega}^p 
\end{pmatrix} e^{i(\omega/v)z} \tag{3.12}
\]

With the wave vector of the particle field $\vec{k}$, defined by
\[
\vec{k} = \vec{k} + \frac{\omega}{v} \vec{n}_z \tag{3.13}
\]

Using equations 3.12 & 3.13, Maxwell's equations (3.8-3.11) are transformed into a set of algebraic equations, which can be solved easily. The homogeneous equations are transformed into a wave equation of the form
\[
\left( \frac{d}{dz^2} + \lambda_{1,2}^2 \right)(E'_\omega, H'_\omega) = 0 \tag{3.14}
\]
where
\[
\lambda^2 = \frac{\omega^2}{c^2} \chi - k^2, \quad \chi = \varepsilon \mu \tag{3.15}
\]

The homogeneous solutions are obtained by utilizing continuity conditions at $z = 0$ and it can be shown that the components $E'_\rho$ and $H'_\rho$ are zero.

The Poynting vector in the direction of the radiation field wave vector is given by
\[
S_R = \frac{C}{4\pi} \left( H'_\phi E'_z \sin \theta + H'_\rho E'_\phi \cos \theta \right) \tag{3.16}
\]

The components of radiation fields in the wave zone is expressed in terms of Bessel function...
\[ E'_{r\rho} = -\frac{e}{\pi v} \int \int \int \frac{k^2 \lambda \eta}{\zeta} J_1(\rho k) d\omega d\phi \times e^{-i\lambda z - i\omega t} \] \[ \text{-----------------------------3.17} \]

where
\[ \zeta = \lambda_2 \epsilon_1 + \lambda_2 \lambda_1 \] \[ \text{-----------------------------3.18} \]
\[ \eta_i = \frac{\epsilon_i}{\lambda \eta} \left( \frac{\nu}{\omega} \right)^2 \frac{1}{\kappa^2 - \lambda_0 \omega^2 / c^2} \] \[ \text{-----------------------------3.19} \]

In terms of radial distance \( R \) and angle \( \theta \),
\[ z = -R \cos \theta \]
\[ \rho = R \sin \theta \] \[ \text{-----------------------------3.20} \]

Finally,
\[ E'_{1\rho} = \frac{e\beta^2}{\pi v R} \sin \theta \cos^2 \theta \left( R\omega(\epsilon_2 \eta_1)^{1/2} - m^2 \right) \zeta_1 \] \[ \text{-----------------------------3.21} \]
\[ E'_{1z} = \frac{e\beta^2}{\pi v R} \sin^2 \theta \cos \theta \left( R\omega(\epsilon_2 \eta_1)^{1/2} - m^2 \right) \zeta_1 \] \[ \text{-----------------------------3.22} \]
\[ H'_{1\rho} = \frac{e\beta^2}{\pi v R} \sin \theta \cos \theta \left( R\omega(\epsilon_2 \eta_1)^{1/2} - m^2 \right) \mu_1 \] \[ \text{-----------------------------3.23} \]

where
\[ \xi_i = \frac{(\epsilon_i^2 + 1)[1 + \beta(\chi_2 - \chi_1 \sin^2 \theta)^{1/2} - \beta^2 \chi_1] - \beta^2 \epsilon_2 \mu_1 (\epsilon_i / \mu_i)^{1/2} - 1}{[\epsilon_i(\chi_2 - \chi_1 \sin^2 \theta)^{1/2} + \epsilon_2 \sqrt{\chi_1 \cos \theta}][1 - \beta^2 \chi_1 \cos^2 \theta][1 + \beta(\chi_2 - \chi_1 \sin^2 \theta)^{1/2}]} \] \[ \text{-----------------------------3.24} \]

### 3.1.1 Vacuum to medium case:

For the vacuum to medium case,
\[ \epsilon_1 = \mu_1 = \mu_2 = 1, \epsilon_2 = \epsilon \]

And for extremely relativistic particles,
\[ \frac{v}{c} = 1 \]

With all these substitutions, equation (3.24) simplifies to:

\[ i = \frac{(e - 1)[(e - \sin^2 \theta)^{1/2}]}{[e - \sin^2 \theta]^{1/2} + e \cos \theta] \sin^2 \theta [1 + (e - \sin^2 \theta)^{1/2}]} \]

and equation (3.23) simplifies to:

\[ I'_{ip} = 1.06 \times 10^{-27} \times \frac{\sin \theta \cos \theta}{R} \times \frac{(e - 1)[(e - \sin^2 \theta)^{1/2}]}{[e - \sin^2 \theta]^{1/2} + e \cos \theta] \sin^2 \theta [1 + (e - \sin^2 \theta)^{1/2}]} \times \int d\omega e^{iR(\omega/c)-i\omega t} \]

This integral in the above equation is a delta function:

\[ \int d\omega e^{iR(\omega/c)-i\omega t} = \delta \left( \frac{R}{c} - t \right) \]

Thus:

\[ I'_{ip} = 1.06 \times 10^{-27} \times \frac{\cot \theta}{R} \times \frac{(e - 1)[(e - \sin^2 \theta)^{1/2}]}{[e - \sin^2 \theta]^{1/2} + e \cos \theta] \sin^2 \theta [1 + (e - \sin^2 \theta)^{1/2}]} \times \delta \left( \frac{R}{c} - t \right) \]

This equation is used in calculating the induced voltage in the loop antenna used for receiving radio signal. A model calculation is presented in Chapter VI.
References


