Exact S-wave solution of Schrödinger equation generated from generalized Hulthé potential

N Bhagawati* and N N Singh
Department of Physics, Gauhati University, Guwahati 781014, India

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Abstract: We apply extended transformation method to generate three new exact solutions of radial Schrödinger equation from the generalized Hulthé potential in the framework of non-relativistic quantum mechanics. This method is a simple and compact mapping procedure, which consists of a co-ordinate transformation followed by a functional transformation. The exact energy eigenvalues and the normalized eigenfunctions associated with the newly generated exactly solvable potentials are also found out.

Keywords: Exactly solvable potential; Schrödinger equation; Generalized Hulthé potential

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1. Introduction

The number of exactly solvable potentials (ESPs) is limited in physics. Researchers always try to find new ESPs as they incorporate new ideas and/or mathematical techniques in different branches of physics such as atomic and molecular physics, nuclear physics, particle physics etc. Morse potential [1–3], Hulthé potential [4, 5], Rosen-Morse potential [6–8], Eckart potential [6], Scarf Potential [6, 9], Pöschl–Teller potential [10–12], modified Rosen-Morse potential [13, 14], pseudoharmonic potential [15, 16] etc. are some well known central potentials for which the exact s-wave solutions for Schrödinger equation are available. So far various methods [1, 17–26] have been developed to get such ESPs.

The extended transformation (ET) method is a simple and efficient method [20, 27–32] that generates new exactly solvable quantum systems (QSs) in arbitrary dimensions from already known exactly solvable QS. It consists of a two step transformation that includes a coordinate transformation followed by a functional transformation and a set of plausible ansatze. In this paper, we have taken generalized Hulthé potential i.e.,

\[ V(r) = \frac{s^2}{1 - e^{\alpha r}} - \left( \lambda^2 + \alpha^2 \right) \frac{s^2}{1 - e^{\alpha r}} - \frac{2\alpha^3}{(1 - e^{\alpha r})^2} \]

as our input reference potential. The Hulthé potential has already been solved for the bound states with Nikiforov-Uvarov method [4], Supersymmetry (SUSY) method [5], shifted 1/W expansion method [33] and asymptotic iteration method [21]. It has been widely used to describe the bound state or continuum states of the interaction systems. In this work, we treat the Hulthé potential within the framework of the ET method and generate new ESPs which give rise to bound state solutions of D-dimensional radial Schrödinger equation. When ET method is applied to non power-law type ESPs, the newly generated ESPs are always Sturmian (i.e., principal quantum number n-dependent potential). As there is no specific technique to convert a Sturmian ESP to normal ESP, we employ different QS specific regrouping techniques to produce a normal QS.

2. The extended transformation method

For a QS, say A-QS, the radial part of the Schrödinger equation [34, 35] for the potential \( V_A(r) \) in \( D_A \) dimensional Euclidean space (in natural units \( \hbar = 1 = 2m \)) is

\[ \psi''_A(r) + \frac{(D_A - 1)}{r} \psi'_A(r) + \left( E_A - V_A(r) - \ell \left( \ell + D_A - 2 \right) \right) \psi_A(r) = 0 \] (1)

where \( r \) is a dimensionless spatial coordinate.
Exact solution of $D$-dimensional Schrödinger equation generated from certain central power-law potentials

Nabaratna Bhagawati*

Department of Physics, Gauhati University, Guwahati-781014, India

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Abstract: By applying Extended Transformation method we have generated exact solution of $D$-dimensional radial Schrödinger equation for a set of power-law multi-term potentials taking singular potentials $V(r) = ar^{-\frac{1}{4}} + br^{-\frac{1}{3}}$, $V(r) = ar^{-\frac{1}{3}} + br^{-\frac{1}{2}} + cr^{-\frac{1}{6}}$, $V(r) = ar + br^{-1} + cr^{-2}$ and $V(r) = ar^2 + br^{-2} + cr^{-4} + dr^{-6}$ as input reference. The restriction on the parameters of the given potentials and angular momentum quantum number $l$ are obtained. The multiplet structure of the generated exactly solvable potentials are also shown.

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Keywords: exactly solvable potential • Schrödinger equation

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1. Introduction

In quantum mechanics, it is important to seek new exactly solvable potentials of Schrödinger equation as they incorporate new ideas and/or mathematical techniques in different branches of physics such as atomic and molecular physics, nuclear physics, particle physics etc. Also, exactly solvable potentials are essential for successful implementation of approximate methods in the study of practical quantum systems. Starting from the bound state solutions of hydrogen atom ($\frac{1}{2}$ type potential) and harmonic oscillator ($r^2$ type potential) problems, many researchers have contributed several new exactly solvable potentials of Schrödinger equation. So far, various methods e.g. the traditional method [1, 2], the Nikiforov-Uvarov (NU) method [3], the ansatz for the eigenfunctions method [4, 5], the Laurent series ansatz for the eigenfunctions method [6], the Extended Transformation (ET) method [7], the supersymmetric (SUSY) method [8], the asymptotic iteration method (AIM) [9] etc. have been developed for the exact solution of the quantum mechanical systems.

Considerable effort has been made in the past several decades to solve the time-independent Schrödinger equation for central multi-term potentials containing negative powers of the radial coordinate in two as well as three dimensions [6, 10–25]. However, analysis of such problems in arbitrary $D$ dimensions is limited to the best of our knowledge [22, 23, 26, 27]. In pursuit of this goal, we have used an efficient transformation method called the extended transformation (ET) method [7, 28–31] that generates new exactly solvable quantum systems (QSs) in arbitrary dimensions from already known exactly solvable QS. The ET method is a two step transformation that includes a coordinate transformation followed by a functional trans-
Exactly solvable central potentials related to Romanovski polynomials

Nabaratna Bhagawati
Department of Physics, Gauhati University, Guwahati-781014, India
E-mail: nabaratna2008@gmail.com

Abstract. In this paper, we apply a simple transformation method to construct exactly solvable potentials of Schrödinger equation in any arbitrary dimensional Euclidean space. The normalized wave functions of the constructed potentials are obtained in terms of Romanovski polynomials.

1. Introduction
Exact solution of Schrödinger equation with a physical potential is of utmost importance in non-relativistic quantum mechanics. Successful solution of Schrödinger equation provides analytical form of the normalized wave function and quantized energy eigenvalues. However, a very few quantum systems yield exact solutions for potentials of physical interest. Along the years, many authors have tried different methods to obtain the exact solution of the Schrödinger equation [1-6]. To get the bound state wave functions in terms of Romanovski polynomials, we apply a simple transformation method [6] which comprises of a co-ordinate transformation followed by a functional transformation. By applying this method, we transform the second order ordinary differential equation satisfied by Romanovski polynomials to standard radial Schrödinger equation in D-dimensional Euclidean space and thus try to construct exactly solvable potentials. The motivation for doing this work comes from the fact that exact solutions of Schrödinger equation in terms of Romanovski polynomials is not so widespread [7,8] like other orthogonal polynomials [1-5,9,10].

The paper is organized as follows. In section 2, the formalism of the theory is given. In section 3, exact solution of Schrödinger equation in terms of Romanovski polynomials is discussed. The conclusions are discussed in section 4.

2. Formalism
We start with the linear second order differential equation satisfied by a special function $Q(r)$

$$Q''(r) + M(r)Q'(r) + J(r)Q(r) = 0 \quad (1)$$

where a prime denotes differentiation with respect to its argument. $Q(r)$ will later be identified as one of the orthogonal polynomials.

The transformation method comprises of the following two steps

$$r \rightarrow g(r) \quad \text{and} \quad \psi(r) = f^{-1}(r)Q[g(r)] \quad (2)$$
EXACTLY SOLVABLE EXTENDED POTENTIALS
IN ARBITRARY DIMENSIONS

NABARATNA BHAGAWATI

Department of Physics, Gauhati University, Guwahati-781014, Assam, India
nabaratna2008®gmail.com

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We apply a simple transformation method to construct a set of new exactly solvable potentials (ESP) which gives rise to bound state solution of the $D$-dimensional Schrödinger equation. The important property of such exactly solvable quantum systems is that their normalized eigenfunctions can be written in terms of recently introduced exceptional orthogonal polynomials (EOP).

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1. Introduction

The exact solutions of fundamental dynamical equations are important in different areas of physics and chemistry. Researchers always try to find a new exact solution of Schrödinger equation as it is possible only for a few potentials. Also, exactly solvable potentials are essential for the successful implementation of approximate methods in the study of practical quantum systems.

Since the early days of quantum mechanics, classical orthogonal polynomials (COP) such as Laguerre, Legendre, Jacobi, Hermite etc. play an important role, as the bound state eigenfunctions are expressible in terms of these polynomials. The factorization method [1, 2] initiated by Schrödinger is the pioneering work in this regard. Thereafter, researchers employed various methods e.g. the point canonical transformation (PCT) method [3-5], the supersymmetric (SUSY) method [6, 7], the Nikiforov–Uvarov (NU) method [8-10], the Extended Transformation (ET) method [11, 12], the asymptotic iteration method (AIM) [13], the new exact quantization rule [14], the Laplace transforms method [15], the path integral method [16] etc. to solve the Schrödinger equation analytically and they used COPs to express the bound state eigenfunctions.

(15)
A TRANSFORMATION METHOD TO CONSTRUCT FAMILY OF EXACTLY SOLVABLE POTENTIALS IN QUANTUM MECHANICS

NABARATNA BHAGAWATI\textsuperscript{a}, N. SAIKIA\textsuperscript{b}, N. NIMAI SINGH\textsuperscript{a}

\textsuperscript{a}Department of Physics, Gauhati University, Guwahati-781014, India
\textsuperscript{b}Department of Physics, Chaiduar College, Gohpur-784168, India

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A transformation method is applied to the second order ordinary differential equation satisfied by orthogonal polynomials to construct a family of exactly solvable quantum systems in any arbitrary dimensional space. Using the properties of orthogonal polynomials, the method transforms polynomial differential equation to $D$-dimensional radial Schrodinger equation which facilitates construction of exactly solvable quantum systems. The method is also applied using associated Laguerre and hypergeometric polynomials. The quantum systems generated from other polynomials are also briefly highlighted.

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1. Introduction

The Schrodinger equation plays a pivotal role in modern physics as its solution gives complete information of any given non-relativistic quantum system. Along the years, many authors have tried to obtain the exact solution of the Schrodinger equation for potentials of physical interest [1—12]. This is because, despite the intrinsic interest of the exactly solvable systems, these solutions can be used to get better approximated solutions for potentials which are physically interesting. To enhance the set of exactly solvable potentials, we follow a simple and compact transformation method [10, 13–16] which comprises of a coordinate transformation supplemented by a functional transformation. By applying this method, we transform the second order ordinary differential equation satisfied by special functions to standard Schrodinger equation in arbitrary $D$-dimensional Euclidean space and thus try to construct as many exactly solvable potentials as possible. The method is efficient in generating both power and non-power law type spherically symmetric potentials.

\* nabaratna2008@gmail.com

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