Chapter 6

Conclusions and outlooks

Science is always wrong. It never solves a problem without raising ten more problems. - Bernard Shaw

The main objective of our work, as it is reflected in the title of the thesis, is to generate ESRPs in the nonrelativistic regime. The methods we have discussed in this thesis are based on the ET, which is basically a CT supplemented by a FT, required for obtaining the desired results. The researchers prior to us applied the said transformation successfully for the generation of ESCPs from already known ESCPs of physical QSs, but we have applied for the first time the same method for the generation of ESRPs from already known ESCPs of physical QSs. And eventually the role of FT within the ET and the ansatze considered in our cases got changed. The FT is required to take care of the dimensionality of the generated QSs for generation of central potentials, while it is now required to mould the differential equation, obtained through the application of CT on the Schrödinger radial equation with an ESCP, to the form of Schrödinger polar angle.
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equation. The invocation of expedient ansatze is indispensible for the generation of potentials through the specification of the CT function and to extract desired result, we, of course, have to choose different ansatze in different situations. Coordinate redesignation is a very unusual trick we have introduced here to alter the Schrödinger radial equation environment to the Schrödinger polar angle equation environment and vice-versa. The methods based on the ET not only helps us in generating potentials, but it also generates the solutions of the Schrödinger equation for the generated potentials, no need to solve the Schrödinger equation for obtaining solutions. In our methods based on the ET, the wave functions of the generated systems are transformed from those of the already known potentials of physical QSs; and therefore the transformed wave functions can be shown to be normalizable in a very straight forward manner, solving the issue of normalizability elegantly.

In the second and third chapter, we have shown how ESRPs can be generated and we have succeeded in the generation of some exotic ESRPs along with some already known ring-shaped potentials from exactly solvable central hyperbolic and trigonometric potentials respectively and the wave functions of the generated potentials are shown normalizable. A single stage transformation based on the ET is sufficient to generate ESRPs from hyperbolic central potentials, but a two-stage transformation with successive application of the ET is mandatory to generate ESRPs from trigonometric central potentials. In the chapter 4, we have discussed a completely different approach for construction of ESRPs, where we start with a second-order homogeneous differential equation satisfied by a special function. On application of the ET, the differential equation gets converted to the form of Schrödinger polar angle equation and by invoking suitable ansatze, ring-shaped potentials are constructed. In the chapters 2, 3 and 4, we have seen that the orbital quantum number needs redefined individually in presence of ring-shaped potentials in the QSs and again, the quantum numbers sometimes have to satisfy system specific constraint relations. In the chapter 5, ESRPs are used to generate (regenerate) exact analytic solutions of ESCPs using transformation method based on the ET, revealing the fact that there
is always an inverse transformation to every transformation.

We admit that the methods we have discussed in the chapter 2 and 3 are inadequate to generate ESRPs from already known ESRPs or exactly solvable central power-law potentials (e.g. Coulomb, harmonic oscillator, Kratzer, etc) and new ansatz/techniques must be incorporated to solve this problem. Again, till date, the ET is applied on the Schrödinger radial equation by the researchers in the nonrelativistic regime, so there is a very wide field of applicability in the relativistic regime with the Klein-Gordon or the Dirac equation as the starting points. In the chapter 5, we have made the ansatze equal to only certain fixed values, so there is a high possibility of choosing other values for the ansatze for generation of more and more number of potentials. Again, in that very chapter, we consider only Jacobi polynomial and hypergeometric function as special functions, so here is again a possibility of the application of the method considering other special functions, e.g. Romanovski polynomial, exceptional polynomial, etc. for the construction of exotic potentials. The technique applied in that chapter can also be fruitfully applied for generation of ESCPs and the essence of the technique can be extrapolated to the other branch of science. In physics, we can not ignore the application of a theory, for benchmark, in physical systems and in our present work, we have totally remain disconcerned with that fact and so, the analytical study of the energy spectra of QSs with our newly generated ring-shaped potentials may shed more light on the properties of those systems and may expose new physics and/or mathematical techniques in front of us.