Chapter 1

Introduction

Particle physics is concerned with the study of the elementary constituents of matter which make up the universe and their interactions. An elementary particle is a particle without any internal structure and is not composed of other particles. Elementary particles can be found in cosmic rays. They can be produced in the laboratory in collisions between high-energy particle beams in accelerators. For this reason Particle Physics is also called High-Energy Physics. Like much of fundamental research, it is impossible to know exactly what benefits might be realized from particle physics. However, it is worth noting that in 1897, the quest to understand the universe led J.J. Thomson [1] to discover electron and this discovery undoubtedly created a history in particle physics.

1.1 The Standard Model of Particle Physics

At present, elementary particle physics is described by the Standard Model (SM) theory, which was developed during the last century. The Standard Model of Particle Physics (SM) is a quantum field theory, which describes the knowledge of the fundamental particles and three of the four fundamental interactions, namely the electromagnetic interaction, the weak interaction and the strong interaction. The fourth interaction, gravity, is not in-
1.1. The Standard Model of Particle Physics

eluded, due to its non-re-normalizable terms in the theoretical description of the interaction. All fundamental particles forming matter in the SM are fermions with spin 1/2, while the interaction fields are mediated by bosons with an integer spin quantum number. The fermions can be divided into two classes, quarks and leptons. Quarks have an additional strong charge, colour and thus can interact through the strong force, while the leptons are colour neutral and not affected by the strong interaction. Both classes consist of 6 particles, divided into three hierarchic ordered generations, which differ only by their mass and their flavour quantum number. The first family of the leptons is formed by the electron ($e$) and the electron-neutrino ($\nu_e$), the second family consists of the muon ($\mu$) and the muon neutrino ($\nu_\mu$), and the heaviest generation are the tau lepton ($\tau$) and the $\tau$ neutrino $\nu_\tau$. The three interactions are introduced into the SM by a $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. Here, $C$ represents the color of the strong interaction and $L$ denotes the fact, that the electroweak force couples only to the left-handed states of the particles. $Y$ represents the weak hyper-charge, which results in the unification of the weak and electromagnetic interaction in the electroweak unified theory given by Glashow, Weinberg and Salam. The interactions are mediated by gauge bosons. The hypercharge acts as the carrier of the electromagnetic force, whereas $W^\pm$ and $Z_0$ mediate the weak force. The strong interaction is mediated by eight interaction bosons, the gluons.

1.1.1 The Particles that Constitute Matter: Quarks and Leptons

At present, the elementary particles that make up matter are thought to be the quarks and the leptons along with their antiparticles. We consider them to be elementary because, so far, we do not have any indication of other particles inside them. The up ($u$) quark, charm ($c$) quark and top ($t$) quark, all have electric charge $+\frac{2}{3}$ (on a scale where the electron has charge $-1$), and the down ($d$), strange ($s$) and bottom ($b$) quarks, all have charge $-\frac{1}{3}$. The three leptons all with charge $-1$ are the electron ($e$), muon ($\mu$) and tau ($\tau$) leptons and the
three corresponding neutrinos all with charge 0 are $\nu_e$, $\nu_\mu$, and $\nu_\tau$.

\[
\begin{align*}
\text{Quarks} : & & \text{charge } + \frac{2}{3} : & \begin{pmatrix} u \\ c \\ t \end{pmatrix} \\
& & \text{charge } - \frac{1}{3} : & \begin{pmatrix} d \\ s \\ b \end{pmatrix} \\
\text{Leptons} : & & \text{charge } 0 : & \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \\
& & \text{charge } -1 : & \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}.
\end{align*}
\]

(1.1)

In the following Table(1.1), we tabulate the masses of the fundamental particles- the quarks and leptons. However, for the quarks we list two different masses. The current mass is the mass that appears in the Lagrangian to describe the self-interaction of the quark and is not directly observable. The constituent mass is the effective mass of the quark, when it is bound inside a hadron. The numbers for the constituent quark masses are approximate here because they depend on the hadron model used. The mass parameter is much like a coupling constant in quantum field theory and is technically dependent on the momentum scale and the renormalization scheme and is scale-dependent.

Table 1.1: Masses of the quarks and leptons. The current quark masses and lepton masses are taken from PDG[2]

<table>
<thead>
<tr>
<th>Quark</th>
<th>Quark Mass</th>
<th>Lepton</th>
<th>Lepton Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Constituent</td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>$2.3^{+0.7}_{-0.5}$ MeV</td>
<td>$\sim 330$ MeV</td>
<td>$\nu_e$</td>
</tr>
<tr>
<td>$c$</td>
<td>$1.275 \pm 0.025$ GeV</td>
<td>$\sim 1.5$ GeV</td>
<td>$\nu_\mu$</td>
</tr>
<tr>
<td>$t$</td>
<td>$173 \pm 0.6 \pm 0.8$ GeV</td>
<td>$\sim 180$ GeV</td>
<td>$\nu_\tau$</td>
</tr>
<tr>
<td>$d$</td>
<td>$4.8^{+0.7}_{-0.3}$ MeV</td>
<td>$\sim 330$ MeV</td>
<td>$e$</td>
</tr>
<tr>
<td>$s$</td>
<td>$95 \pm 5$ MeV</td>
<td>$\sim 500$ MeV</td>
<td>$\mu$</td>
</tr>
<tr>
<td>$b$</td>
<td>$4.65 \pm 0.03$ GeV</td>
<td>$\sim 5$ GeV</td>
<td>$\tau$</td>
</tr>
</tbody>
</table>

The masses of the quarks are generated through a symmetry breaking phase transition of the electro-weak interactions (a transition similar to that of a normal conductor to superconductor in condensed matter physics, in which an effective mass for the photon is
produced) in the standard model. The detailed aspects of the symmetry breaking, such as the existence of Higgs bosons are yet to be confirmed in experiments at high-energy colliders[3] though CMS [4] and ATLAS [5] in LHC have recently updated “a 125 GeV” particle as the Higgs boson.

Quarks are strongly interacting fermions and by convention of quark model, quarks have positive parity where as antiquarks have negative parity. In addition to their electric charge, each quark has an additional “charge” referred to as colour (but nothing to do with the colours of the everyday world). There are three possible values of a colour charge, plus three anti-colours for the antiquarks. It appears to be a property of nature that coloured objects cannot exist freely by themselves, so quarks are confined inside hadrons in configurations that produce an object without any colour.

1.1.2 The Hadrons

The six quarks and six leptons (plus their antiparticles) may make up matter but only three of them make up the everyday matter around us. We normally do not “see” the quarks and gluons in low-energy experiments. What we usually observe in experimental apparatus are hadrons and nuclei which are bound states of these basic building blocks. It was M. Gell-mann [6] and G. Zweig [7, 8] who in 1964, put forward the quark model according to which the hadrons are composed of a more variety of pointlike objects called quarks.

Baryons and mesons are the two groups to find under the classification of Hadrons. Baryons are made up of three quarks \((qqq)\) and hence an anti-baryon would be made up of three antiquarks. The other known type of structure is the meson, which is made up of a quark and an antiquark, \(q\bar{q}\) (so an anti-meson is just a meson). The complicated structure of QCD means that the groups of quarks can be bound together to form a hadron which is possible only for certain configurations which can have no net colour (so they are in a colour singlet state). It also means that the attractive force between coloured objects is huge, so
they are always confined together into colourless objects. This specific property of quark due to which the strong interaction has got its own importance is known as confinement.

Thus the colour part of the baryon's state function is an $SU(3)$ singlet, a completely antisymmetric state of the three colours. The ordinary baryons are made up of $u$, $d$, and $s$ quarks and belong to the multiplets on the right side of

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1.$$ 

The decuplet is symmetric in flavour, the singlet is antisymmetric and the two octets have mixed symmetry.

Following $SU(3)$, the nine states (nonet) made out of a pair $q\bar{q}$ (meson) states containing the $u$, $d$, and $s$ quarks can be decomposed into the trivial representation of singlet and octet. The notation for this decomposition is

$$3 \otimes 3 = 8 \oplus 1.$$ 

The parity of a meson state is given by $(-1)^{l+s}$, where $l$ is the orbital angular momentum and its spin is either 0 (antiparallel quark spins) or 1 (parallel quark spins). The $C$-parity (charge conjugation), which is defined only for the $q\bar{q}$ states is given by the relation $C = (-1)^{I_{iso}}$. The $C$-parity can also be generalised to the $G$-parity defined by $G = (-1)^{I_{iso}+s}$ for the mesons made of quarks and their own antiquarks, where $I$ is the isospin quantum number.

It can also be noted in this context that the mesons are classified in $J^{PC}$ multiplets. The $l = 0$ states give the pseudoscalar ($0^{-+}$) and vector ($1^{--}$) mesons where as $l = 1$ states are the scalars ($0^{++}$), the axial vectors ($1^{++}$) and ($1^{--}$) and the tensors ($2^{++}$). Depending upon the quark-antiquark combinations, three types of terminology to categorise the mesons are being widely used in the literature. They are:

a) Light-light mesons, where both the quark and antiquark are light ($u$, $d$ or $s$ only). $\pi$ and $K$ mesons play the dominant role in this sector.
1.1. The Standard Model of Particle Physics

b) Heavy-light mesons, where one quark or antiquark is heavy (c, b or t) and the other is light. $D$ and $B$ mesons are characterised in this category and

c) Heavy-heavy mesons, where both the quark and antiquark are heavy. The particles of the cutting edge study of Charmonium and Bottomonium spectroscopy like $\eta_c$, $\Upsilon$, $\eta_b$ etc. are studied under this category.

In this thesis work, however, we put our special emphasis on the pseudoscalar heavy-light mesons of $l = 0$ state. In Table 1.2, we show the different heavy-light pseudoscalar mesons with their experimental masses which will be involved in this work.

Table 1.2: The pseudoscalar heavy-light mesons and their experimental masses from PDG [2]. $B_c$ meson is included as a heavy-light meson due to its different flavour.

<table>
<thead>
<tr>
<th>Mesons</th>
<th>Quark composition</th>
<th>Meson masses (in GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0$</td>
<td>$\bar{c}u$</td>
<td>1.8649</td>
</tr>
<tr>
<td>$D^+$</td>
<td>$c\bar{d}$</td>
<td>1.8696</td>
</tr>
<tr>
<td>$D_s$</td>
<td>$c\bar{s}$</td>
<td>1.9685</td>
</tr>
<tr>
<td>$B^0$</td>
<td>$d\bar{b}$</td>
<td>5.2796</td>
</tr>
<tr>
<td>$B^+$</td>
<td>$u\bar{b}$</td>
<td>5.2793</td>
</tr>
<tr>
<td>$B_s$</td>
<td>$s\bar{b}$</td>
<td>5.3668</td>
</tr>
<tr>
<td>$B_c$</td>
<td>$c\bar{b}$</td>
<td>6.277</td>
</tr>
</tbody>
</table>

1.1.3 Role of CKM in the Standard Model

In 1963, N. Cabibbo proposed the Cabibbo theory [9] of quark mixing to explain the suppression of $\Delta S = 0$ decay over $\Delta S = 1$ decay. According to this theory, the weak eigenstates can be represented as the combinations of flavour eigen states i.e. the quarks in strong interaction are not the same as the ones in the weak interaction. In 1972, Kobayashi and Masakawa [10] extended the idea of Cabibbo Model to six quarks. Thus in the Standard Model, quark flavour mixing is described by a $3 \times 3$ unitary matrix, the so called Cabibbo Kobayashi-Masakawa (CKM) matrix. The CKM matrix can be regarded as a rotation from the quark mass eigenstates $d$, $s$, and $b$ to a set of new states $d'$, $s'$, and $b'$ with diagonal
The Standard Model of Particle Physics
couplings to $u$, $c$, and $t$. The standard notation of CKM matrix is

$$
\begin{pmatrix}
d' \\
s' \\
b'
\end{pmatrix}
= \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}.
$$

(1.2)

Although the quark couplings to the W-boson are non-universal, the departure from universality is constrained in the Standard Model by the unitarity of the CKM matrix. Unitarity is the only powerful constraint on CKM matrix $V_{CKM}$. Without loss of generality, the matrix can be parametrised in terms of three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase $\delta$:

$$
V_{CKM} = \begin{pmatrix}
C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\
-S_{12}C_{23} - S_{12}S_{23} & C_{12}C_{23} - S_{12}S_{23} & S_{23}C_{13} \\
S_{12}S_{23} & C_{12}S_{23} & C_{23}C_{13}
\end{pmatrix},
$$

(1.3)

where $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$ for $(ij = 12, 23 \text{ and } 13)$. The phase term $\delta$ is the unique source of CP violation in quark flavour changing processes within the Standard Model. This term does not appear for two generation of quarks. The known values of CKM elements motivated Wolfenstein [11] to parametrise the CKM matrix in terms of four independent parameters $A, \lambda, \rho, \eta$:

$$
V_{CKM} = \begin{pmatrix}
1 - \frac{1}{2}A^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-A & 1 - \frac{1}{2}A^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
$$

(1.4)

This parametrisation is based on the expansion of the small parameter $\lambda = \sin\theta_c \approx 0.22$, where $\theta_c$ is the Cabibbo angle. The recent values of these parameters, provided by the CKMfitter and UTfit [12] groups for Particle Data group [2] are $\lambda = 0.22535 \pm 0.00065$, $A = 0.817 \pm 0.015$, $\rho = \rho(1 - \frac{5}{2} + ...) = 0.136 \pm 0.018$ and $\eta = 0.348 \pm 0.014$. 
The unitarity condition of CKM matrix imposes the relations $\sum_i V_{ij} V_{ik}^* = \delta_{jk}$ and $\sum_j V_{ij} V_{kj}^* = \delta_{ik}$. The six vanishing combinations which are valid from these relations are

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0,$$  \hspace{1cm} (1.5)

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0,$$  \hspace{1cm} (1.6)

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0,$$  \hspace{1cm} (1.7)

$$V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0,$$  \hspace{1cm} (1.8)

$$V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0,$$  \hspace{1cm} (1.9)

and

$$V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0.$$  \hspace{1cm} (1.10)

By using the Wolfenstein parametrisation, these relations can be visualised as triangles of equal area (which are proportional to the magnitude of CP violation) in the complex plane $(\rho, \eta)$ as illustrated in Fig. 1.1 for the most commonly used Eqn.1.7.

Figure 1.1: Unitarity triangle from Eqn.1.7
The three angles of this triangle are [13]

\[ \alpha = \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{us} V_{cb}^*} \right) = \tan^{-1} \left( \frac{\eta}{\rho^2 + \rho (\rho - 1)} \right), \] (1.11)

\[ \beta = \arg \left( \frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right) = \tan^{-1} \left( \frac{\eta}{1 - \rho} \right), \] (1.12)

\[ \gamma = \arg \left( \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) = \tan^{-1} \left( \frac{\eta}{\rho} \right). \] (1.13)

In general, the real side of the triangle from Eqn. 1.7, is normalised to one using

\[ \rho + i \eta = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \] (1.14)

and results in sides

\[ R_b = \sqrt{\rho^2 + \eta^2} = \frac{1 - \lambda^2/2}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| \] (1.15)

\[ R_t = \sqrt{(1 - \rho^2) + \eta^2} = \frac{1}{\lambda} \left| \frac{V_{ud}}{V_{cb}} \right|. \] (1.16)

In the Standard Model, \( R_t \) is the least known length of unitarity triangle, which can be measured through the mixing\(^1\) of \( B \) meson[14]. Considering the experimental average values of the CKM elements, from PDG2012 [2], we can express \( V_{CKM} \) as:

\[ V_{CKM} = \begin{pmatrix} 0.9742 \pm 0.0002 & 0.2252 \pm 0.0009 & (4.15 \pm 0.49) \times 10^{-3} \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & (40.9 \pm 1.1) \times 10^{-3} \\ (8.4 \pm 0.6) \times 10^{-3} & (42.9 \pm 2.6) \times 10^{-3} & 0.89 \pm 0.07 \end{pmatrix}. \] (1.17)

The main aim of precession of CKM physics is threefold: (a) to measure the mixing and CP violating parameters of \( V \) as accurately as possible; (b) to test the self-consistency of the CKM picture for quark mixing and CP violation and (c) to search for possible new physics beyond the CKM mechanism. It is therefore, important to measure very precisely

\(^1\)Mixing of \( B \) and \( B_s \) mesons are discussed in Chapter 3
the various entries of the CKM matrix.

### 1.2 Weak decay of Mesons

During a weak decay a fermion (quark or lepton) transforms into its doublet partners by emission of a charged boson $W^\pm$. The $W^\pm$ can then either materialize into a fermion anti fermion pair or couple to another fermion and transform into its doublet partner. Therefore a weak decay can be represented as the interaction of two fermion currents, mediated by a charged $W^\pm$ bosonic current. Obviously, a weak decay can occur only if the parent fermion has a larger mass than the daughter fermion and hence the $u$ quark and $e$ lepton being the lowest mass quark and lepton do not decay. There are three types of weak decays which are extensively studied in the literature. They are leptonic decay, semileptonic decay and non-leptonic decay. In Table 1.3, we tabulate the three types of weak decays and their product particles.

Table 1.3: The three types of weak decay and their decay products.

<table>
<thead>
<tr>
<th>Types of weak decay</th>
<th>Products</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptonic decay</td>
<td>Leptons only</td>
<td>$D^+ \to l^+\nu$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pi^+ \to l^+\nu$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K^+ \to l^+\nu$ etc.</td>
</tr>
<tr>
<td>Semileptonic decay</td>
<td>Both leptons and hadrons</td>
<td>$B^+ \to D^* l^+\nu$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B^- \to \pi^0 l^-\bar{\nu}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B_c \to J/\psi l^+\nu$ etc.</td>
</tr>
<tr>
<td>Non-leptonic decay</td>
<td>Hadrons only</td>
<td>$B^0 \to D^-\pi^+$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{B}^0 \to D^+\phi^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{B} \to KJ/\psi$ etc.</td>
</tr>
</tbody>
</table>

#### 1.2.1 Leptonic and Semileptonic decay of mesons

In this section, we present an overview of leptonic and semileptonic decays, which are useful for both charm and bottom hadrons.
In the diverse phenomenology of weak interactions, leptonic and semileptonic decays of hadrons have a special standing, since the final state particles include a single charged lepton, the clearest experimental signature for a weak process mediated by the W-boson. From the theoretical perspective, these decays are relatively simple and provide a means both to measure the fundamental Standard Model parameters and to perform detailed studies of decay dynamics.

Historically, the semileptonic process of nuclear decay opened the era of weak interaction physics and presented physicists with the mystery of the electron's undetected partner, the neutrino [15]. The process underlying $\beta$ decay is the W-boson mediated weak transition $d \rightarrow uW$, $W \rightarrow e^+\nu_e$; where the decay of a d-quark into a u-quark transforms a neutron ($udd$) into a proton ($uud$). $\beta$ decay was the only known weak process from the turn of the century until the late 1930s and 1940s, when muons, pions and kaons were discovered in cosmic rays.

A key feature of leptonic and semileptonic decays is their relative simplicity, a consequence of the fact that here the effects of the strong interactions can be isolated. The decay amplitude for either type of decay can be written as the product of a well-understood leptonic current for the system and a more complicated hadronic current for the quark transition. In leptonic decays, the hadronic current describes the annihilation of the quark and antiquark in the initial-state meson, whereas in semileptonic decays it describes the evolution from the initial to final state hadrons. Because strong interactions affect only one of the two currents, leptonic and semileptonic decays are much more tractable theoretically than hadronic decays, in which the decay products of the W are also hadrons. A further complication of hadronic decays is that the hadrons in the final state can interact strongly with each other. Leptonic and semileptonic decays therefore, provide a means for studying the strong interactions in a relatively simple environment. Perhaps more important, the effects of strong interactions in these processes can be understood sufficiently well that the underlying weak couplings of quarks to the W boson can be determined.
The standard model successfully accounts for flavour-changing quark transitions in terms of a V-A charged weak current operator \( J^\mu \) that couples to the W-boson according to the interaction Lagrangian [17]

\[
L_{\text{int}} = -\frac{g}{\sqrt{2}} (J^\mu W^+_\mu + J^\mu L^-_\mu) \tag{1.18}
\]

where

\[
J^\mu = \sum_{q_iq_j} J^{\mu}_{q_iq_j} = \sum_i \gamma^\mu \frac{1}{2}(1 - \gamma_5)V_{q_iq_j} d_j. \tag{1.19}
\]

The indices \( i \) and \( j \) run over the three quark generations, so that the field operators \( u_i \) (\( i = 1, 2, 3 \)) annihilate (or create their antiparticles) and the \( d_j \) annihilate. Thus the amplitudes of the decay processes are proportional to the CKM element \( V_{q_iq_j} \).

Purely leptonic decays are considered to be the simplest and the cleanest decay modes of the pseudoscalar charged meson. To obtain transition amplitudes, the quark and lepton current operators must be sandwiched between physical states [16]. For the leptons, this calculation yields directly an expression in terms of Dirac spinors. The hadronic current, however, cannot be so easily evaluated, since the quarks in the hadrons are not free and nonperturbative strong-interaction effects are important in describing the physical states. In general, the long-distance effects, present in the formation of the bound meson state (hadronisation) in hadronic interactions are parameterised with so called **form factors**. These form factors are functions of the momentum transfer and polarization states of the hadrons involved in the interaction. For leptonic decays, the initial state is unpolarised, and the momentum transfer is constant \( q^2 = m^2 \) and hence the form factor becomes a constant \( f_p \), the **decay constant** of the meson.

Mathematically, the amplitude for a leptonic decay can be written as [17]

\[
\mathcal{A}(M_{q\bar{q}} \rightarrow \ell^+\nu) = -i\frac{G_F}{\sqrt{2}} V_{q\ell} U^\mu H_\mu \tag{1.20}
\]
where the leptonic current $L^\mu$ can be written in terms of Dirac spinors $u_l$ and $v_\nu$,

$$L^\mu = \bar{u}_l \gamma^\mu (1 - \gamma_5) v_\nu .$$  \hspace{1cm} (1.21)

The hadronic current for leptonic decay is very simple, since the only four vector available to be constructed with the leptonic current is $q^\mu$. i.e.

$$H^\mu = \langle 0 | \bar{\gamma}^\mu (1 - \gamma_5) Q | M \rangle = i f_p q^\mu .$$  \hspace{1cm} (1.22)

Here $f_p$ is parametrised to absorb all the strong interaction effects, which is called the decay constant. Since the two initial quarks must annhilate, the matrix element is sensitive to $f_p$, which measures the amplitude for the quarks to have zero separation.

For semileptonic decay of a meson $M$ into a meson $X$, the amplitude takes the form\[18, 19\]

$$\mathcal{A}(M_{Q\bar{q}} \to X q \ell^+ \nu) = -i G_F \sqrt{2} V_{qQ} L^\mu H_\mu .$$  \hspace{1cm} (1.23)

Here the hadronic current

$$H^\mu = \langle X | \bar{\gamma}^\mu (1 - \gamma_5) Q | M \rangle \hspace{1cm} (1.24)$$

is not calculated in a simple manner as is done in leptonic decay, since $q^2$ is different from event to event. Thus $H^\mu$ can be expressed in terms of different form factors, which isolate the effects of strong interactions on the amplitude. Unlike the case of electromagnetic interaction, here the normalisation of weak form factors are in general unknown. However, in the limit of infinitely heavy quark masses $m_Q \to \infty$, a new heavy flavour symmetry appears in the effective Lagrangian of the standard model which provides the model independent normalisation of the weak form factors and the necessity of HQET(Heavy Quark Effective Theory)[20] enters into the literature. In this heavy quark symmetry, the form factors (two for pseudoscalar to pseudoscalar transition and four for pseudoscalar to vector transition) of heavy-light mesons in semileptonic decay can be expressed in terms a single form factor

T1522.
which is termed as Isgur-Wise function [21].

In heavy-quark decays, semileptonic modes are generally much more accessible experimentally than leptonic modes, simply because semileptonic branching fractions are larger. Considering its simplicity and great importance, Leptonic and semileptonic decays have been widely studied in the literature [22, 23, 24, 25, 26, 27]. For a review one can see Ref.[16] and the references there in.

1.2.2 Status of decay constant $f_p$ in leptonic decay

Measurement of purely leptonic decay branching ratios of heavy-light mesons are important since it allows an experimental determination of the product $|V_{qQ}| f_p$. If the CKM element $V_{qQ}$ is well known from other measurements, then $f_p$ can be well measured. If, on the other hand, the CKM element is less well or poorly measured, then having the theoretical input on $f_p$ can allow a path to determine the CKM element. These decay constants are accessed both experimentally and through lattice Quantum Chromodynamics (lQCD) simulations. While for $f_{\pi}$, $f_K$, $f_D$, experimental measurements agree well with lattice QCD calculations, a discrepancy is seen for the value of $f_{D_s}$: The 2008 PDG average for $f_{D_s}$ is $273 \pm 10 \text{MeV}[28]$, about $3\sigma$ larger than the most precise $N_f = 2 + 1$ lQCD result from the HPQCD/UKQCD collaboration [29], $241 \pm 3 \text{MeV}$. On the other hand, experiments and lQCD calculations agree very well with each other on the value of $f_D$, $f_D^{(expt)} = 205.8 \pm 8.9 \text{MeV}$ and $f_D^{(lQCD)} = 207 \pm 4 \text{MeV}$. The discrepancy concerning $f_{D_s}$ is quite puzzling because whatever systematic errors have affected the lQCD calculation of $f_D$, they should also be expected for the calculation of $f_{D_s}$ [30]. It is being argued that for $D_{s+}$ decays, beyond the Standard Model there is existence of a charged Higgs boson or any other charged object which would modify the decay rates but would not necessarily be true for the $D^+$ decay [31, 32].

However, the discrepancy is reduced to $2.4\sigma$ with the new (updated) data from CLEO
[33, 34] and Babar [35], together with the Belle measurement [36] and the PDG(2010) average is $f_{D_s} = 257.5 \pm 6.1$ MeV [37]. Lately the HPQCD collaboration has also updated its study of the $D_s$ decay constant [37]. By including additional results at smaller lattice spacing along with improved tuning of the charm and strange quark masses, a new value for the $D_s$ decay constant has been reported as $f_{D_s} = 248.0 \pm 2.5$ MeV which has lowered this discrepancy with the latest PDG average $f_{D_s} = 260.0 \pm 5.4$ MeV [2].

In studying the leptonic branching ratio of $B$ meson, the largest uncertainty arises from the unknown decay constant $f_B$. In principle $f_B$ can be measured in the annihilation process of $B^- \rightarrow l^-\bar{\nu}$, since the decay rate is proportional to the product of $f_B^2 |V_{ub}|^2$. But it is a very difficult process to measure and even if this were done, the uncertainty on $|V_{ub}|$ will not lead to a precise result. Thus the best hope is to rely on unquenched lattice QCD or Potential model, which can use the measurements of the analogous $D^+ \rightarrow \mu^+\nu$ as a check. The knowledge of $f_{B_s}$ is also important, but it cannot be measured directly since $B_s$ does not have leptonic decay and so the violation of $f_{B_s} = f_B$ must be estimated theoretically[2].

The decay of $B_s$ mesons is also important for studying CP violation, which has nothing to do with $D$ mesons. Thus determination of $f_B$ and $f_{B_s}$ in conjunction with that of $D$ mesons becomes crucial to study whether there is new physics[NP] [38, 39, 40] beyond the Standard Model or not.

### 1.2.3 CP violation in meson decays

The CP transformation of a particle refers to the combination of charge conjugation $C$ with parity $P$. Under $C$ transformation a particle interchange to an antiparticle and vice-versa. Under $P$ transformation, the handedness of space is reversed $i.e \; x \rightarrow -x$. Thus, for example the combined effect of CP transformation a left-handed electron $e^-$ is transformed into a right-handed positron $e^+$. If CP transformation were an exact symmetry, the laws of Nature would be the same for
1.2. Weak decay of Mesons

matter and antimatter. Most of the observed phenomenon are CP symmetric. Particularly, the three interactions gravitational, electromagnetic and strong respects these symmetries. On the other hand, the weak interactions, violate C and P symmetries in the strongest possible way. While C and P symmetries are violated separately, the combined effect of CP is still preserved in most weak interaction processes. However, in certain rare processes, as discovered in neutral K meson decays in 1964 [41] and in neutral B meson decays in 2001 [42, 43], CP symmetry is found to be violated. The decay rate asymmetry for K meson decay is found to be at 0.003 level, while as this CP violating effect is quite larger for $B^0$ decays which is about about 0.70. Hence the study of CP violation in charmless charged B decays provides a stringent tests of the CKM picture of CP violation in the Standard Model. However, it cannot be excluded for the moment that CP violation is generated by a mechanism beyond the Standard Model [44].

1.2.4 Non-leptonic decay of mesons

Non-leptonic decays, in which only hadrons appear in the final state, are strongly influenced by the confining colour forces among the quarks. Whereas in semileptonic transitions the long-distance QCD effects are explained by some form factors parametrising the hadronic matrix elements of quark currents, non-leptonic processes are complicated by the phenomenon of quark rearrangement which occurs due to the exchange of soft and hard gluons. The theoretical analysis involves matrix elements of local four-quark operators, which are more complex to deal with than current operators. These strong-interaction effects prevented the coherent understanding of non leptonic decays for a long time. However, a factorization prescription for reducing the hadronic matrix elements of four-quark operators to products of current matrix elements provided a path onto the dynamics of non-leptonic processes [45, 46]. Later on, non-leptonic two body decays of $B$ and $D$ mesons were studied in the approximation of factorisation method [47, 48, 49, 26], where the com-
plicated non-leptonic decay amplitudes are related to products of meson decay constants and hadronic matrix elements of current operators, which are similar to those encountered in semileptonic decays.

In many respects, non-leptonic decays of heavy mesons are an ideal instrument for exploring the most interesting aspect of QCD like CP violation. In studying CP violation for mixing of $B$ and $D$ mesons the best bounds come from the measurement of a particular non-leptonic decay. For example, in case of $D - \bar{D}$ mixing the bounds come from the measurements of $D \to K^+\pi^-$ [50, 51].

Regarding the recent updates of non-leptonic decay in $B$ meson, in March 2012, the LHCb collaboration reported an observation for CP violation in $B^\pm \to DK^*$ decay. Recently, in 2013 the same collaboration has announced a similar observation for the first time with a significance of more than $5\sigma$ marks for the $B_s$ mesons [52].

### 1.3 Quantum Chromodynamics (QCD)

Quantum Chromodynamics, familiarly called QCD is the sector of the Standard Model (SM) which describes the action of the strong force. It is obtained from the full SM by setting the weak and electromagnetic coupling constants to zero and freezing the scalar doublet to its vacuum expectation value. What remains is a Yang-Mills (YM) theory with local gauge group $SU(3)$ (colour) vectorially coupled to six Dirac fields (quarks) of different masses (flavours). The vector fields in the YM Lagrangian (gluons) live in the adjoint representation and transform like connections under the local gauge group whereas the quark fields live in the fundamental representation and transform covariantly. The QCD Lagrangian reads

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{(q)} \bar{q} \left( i\gamma^\mu D_\mu - m_q \right) q,$$  \hspace{1cm} (1.25)
where \( \{q\} = u, d, s, c, b, t \), \( F_\mu^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \), \( D_\mu = \partial_\mu - i T^a A^a_\mu \). \( f^{abc} \) are the \( SU(3) \) structure constants and \( T^a \) form a basis of the fundamental representation of the \( SU(3) \) algebra. When coupled to electromagnetism, gluons behave as neutral particles whereas \( u, c \) and \( t \) quarks have charges \( +(2/3)e \) and \( d, s \) and \( b \) quarks have charges \( -(1/3)e \).

The main properties of QCD follow:

- It is invariant under Poincaré transformation\(^2\), parity transformation, time reversal and (hence) charge conjugation. In addition, it conserves quark flavour.

- Being a non-abelian gauge theory, the physical spectrum consists of colour singlet states only. The simplest of these states have the quantum numbers of quark-antiquark pairs (mesons) or of three quarks (baryons), although other possibilities are not excluded.

- The QCD effective coupling constant \( \alpha_s(q) \) decreases, as the momentum transfer scale \( q \) increases (asymptotic freedom) \([53, 54]\). This allows perturbative calculations in \( \alpha_s \) at high energies.

- At low energies it develops an intrinsic scale, which is usually referred as \( \Lambda_{\text{QCD}} \) and provides the main contribution to the masses of most hadrons. At scales \( q \sim \Lambda_{\text{QCD}} \), \( \alpha_s(q) \sim 1 \) and perturbation theory cannot be used. Nonperturbative techniques are being used at this scale, the best established of which is lattice QCD (lQCD).

### 1.3.1 Perturbative and Nonperturbative QCD

Asymptotic freedom turned out to be a useful tool in understanding high energy QCD.

The short distance behaviour of quarks and gluons can be described with a perturbative
expansion in the small value of the coupling constant and allows the calculation of physical properties from first principles. However, understanding of the strong interactions are far from complete. Properties of medium and low energy QCD still present challenges to particle physicists and remain to be understood. Perturbation theory, which proved very useful for the high energy region, is not applicable at low energy scales and no other analytical tool has been developed so far. Quark confinement, chiral symmetry breaking, dynamical mass generation (ie. the hadron spectrum and the origin of the hadron mass), hadron high energy scattering are fundamental strong interaction phenomena at low energy but they are inherently non perturbative and have not yet been proven analytically from the QCD Lagrangian.

The only reliable method of studying the physical properties of low energy QCD is the lattice formulation of gauge theory proposed by Wilson [55] and independently by Polyakov [56, 57] and Wegner [58]. This theory provides a non perturbative description of QCD and indeed numerical simulations of QCD on Euclidean lattices give strong evidence for colour confinement and spontaneous chiral symmetry breaking as well as describing dynamical mass generation from the QCD Lagrangian [59, 60, 61]. But lattice QCD is limited to the Euclidean formulation of QCD and cannot be applied in Minkowski space time to simulate high energy reactions in which the particles are inherently moving near the light cone. It is also difficult to understand from simulations the important QCD mechanisms that lead to colour confinement. The numerical integrations required in this approach are also extremely computationally expensive. Even with the use of efficient Monte Carlo methods, approximations must be made in order to obtain results with the computational technology available today. Even then, however, several lattice gauge theory calculations are being performed and have already made an impact and preliminary understanding has been achieved on the sources of error in these studies.

The only other way to proceed in the non perturbative regime of QCD is by inventing and using phenomenological models that capture the most important features of strong
1.3. Quantum Chromodynamics (QCD)

QCD. A great variety of models have been developed during thirty years of QCD. Among them are the constituent quark models, light cone QCD [62], and various effective field theories such as Heavy Quark Effective Theory (HQET) and Chiral Perturbation Theory (ChPT) [63, 64] besides QCD Sum rules [65, 66].

In Constituent quark models (CQM), hadrons are considered to be bound states of three valence quarks (baryons) and a quark and an antiquark (mesons). Those quarks are quasiparticle degrees of freedom with the same quantum numbers as QCD quarks, but differing from the latter in their masses and in the fact that they could have an internal structure. Among CQM, non-relativistic quark models (NRQM) have shown to be phenomenologically very successful. In these models, constituent quarks are treated non-relativistically and they interact through potentials that mimic QCD asymptotic freedom and confinement. In NRQM, the dynamical effects of gluon fields on hadron structure and properties are ignored. The quarks are considered as non-relativistic objects interacting via an instantaneous adiabatic potential provided by the gluons. NRQM could account satisfactorily for the magnetic moments of the octet baryons [67]. Isgur and Karl [68, 69, 70, 71, 72, 73] studied baryon spectra within NRQM with the non-relativistic point-like quarks moving in a flavour independent confining potential and with the help of colour hyperfine interactions was able to explain the main features of the spectra. Meson spectra was explained within NRQM [74] by the generation of colour hyperfine interaction among quarks generated through the one gluon exchange potential introduced in [75].

1.3.2 Effective Field Theory (EFT) and Non-relativistic QCD (NRQCD)

In studying different static and dynamic properties of hadrons, relativistic and non-relativistic treatment of the quarks are found to be equally useful with their own success and failure. However, a proper relativistic treatment of the bound state based on the Bethe-Salpeter equation [76] has been found to be very difficult. The entanglement of all energy modes in
a fully relativistic treatment is more an obstacle than an advantage for the factorization of physical quantities into perturbative and non-perturbative contributions. To overcome this problem, semi-relativistic models have been adopted to study the hadronic properties, but due to the uncontrolled approximation it loses contact with QCD.

A non-relativistic treatment, offered by the large mass of the heavy quarks on the other hand has clear advantages. The basis of non-relativistic treatment is that in the center of mass frame of the heavy quark-antiquark system, the momenta $p$ of quark and antiquark are dominated by their rest mass. It can also be noted that the relativistic theory, like the light front approach, reproduces the results of non-relativistic potential models under non-relativistic approximation [77].

Within the non-relativistic approach, three scale parameters are found to play an important role in studying the bound states of a heavy quark and antiquark. These scale parameters include the heavy quark mass $m$ (hard scale), the momentum transfer $mv$ (soft scale), which is inversely proportional to the typical size of the system $r$ and the binding energy scale $mv^2$ (ultrasoft scale), which is proportional to the typical time of the system. Here $v$ is the typical heavy-quark velocity in the center of mass frame and the scales $mv$ and $mv^2$ are dynamically generated. For a non-relativistic system, $v \ll 1$ and the above scales are hierarchically ordered: $m \gg mv \gg mv^2$. For bottomonium study $v^2 \approx 0.1$ whereas for charmonium $v^2 \approx 0.3$. It is useful to study the physics at each of these scales separately. The wide range of involved energy scales also makes lattice calculation extremely challenging. Generally speaking, lattice QCD(QCD) can encompass only a limited range of scale and hence, it become more tractable after scale separation.

Effective field theories(EFT) are the convenient quantum tool to separate these scales. It describes the low-momentum degrees of freedom in the original theory. To construct an effective field theory, the high momentum degrees of freedom are integrated out in that theory. For low energy EFT, such integration is done in a matching procedure which enforces the equivalence between EFT and QCD at any given order of the expansion in $v$. Thus a
1.3. Quantum Chromodynamics (QCD)

prediction of the EFT is a prediction of QCD with an error of the size of the neglected order in $\nu$.

**Non-relativistic QCD (NRQCD)** [78, 79] is an effective field theory which follows from QCD and is obtained by integrating out the hard scale $m$ [80, 81]. Taking into account that $m$ is much larger than the remaining scales of the system, the velocity of heavy quark is chosen as the expansion parameter [82] here. Thus, NRQCD has a UV cutoff scale $\Lambda \approx m$. Since $m \gg \Lambda_{QCD}$, it can be made equivalent to QCD at any desired order in $1/m$. In a proper way of speaking, it can reproduce QCD for processes with $p < \Lambda_{QCD}$ where as processes with $p > \Lambda_{QCD}$ are not manifested in NRQCD. Thus $\Lambda_{QCD}$ becomes the factorization scale between the soft and hard physics.

Following NRQCD, there is another Effective Field Theory known as potential Non-relativistic QCD (pNRQCD), which integrates out the soft scale $mv$. It distinguishes two situations: 1) weakly coupled pNRQCD when $mv \gg \Lambda_{QCD}$, where the matching from NRQCD to pNRQCD is performed in perturbation theory and 2) strongly coupled pNRQCD when $mv \approx \Lambda_{QCD}$, where the matching is non perturbative [83].

It is not necessary that the heavy quarks $Q$ and $Q'$ in the bound state of mesons have similar masses in NRQCD but the masses must be large compared to $\Lambda_{QCD}$. In systems containing a heavy quark with mass much larger than the QCD scale ($\Lambda_{QCD}$) i.e $m_Q \rightarrow \infty$, a new symmetry known as Heavy Quark Symmetry arises [84, 87, 88, 89, 90, 85, 86].

Heavy Quark Symmetry is an approximate $SU(2N_{HF})$ symmetry of QCD, $N_{HF}$ being the number of heavy flavours ($c, b, ...$) that appears in systems containing heavy quarks with masses much larger than the typical quantities ($\Lambda_{QCD}, m_u, m_d, m_s, ...$) which set up the energy scale of the dynamics of the remaining degrees of freedom. In that limit, the dynamics of the light quark degrees of freedom becomes independent of the heavy quark flavour and spin. This infinite mass limit of QCD leads to another well defined effective field theory—**Heavy Quark Effective Theory (HQET)** [20] that allows a systematic, order by order evaluation of corrections to the infinite mass limit in inverse powers of the heavy
quark masses.

The concept of a new flavour symmetry for hadrons, containing a heavy quark was first introduced by Shuryak in 1980 [91], who later studied many properties of heavy mesons and baryons with QCD sum rules [62]. But a clear model independent formulation of the physical ideas of the spin flavour symmetry was developed by Nussinov and Wetzel [84], Voloshin and Shifman [87, 88], Politzer and Wise [89, 90], Isgur and Wise [85, 86] and Grinstein [92], until finally Georgi [20] reformulated the low energy effective Lagrangian for a heavy quark in a covariant way in a theory called Heavy Quark Effective Theory (HQET). Heavy Quark Symmetry and HQET have proved to be very useful tools to describe the dynamics of systems containing a heavy \( c \) or \( b \) quark [21, 93].

### 1.4 QCD Potential

QCD potential between a quark and antiquark has been one of the first important ingredient of phenomenological models to be studied in quarkonium physics. In a non-relativistic potential model, one ignores the dynamical effects of gluon fields on the hadron structure and properties. Quarks are considered as non-relativistic objects interacting via an instantaneous adiabatic potential provided by gluons[94]. The force between a heavy quark and a heavy/light quark is due to the static quark antiquark potential, since the heavy quark is static with respect to the light quark. Vairo [95] defines the potential as the function \( V \) into the Schrodinger equation describing the quark-antiquark bound state \( \psi \):

\[
E\psi = \left( \frac{p^2}{2m} + V \right)\psi,
\]

\( p \) being the momentum of the quark-antiquark pair in the centre of mass frame and \( E \) is its binding energy.

The QCD potential is based on the two important facts of QCD: confinement, which
means that the force between quarks does not diminish as they are separated and asymptotic freedom, which means that in very high-energy reactions, quarks and gluons interact very weakly.

At low energy or large distance scale, colour confinement one of the prominent feature of QCD comes into play. In case of a quark antiquark pair in the colour singlet state, when one tries to separate the quark from the antiquark by pulling them apart then the interaction between the quarks gets stronger as the distance between them gets larger, similar to what happens in a spring. In fact, when a spring is stretched beyond the elastic limit, it breaks to produce two springs. In the case of the quark pair, a new quark-antiquark pair will be created when pulled beyond a certain distance. Part of the stretching energy goes into the creation of the new pair, and as a consequence, one cannot have quarks as free particles. To understand really what happens, one must make calculations in QCD at large distance scales where, according to the renormalization group equation, the coupling becomes very strong. At present time, such a calculation is found to be very difficult.

Figure 1.2: Electric lines between the positive and negative charges in QED.
In contrast to QED, where the electric lines between positive and negative charges spread all over the space (as shown in Fig. 1.2) and generates a $\frac{1}{r}$ potential, in QCD the vacuum acts like a dual superconductor which squeezes the color electric field to a minimal geometrical configuration—a narrow tube as shown in Fig. 1.3.

![Figure 1.3: Flux tube due to chromoelectric lines of force in QCD.](image)

The tube has approximately a constant cross section and constant energy density. Because of this feature, the energy stored in the flux increases linearly with the length of the flux. The qualitative picture is that the chromo-electric lines of force bunch together into a flux tube which leads to a distance independent force or a potential of the form

$$V(r) \sim constant \times r$$ \hspace{1cm} (1.27)

for $r > M^{-1}$, where $M$ is a typical hadronic mass scale. For hadron size of $1 fm = 5 GeV^{-1}$, $M = 200 MeV$. Nambu provided a connection between linear energy density and a linear Regge trajectory with the string model of hadrons [96] and calculated the potential to be

$$V(r) = br$$ \hspace{1cm} (1.28)

where $b$ is known as the QCD string tension which is also known as the slope of the potential. The linear form for the long range part of the QCD potential has been validated by Lattice QCD calculations. Phenomenologically, almost all the potential models have found $b = 0.18 GeV^2$. 
At short distance (in the weak-coupling limit), on the other hand quarks are free, which can be represented by a Coulomb potential with an asymptotically free coupling constant. In this scale the quark-gluon interaction is similar to the electron-photon interaction in quantum electrodynamics with the Born term for the $qq$ or $q\bar{q}$ interaction being the familiar $\frac{1}{r}$ form.

\begin{equation}
V(r) \sim \frac{1}{r}
\end{equation}

The naive idea of this Coulomb like potential is that the exchange of a gluon gives rise to a force between the colour states. They are attractive in the color singlet channel and repulsive in the color octet channel but are spin and flavor independent. In contrast with QED the gluon self-coupling results in a slow decrease of the effective coupling strength at short distance. For hadrons, the one gluon exchange contribution in the colour singlet channel is given by:

\begin{equation}
V(r) = -\frac{4}{3} \alpha_s \frac{1}{r}
\end{equation}

where $\alpha_s$ is the strong running coupling constant and the factor $\frac{4}{3}$ in Eqn.1.30 arises from the SU(3) colour factors.

It is, however not a proper justification to consider the dominating role of one gluon exchange at short distances. Indeed the studies of the static potential by using LQCD in Ref.[97] suggests that the one gluon exchange can dominate only at very small distances which is hardly accessible from the lattice data and even after including the perturbative higher order correction, only a small part of the static potential is described by perturbative QCD. Generally speaking, Coulomb like potential in phenomenological models covers a large range of distances and should probably be considered only as a phenomenological description that the Coulomb like potential together with the linear contribution provides the medium range potential responsible for the bound states. This is manifested by the fact that $\frac{4\alpha_s}{3}$ is not necessarily small. In the Cornell model of quarkonia, it is in fact large [98].
Numerous variations of the resulting Coulomb plus linear potential exist in the literature. Some of the better known ones are

a) The Cornell potential

The Cornell Potential [99], which was initially proposed to describe masses and decay widths of charmonium states is given by

\[ V(r) = -\frac{\beta}{r} + \frac{r}{a^2} + V_0 \]  (1.31)

where the coefficients \(a, \beta\) and \(V_0\) are adjusted to fit the charmonium spectrum.

b) Screened Cornell potential

To include the effect of saturation of the strong interaction at long distances, a variation of the Cornell potential, which is called the Screened Cornell potential also appears in Lattice as [100]

\[ V(r) = (-\beta/r + r/a^2) \left( \frac{1 - e^{\mu r}}{\mu r} \right) \]  (1.32)

where \(\mu\) is the screening parameter. This potential behaves like a Coulomb potential at short distances but, unlike in the previous model, it tends to a constant value for large \(r\) (namely, for \(r >> \mu^{-1}\)). \(\beta\) and \(\mu\) are intrinsic to the model, while \(a, m_c\) and \(m_b\) were fixed by the authors. In this Potential the linearly growing confining potential flattens to a finite value at large distances, corresponding to the saturation of \(\alpha_s\) to a finite value for decreasing \(Q^2\) [101]. This effect should be due to the creation of virtual light quark pairs that screen the interaction between the bound quarks at long distances.

c) Richardson’s potential

The Richardson’s potential [102] incorporates the features of asymptotic freedom at short distances and linear confinement at long distances with a minimal interpolation between
these two asymptotic behaviours given by:

\[ V(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left[ \Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right] \]  

(1.33)

Here \( n_f \) is the number of light quarks relevant to the renormalization scale which is taken equal to three, while \( \Lambda \) is the scale interpolation between the two asymptotic regimes.

d) Power law potential

This potential belongs to the special choices of the generality of the potential [103, 104, 105]

\[ V(r) = -Cr^\alpha + Dr^\beta + V_0. \]  

(1.34)

With \( V_0 = 0 \) and \( \alpha = -1 \), one gets

\[ V(r) = -\frac{C}{r} + Dr^\beta. \]  

(1.35)

With the power \( \beta = 1 \), one obtain the simple Cornell potential Eqn.(1.31). However there is variation of \( \beta \) in different models. Martin potential[106] corresponds to \( \beta = 0.1 \), Heikkila, Tornquist and Ono [107] potential corresponds to \( \beta = 2/3 \) where as Vinodkumar et al [108] explores within the range \( 0.5 \leq \beta \leq 2.0 \).

Overall, the spin-independent features of quarkonium spectroscopy are well described by the potentials just described. The difficulty resulting from the consideration of different potentials is that the number of parameters to be fixed by comparison with experimental data is almost the same as the number of experimental data. Only qualitative arguments can be made to introduce a new potential form and hence it is more judicious to consider simpler models than to explain experimental data and to find out the limits of the models.

The Coulomb-plus-linear potential, so called the Cornell potential, has received a great deal of attention both in particle physics, more precisely in the context of meson spec-
troscopy where it is used to describe systems of quark and antiquark bound states, and in atomic and molecular physics where it represents a radial Stark effect in hydrogen. The addition of the linear term in the potential makes the “funnel” of the potential narrower and can be seen in Fig.1.4.

Figure 1.4: The potential \( \frac{4}{3r} \alpha_s \) (coulombic), \( br \) (linear) and the potential \( \frac{4}{3r} \alpha_s + br \) (Coulomb+linear), plotted against \( r \) (in \( GeV^{-1} \)) with \( b = 0.183 \ GeV^2 \) and \( \alpha_s = 0.4 \).

This potential was used with considerable success in models describing systems of bound heavy quarks [109, 110, 111]. All of our results presented in this work will be based on the Cornell potential with a constant shift \( c \)

\[
V(r) = -\frac{4}{3r} \alpha_s + br + c.
\]  

(1.36)

The parameter \( c \) which we call the constant shift of the potential is also known as the quarks self energy[112]. This parameter is needed to reproduce the correct masses for heavy-light
meson system and is found to have a variation in its numerical values. For example in the work of relativistic quark model by Faustov et al $c = -0.3 GeV$\cite{113} is used, in Ref.\cite{99}, it is taken to be $c = 0.50805 GeV$, Mao Zhi Yang has taken $c = -0.19 GeV$ \cite{114}, Scora and Isgur \cite{115} obtained $c = -0.81 GeV$, H M Choi et al in Ref.\cite{116} have considered $c = -0.5575, -0.6664 GeV$ whereas Grant and Rosner \cite{117} considered a large negative value of $c = -1.305 GeV$ in a power law potential.

1.5 The Work of this Thesis

We mentioned above that perturbative QCD does not work at low energies, and that non-perturbative calculations have yet to produce detailed results. (In fact, the problem arises with any strongly-interacting field theory.) Until we can make quantitative predictions of hadronic properties using QCD, we cannot say that we understand the theory, nor can we make significant headway in this important regime. The main motivation for the present dissertation is to investigate meson properties in the quark model to understand the model applicability and generate possible improvements. Certain modifications to the model are suggested which have been inspired by fundamental QCD properties (such as running coupling or spin dependence of strong interactions). These modifications expand the limits of applicability of the constituent quark model and illustrate its weaknesses and strengths.

The present work is an endeavour to formulate meson wave functions with Coulomb plus linear potential incorporating relativistic correction (in a free Dirac way) and thereby to study the static and dynamic properties of heavy-light flavoured mesons using this wave function. The wave function has been tested initially in Ref.\cite{118, 119} to study the properties of heavy-light mesons such as form factors, decay constants and charge radii. Further, the slope and curvature of Isgur-Wise function were studied in Ref.\cite{120, 121} by incorporating two loop static potential in V-Scheme \cite{122, 123}. However, application of V-Scheme was found to be successful in studying the Isgur-Wise function of $D$ and $D_s$ mesons but
was not so successful in studying the $B$, $B_s$ and $B_c$ mesons. Also the wave function shows singularity at the origin, which is required to study the leptonic decay constant in the non-relativistic limit.

Considering these two facts, we further improve the wave function and examine two general areas in the quark model: models of meson decay and final state interactions. The meson properties studied include meson masses, decay constants, form-factors and Isgur-Wise functions. The results are then compared to the experimental data, lattice gauge theory calculations (lQCD) and other theories.

Our motivation to study heavy-light mesons, containing at least one heavy quark is natural not only because they are being intensively studied by current experiments, but also because their decay dynamics are significantly different from particles containing only light quarks.

In Chapter 2, we compute the ground state masses and leptonic decay constants of Open flavour charm Mesons ($D$, $D_s$ and $B_c$ using the QCD Potential model described above. Here we introduce a short distance scale to regularise the wavefunction at the origin and the strong running coupling constant is taken from the $\overline{MS}$ scheme with $\alpha_s = 0.39$ for $D$ and $D_s$ mesons and $\alpha_s = 0.22$ for $B_c$ meson.

In Chapter 3, we try to incorporate the effect of short distance scale into the $B$ sectors of mesons with a scale dependent $\alpha_s[125]$ and compute the Oscillation Frequency of $B_d$ and $B_s$ mesons.

In Chapter 4, we transform the wavefunction from co-ordinate space to momentum space by using Fourier transform and study the masses and Decay constants without applying the short distance scale. We use the same prescription of strong running coupling constant($\alpha_s$) as is used in chapter 3 and compute the branching ratio for different leptonic channels to compare with the experimental data.

In Chapter 5, we study the Isgur-Wise function and its derivatives for $B$ and $D$ mesons in this version of our model. We put our special emphasis to study the semileptonic decay of
1.5. The Work of this Thesis

$B$ meson and compute the CKM element $V_{cb}$ by using HQET. In this chapter, we also show that the leptonic and semileptonic decay are not controlled by the same scale of parameter $\Lambda_{QCD}$.

In Chapter 6, we explore $B_c$ meson as a heavy-light meson and study its semileptonic decay to $c\bar{c}$ states. We also use another approach of the model with Coulombic part of the potential as Perturbation to study the same.

In Chapter 7, we present our concluding remarks as well as our future outlook.